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## Artificial Neural Networks with Radial-Basis Transmission Functions for Modelling Economic Phenomena and Processes

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**Abstarct:**

**Purpose:** The paper presents the possibility of using artificial neural networks (ANN) with radial-basis transmission function (RBF) for modeling economic phenomena and processes.

**Design/Methodology/Approach:** The basic characteristics and parameters of an ANN with RBF are shown and the advantages of using this type of ANN for modeling economic phenomena and processes are emphasized. Using an ANN with RBF, together with official statistics for 2010-2017, the modeling of the influence caused by work efficiency indicators of the customs authorities of Ukraine on the indicators of economic security of Ukraine was carried out. These eighteen indicators of economic security of Ukraine, which comprehensively characterize the economic status of the country in terms of production, social, financial, food, transport, energy, and foreign economic security, were chosen as the most informative indicators.

**Findings:** The results of the study showed that Artificial neural networks with Radial-basis transmission function well describe the trend of changing state economic security indicators under the influence of changing performance indicators of customs authorities. This allows us to recommend this type of artificial neural networks for analysis, evaluation and forecasting of economic phenomena and processes.

**Practical Implications:** The influence of the efficiency of fiscal bodies on the main indicators of economic security of the state is proved. The analytical and prognostic properties of ANN with RBF for estimating the relationships between economic phenomena and processes, under the conditions of non-stationary processes and nonlinearity of relationships between indicators, are shown.

**Originality/Value:** The results obtained showed good analytical and prognostic properties of an ANN with RBF when modeling the impact of customs authorities' performance on the state's economic security indicators.

**Keywords:** Economic security of the state, artificial neural networks with radial-basis transmission functions, customs system, indicators of economic security of the state, macroeconomic forecasting.

**JEL codes:** C6, I15, M5.

**Paper type:** Research article.

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## **1. Introduction**

In the context of the constantly changing global economic development trends and the daily occurrence of internal and external threats that can suddenly change the trajectory of each country's economic development (e.g., coronavirus pandemic, dramatic change in energy prices, etc.), not only the current state of the national economy is an important task for both science and practice, but also the prediction of its values for the near future. Such a prediction is not possible without taking into account the influence of various factors both on particular branches and on the general state of the economy of the state. Developing an appropriate scientific and methodological framework will make it possible to quite accurately predict the value of macroeconomic indicators, and consequently to implement appropriate management decisions to minimize the negative impact of certain global processes (Zhuravka, Filatova, Šuleř and Wołowiec, 2020).

The purpose of this study is to scientifically substantiate the structure of models of state economic security indicators in the form of artificial neural networks with radial-basis functions, as well as to confirm the adequacy of such models in solving specific applied tasks.

Most often, regression models are used to model economic phenomena and processes. However, under conditions of non-stationarity of processes, nonlinearity of relationships between indicators that are characteristic of most economic phenomena and processes, as well as in the absence of objective statistics using a stochastic approach, and in particular, regression analysis does not allow to obtain scientific results that would reflect objective economic reality. In addition, applications such as Statistica, MS Excel, which can be used for modeling, require the study of the statistical characteristics of both the studied economic indicators and the factors influencing them, which, in the case of limited statistics, can sometimes be a difficult practical task (Martyniuk and Muravska, 2020).

## **2. Theoretical Background**

Recently, artificial neural networks (ANN) have become widely used to model economic systems. In particular, perceptron-type networks, which provide good approximation capabilities of the model, are used for these purposes. Among the scientists whose studies deal with the problems of using these networks for modeling economic phenomena and processes, the works of Bodiānsky and Rudenko (2004), Nelles (2001), Dyvak, Pukas, and Kozak, (2008) should be mentioned.

Among a large number of types of neural networks (Kohonen Maps, Carpenter and Grossberg Networks, and those of Hopfield), the Radial Basis Function Neural Networks (RBFNs) are a special class from the standpoint of modeling the development of different economic phenomena and processes, as well as the influence of various factors on them. As the results of the studies show, this type of

ANN is not only a powerful means of approximating the nonlinear functions of many variables, but also allows us to predict the processes successfully. Such networks have a fairly simple architecture and high training speed. RBF networks are known as networks with locally configured processing units, that is, networks where the output signal is "local" or "tuned" to some narrow confined area of the input space (Rudenko and Bodiatsky, 2006).

Radial basis transfer function - this type of function accepts as the argument the distance between the input vector and some preset center of the activation function. The closer the input vector is to the center, the higher is the value of this function (Yasnitsky, 2008). Gaussian function in this case can be used as a radial basis:

$$y = \exp - \frac{(S - R)^2}{2\sigma^2}$$

where  $S = \|X - C\|$  is the distance between the center  $C$  and the vector of the input signals  $X$ . The scalar parameter  $\sigma$  determines the decay rate of a function when the vector is distant from the center and is called the width of the window, the parameter  $R$  determines the shift of the activation function along the abscissa axis. Different metrics can be used as a distance between vectors, although it is the Euclidean distance that is most commonly used (Kruglov and Borisov, 2001):

$$S = \sqrt{\sum_{j=1}^N (x_j - c_j)^2}$$

where  $x_j$  is the  $j^{\text{th}}$  component of the vector fed to the input of the neuron, and  $c_j$  is the  $j^{\text{th}}$  component of the vector that determines the position of the center of the transfer function.

The RBF network consists of an input, a single hidden (radial basis) and a linear (output) layer. The input layer consists of sensors (synaptic contacts) that connect the network to the external environment. The neurons of the hidden layer act on the principle of centering on the elements of the training sample. The weighting matrix functions as the centers. There is an area called radius around each center. The radius (network sensitivity) is adjusted using the vector of smoothing coefficients  $(\sigma_1, \dots, \sigma_m)$ . The transformation function (Gaussian function  $f(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}$ ), which takes values in the range from 0 to 1, determines the output of the hidden layer. The source layer contains ordinary linear or sigmoid neurons. The weight setting determines the output of the network.

The properties of such an ANN are completely determined by the radial basis

functions used in the hidden layer neurons and form some basis for the input vectors  $x$ . The radial basis function  $\varphi(x) = \Phi(\|x - c\|, \sigma) = \Phi(r, \sigma)$  – is a multidimensional function that depends on the distance between the input vector  $x$  and its own center  $c$  and the width (scale) parameter  $\sigma$  (Nelles, 2001). Thus each neuron of the hidden layer determines the distance between the input vector and its center and performs some nonlinear transformation  $\Phi(r, \sigma)$  over it.

In most practical cases, the node centers  $c_i$  and width parameters  $\sigma_i$  are fixed and only the synaptic weights  $w_i$  are adjusted. To solve more complex problems, all three sets of parameters  $c_i \in R^n$ ,  $\sigma_i$ ,  $w_i \in R^1$ ,  $i=0,1,2,\dots,h$  are taken into account.

The main advantages of using RBFNs compared to other ANNs are presented below (Bodiansky and Rudenko, 2004):

- there is only one hidden layer in the RBFN, which simplifies the network structure;
- high learning speed;
- training ability on a heterogeneous sample of data;
- possibility of modeling systems with deep instability;
- ability to build models of dynamics of non-stationary objects;
- prognostic properties;
- capability of modeling and predicting random processes.

It is the presence of these advantages that led to the choice of this type of ANN for the study of economic phenomena and processes.

The generalized structure of RBFN is shown in Figure 1, with the following designations:

$X = (x_1, x_2, \dots, x_n)$  – vector of network inputs;

$(W^r)$  – the weighting matrix acting as centers;

(dist) is the block in which the Euclidean distance between the input vector ( $X$ ) and the corresponding center is calculated;

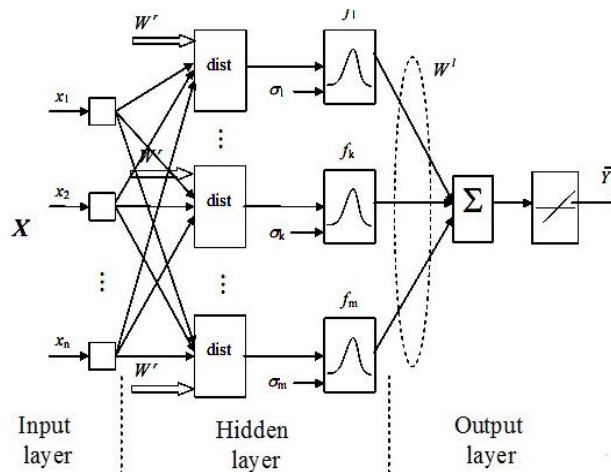
$(\sigma_1, \dots, \sigma_m)$  – smoothing coefficients (parameters of influence, parameter of width of a window of RBF) by means of which the radius (sensitivity) of a network is corrected;

$f_i(x)$  – basis function;

$W^l$  – the weighting matrix of ordinary linear or sigmoid neurons of the source layer that determines the output of the network;

$\bar{Y} = (y_1, y_2, \dots, y_p)$  – network output.

**Figure 1.** Generalized artificial neural network architecture with radial-basis transmission functions



Source: Own elaboration.

### 3. Methodology of Research

The RBFN behavior depends largely on the number and position of the radial basis functions of the hidden layer. Indeed, for any valid  $n$ -dimensional input vector  $x = (x_1, x_2, \dots, x_n)$ , where  $x \in X \subset R_n$  the network output will be determined as follows:  $y_i = \sum_{k=1}^m w_{ik}^l f_k(\text{dist}(x, w_k^r), \sigma_k)$ , where,  $w_{ik}^l \in W^l$ ,  $i = \overline{1, p}$  – the weights of the linear layer, are the centers of radial-basis functions.

The ANN with RBF is characterized by three types of parameters:

- linear weight parameters of the source layer  $w_{ij}$  (included in the network description linearly);
- centers  $c_i$  – nonlinear (included in the description of the neural network in nonlinear way) parameters of the hidden layer;
- deviations (radii of basis functions)  $\sigma_{ij}$  – nonlinear parameters of the hidden layer.

So, as we can see, the advantage of using such ANNs for modeling economic phenomena and processes is a significant simplification of the model structure, since radial basis functions are used, and the training task involves determining the weight coefficients and synoptic connections of the output layer of the network. In this case, the neural network has one hidden layer with nonlinear activation functions having configurable parameters. This ensures the simplicity of the model structure on the

one hand and the approximate and predictive properties, on the other hand, with small "noisy" data samples. That is why, as an experiment with the use of RBFN, we modeled the impact of the customs system of Ukraine performance indices on the indicators of economic security of the state (Wołowiec, Szybowski, and Prokopowicz, 2019).

Based on official statistical information, eight indicators for 2010-2017, which characterize the customs authorities of Ukraine work efficiency were selected as factors of influence (input data) (Among them: the amount of transfer of customs payments to the state budget, the number of cargoes completed, the number of customs declarations issued, the number of vehicles issued, the number of preliminary declarations issued, the number of preliminary notices issued, cases of violation of customs rules, cases of smuggling. Eighteen most effective indicators of economic security of Ukraine (efficiency indicators) in 2010-2017, which comprehensively characterize the economic status of the country in terms of production, social, financial, food, transport, energy, foreign economic security, grouped on the basis of performance, were selected (Martyniuk, 2020).

The list of these indicators is provided by the Methodological recommendations for calculating the level of economic security of Ukraine, which were approved by the Order of the Ministry of Economic Development and Trade of Ukraine of October 29, 2013 No. 1277.

MATLAB was used to train the network and shape its structure, as it allows for the rapid processing of large amounts of statistics, and its tools provide a wealth of data analysis capabilities covering almost all areas of mathematics, including ANN.

Both the one-step Widrow-Hoff training algorithm, and the multi-step learning algorithm are most commonly used for RBFN training. When training a neural network using a one-step algorithm, the network structure is formed in such a way that the number of neurons in the hidden (radial basis) layer is equal to the number of elements of the training sample, and the learning error is zero. A significant disadvantage of the one-step training algorithm is that it forms a network with a number of radial-base-level neurons equal to the number of training sample elements. With this neural network training algorithm, it is not possible to obtain adequate simple models when dealing with large volumes of training samples. Therefore, in our study, we used a multi-step learning algorithm (Isik and Ozden, 2012).

The multi-step network learning algorithm generates an RBFN model with optimal number of hidden level neurons. It creates a bi-level network. The first level consists of radial-base neurons, and calculates its weighted inputs using the Euclidean distance *dist* function as well as its specific inputs. The second layer consists of simple linear neurons ( $y = f(x) = x$ ) and calculates its weighted input and its

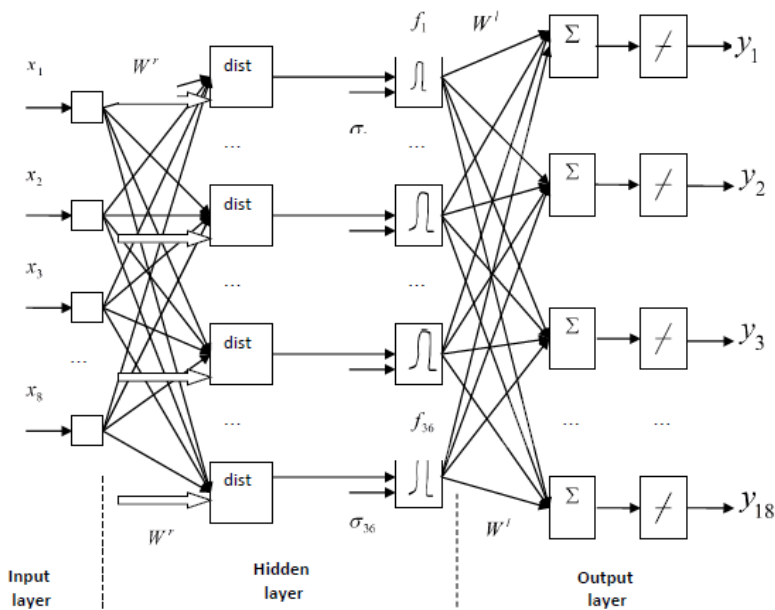
specific inputs using the appropriate functions.

At the beginning of the algorithm, the radial basis level does not contain neurons. Neurons are added to the hidden layer until the sum of squares of the mean square errors of the network is less than the specified value or the maximum number of neurons is used. In the next step, the network forecast is calculated:

- an input vector (in our case it is the realization of inputs in one or another month) with the largest value of the root mean square error is found;
- a radial basis neuron with weights equal to this vector is added;
- the linear scale weights are reorganized in such a way as to minimize the root mean square error.

Figure 2 shows the initial structure of the RBFN, where  $x_1 - x_8$  – the indicators characterizing the efficiency of the customs system functioning, and  $y_1 - y_{18}$  – indicators of economic security of the state.

**Figure 2.** The initial structure of RBFN (the root mean square error of training is zero)

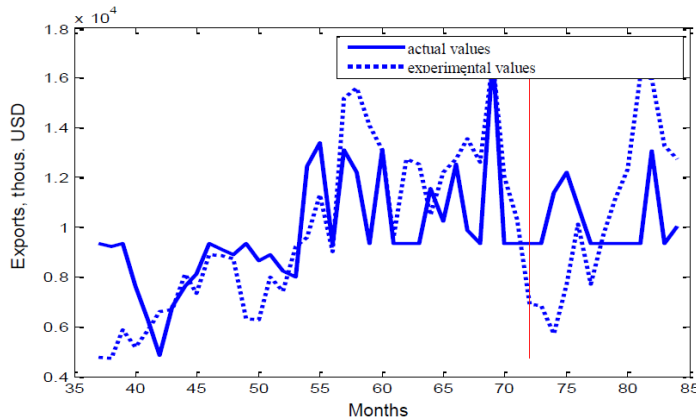


**Source:** Own elaboration.

As the figure shows, the structure of RBFN 8:36:18 is formed when the root mean square error of training is set to equal to zero. The number of neurons in the hidden level is equal to the number of elements of the training sample. From the conducted research it is possible to conclude that the given structure of a network is too complicated (Wołowiec and Suseł, 2010).

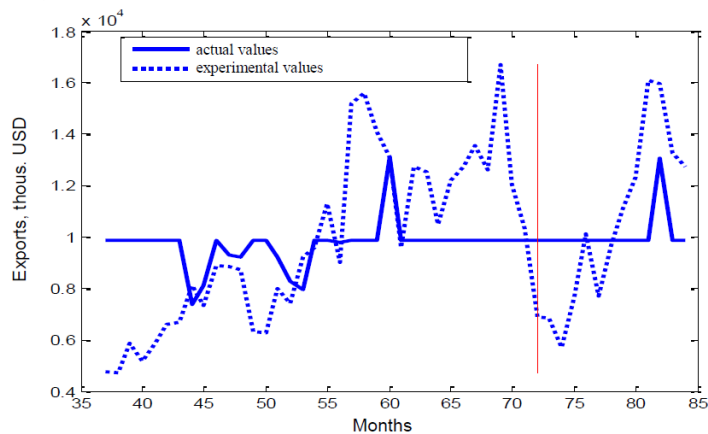
With a root mean square error of network training of 10% the number of neurons of the hidden level will be equal to 30; an error of 20% equals to 21 neurons; errors of 30%, 40%, and 50% result in 14, 6 and 2 neurons of the hidden level respectively. Although at 40% and 50% network training error, the RBFN structure is optimized to 6 and 2 hidden level neurons, however, as the results of the experiments show (Figures 3, 4), this structure does not provide sufficient prognostic properties.

**Figure 3.** Export forecasting based on radial-type ANN structure with a given learning error of 0.4<sup>3</sup>



Source: Own elaboration.

**Figure 4.** Export forecasting based on radial-type ANN structure with a given learning error of 0.5



Source: Own elaboration.

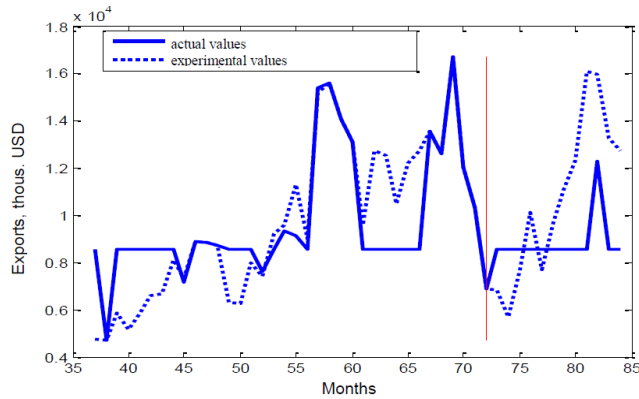
<sup>3</sup>In this and all the following figures, the “experimental values” indicate the data on which the network was trained, and the “actual values” indicate the results the ANN showed after the training.



The formation of the network structure is also influenced by the smoothing factor (the width parameter of the RBF window), which is usually chosen experimentally. The parameter value must be large to override the active areas of the basis functions. This provides the necessary smoothness of the approximating curves and precedes the situation of retraining the network. However, the influence parameter value should not be so large that the radial basis function shows all the input values equal (Liu and Peng, 2010).

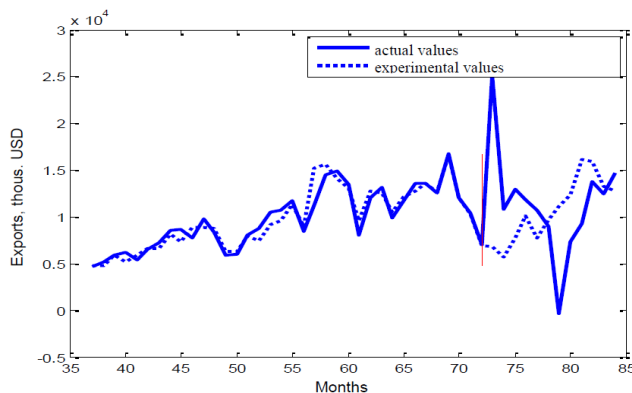
The choice of the smoothing coefficient is completely empirical, and when incorrectly set its value complicates the RBFN structure and deteriorates its prognostic properties. Figures 5 and 6 illustrate using the example of the export indicator how a change in the width of the RBF window affects the structure of RBFN and its prognostic properties (Kovel, 2007).

**Figure 5.** Export forecasting based on RBFN structure with too small impact parameter



Source: Own elaboration.

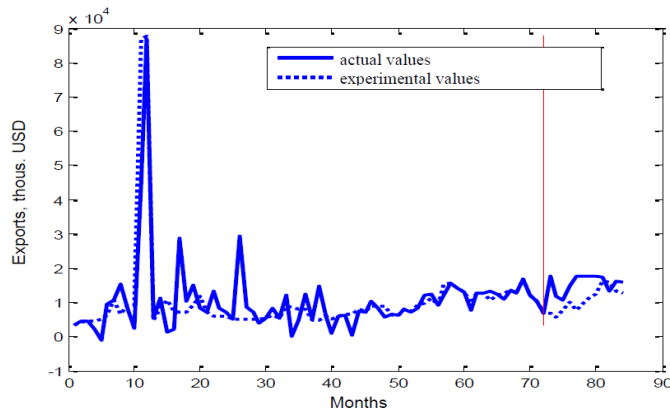
**Figure 6.** Export forecasting based on RBFN structure with too large impact parameter



Source: Own elaboration.

As a result of the conducted research, the optimal value of the impact parameter (RBF window width parameter) was set at 600000, which made it possible to predict rather complicated dynamics of economic security indicators, depending on the factors characterizing the activity of the customs authorities of Ukraine. An example of such a model is shown in Figure 7.

**Figure 7.** Forecasting the ES export indicator values



*Source:* Own elaboration.

#### 4. Conclusions

As a result of the research, the optimal RBFN structure is 8:14:18 (the allowed root mean square error of network training is 30%, the influence parameter is 600000).

To further check the adequacy of the obtained model structure, a computational experiment was conducted: an attempt was made to forecast the values of economic security indicators of Ukraine with the help of RBFN for 2018-2019 based on a training sample of data (performance indicators of the customs authorities of Ukraine and economic security indicators of Ukraine) for 2010-2017.

The results of forecasting the values of economic security indicators of Ukraine were compared with their actual values given in official statistical sources. As the results of the experiment show, there is no significant error in the obtained results of forecasting the values of the economic security indicators of Ukraine. The neural network, in training, describes well the actual trend of individual economic security indicators of Ukraine.

On the whole, the obtained results of the conducted experiments give grounds to conclude that the given RBFN structure well describes the tendency of change of economic security indicators of the country for future periods. This, in turn, will allow future studies to accurately establish the relationship and interaction between performance indicators of the customs system and certain economic security indicators of the state (Krausmann *et al.*, 2013).

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