The Impact of MT Strategies on Risk and Value Distribution of Unit-linked Insurance Portfolio

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Abstract:

Purpose: The analysis conducted demonstrates that in the case of unit-linked insurance, unlike classic insurance, the composition of the reference portfolio gains much more significance than the insurance period.

Design/Methodology/Approach: Hence, this type of insurance cannot be thought of only in the long term but, above all, the investment strategy should be adapted to market realities. Knowledge of the impact of the use of MT strategies on the parameters of the distribution of the portfolio value will enable the insured to control and possibly change the strategy of conduct during the insurance period by adjusting the composition of the portfolio to the market situation, and thus ensuring a payment tailored to their own needs.

Findings: In Poland, in the case of unit-linked insurance, the financial risk is mainly borne by the insured who is responsible for any negative effects of their investment decisions. Therefore, changes in the actuarial value of a unit-linked insurance portfolio have been examined depending on the managers' use of market-timing (MT) strategies.

Practical Implications:

Originality/Value:

Keywords: UFK insurance, market-timing strategies, MC method, flow valuation, portfolio value.

Paper Type: Research paper.

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1. Introduction

Unit-linked insurance (ULIP), as a complex product, requires some activity from the policyholder themselves in terms of determining their own needs and analysing the economic reality. It is only making the right decision as to the optimal scope of insurance and adopting a specific funds investing strategy can guarantee the insured an appropriate payment resulting from the policy value. Importantly, its amount is not guaranteed by the insurer and depends on the price of assets in which funds are invested on the market, and thus it is a random variable.

Since ULIP are transparent contracts of an open structure, they provide the insured with the opportunity to decide on the composition of the portfolio during its duration. It should be emphasized that these insurances offer many ways to invest, but there is always a financial risk that determines the value of the policy. This problem has been addressed in the literature considering the formal, legal, actuarial, or financial aspects, primarily from the insurer's point of view.

On the other hand, the specificity of unit-linked insurance in Poland resulting from the fact that the investment risk is entirely borne by the insured who is responsible for the possible negative effects of their decisions, was a prerequisite for analysing this type of insurance from the point of view of the insured themselves.

Therefore, the actuarial value of the unit-linked insurance reference portfolio has been examined depending on the managers' use of market-timing strategies, which is the strategy of making buying or selling decisions of financial assets by attempting to predict future market price movements.

Thus this analyse providing information enabling the insured to control the effectiveness and risk of funds resulting from the use of MT strategies. At the same time, it constitutes the basis for making optimal decisions as to how to invest and protect funds, in terms of the possibility of using market-timing strategies in periods of increased risk, thus guaranteeing optimization of the amount of withdrawal in the future.

2. Capital Fund Insurance

Insurance with a unit-linked capital fund belongs to the group of the so-called Universal Life insurance, however, it is fundamentally different from the so-called classic life insurance. It combines the element of insurance and saving in an extremely flexible way, creating many possibilities in the field of capital disposal. The idea of unit-linked insurance is related to its savings and investment character.

The most important feature of life insurance with a capital fund, referred to as unitlinked, is its association with a separate fund or funds in which money from premiums is invested. Nonetheless, unlike classic life insurance, in which the cost of insurance (expressed in the premium paid) is the same throughout the insurance period and does not result from the risk each year but from the average risk of the entire insurance period, in unit-linked insurance this cost depends on the age of the insured and varies depending on the payments, interest rate, unit prices, administrative and death risk burden costs, etc.

Thus, the insured person invests some or all the premiums paid in funds offered by insurance companies that differ in the level of risk and derive profits from this, with the clients choosing the fund themselves. Many aspects have been considered in the literature regarding the fair valuation of ULIP insurance (Karim and Dhaene, 2019; Melnikov, 2011; Dahl and Møller, 2006; Bacinello, 2003).

In Poland, contracts of this type allow to accumulate savings in an individually created investment portfolio consisting of various investment funds run by external investment fund companies, independent of the insurer. It is also important that, because policies of this type have an open structure and are transparent, they give the opportunity to adjust the composition of the portfolio on an ongoing basis depending on the changing market situation. It is a crucial element in a situation of unstable market economy.

Contracts of this type allow the insured to accumulate savings in an individually created investment portfolio consisting of investment funds which can be divided into two main groups: classic and specialized, for example market timing type (MT).

The essence of classic funds is to reduce investment risk by diversifying the investment portfolio. On the other hand, in the case of specialized funds the basic principle is additional protection of funds and reduction of investment risk with the use of some strategies, which is enabled by a flexible investment policy.

Regardless of the above division, the insurance capital funds offered differ in investment policy and include the following types of funds: balanced, debt securities, money market and stocks, etc. This means that these funds can generate different profits and the insured bears the investment risk, and thus bears responsibility for the possible negative effects of their decisions.

Therefore, in this work, funds from the most common groups differing in the way of investing funds have been selected for the analysis:

- (1) F_EU European stocks,
- (2) F_AA mixed foreign active allocation,
- (3) F DU global debt universal

dividing them additionally due to the use of MT strategies into a group of classic funds (C) and market-timing funds (MT).

3. Actuarial Value of the Unit-linked Insurance Portfolio

Due to the protective and economical nature of unit-linked insurance, the insurance company undertakes to pay the benefit both in the event of death during the insurance period, as well as in the event of the insured person's survival until the expiry of the insurance. Unlike traditional life insurance, in this type of insurance it is not only the moment of payment that is random but also the size of the benefit paid as it depends on the value of the insurance portfolio and the investment strategy adopted by the insured person.

Therefore, when valuing and calculating unit-linked insurance, it is necessary to consider not only the insurance risk (e.g., death risk) but also the financial risk of the portfolio related to this insurance. Thus, the payout is ta corresponding function of the accumulated investment, i.e., the value of the insurance portfolio (Moller, 2007):

$$B_t = f(FV_t)$$

where: FV_t —the value of the insurance (reference) portfolio at the t time.

Therefore, the value of the insurance portfolio is randomly dependent on the price of units of the selected fund resulting from the strategy adopted by the insured. This value is obtained by investing part of the insurance premium paid at t times, i = 0,1,2,...,n-1, until the policy expires $t_n = T$. The part of the premium invested at the t time is marked as $\pi(t_i)$ and is called the investment bonus. Thus, by investing in selected assets with a price specified as a process S_t the insured person builds the value of the reference portfolio. Thus, the value of the portfolio at the T time is equal to (Homa, 2017):

$$FV_T = \sum_{i=0}^{\tilde{n}(t)-1} S(T) \cdot \frac{1}{S(t_i)} \cdot \pi(t_i)$$

while the value of the portfolio can be written down as:

$$S(T)^{-1} \cdot FV_T = \sum_{i=0}^{\tilde{n}(t)-1} S(t_i)^{-1} \cdot \pi(t_i)$$

where
$$\tilde{n}(t) = min\{i|t_i > t\}$$
.

This means that the discounted (regarding S) value of the portfolio equals the discounted value of all investments. ULIP insurance is a product of a hybrid nature whose the structure is based on classic life or endowment insurance with investment in selected capital funds. Considering this fact, a unit-linked contract can be

interpreted as a traditional insurance with a sum insured G_{Π} with a potential surplus depending on the price of the asset; the value of the portfolio can be determined as follows (Dempsey 2019):

$$FV_{T} = g(\Pi) + \max\left\{0, \sum_{i=0}^{n-1} \frac{s(T)}{s(t_{i})} \cdot \pi(t_{i}) - g(\Pi)\right\} = g(\Pi) + \left(\sum_{i=0}^{n-1} \frac{s(T)}{s(t_{i})} \cdot \pi(t_{i}) - g(\Pi)\right)^{+}$$

Considering the change in the time value of money, the market value of the portfolio is:

$$V_t(FV_T) = \frac{v(T)}{v(t)}g(\Pi) + \frac{v(T)}{v(t)} \left(\sum_{i=0}^{n-1} \frac{S(T)}{S(t_i)} \cdot \pi(t_i) - g(\Pi)\right)^{+}$$

The second component of the sum is the payout function of a European call option with an expiry date oT and a strike price equal to the guaranteed sum (Ballotta, 2006). Exercising the option means that the account of its holder is credited with an amount equal to the difference between the value of the underlying instrument and its strike price. Therefore, the insured person can expect a surplus over the guaranteed sum insured only if the call option is "in the money," which means that the value of the insurance portfolio exceeds the guaranteed value.

Correct valuation of the option will allow not only to achieve a competitive advantage, but also to effectively manage risk. When valuing the UFK portfolio, its actuarial value should be examined considering the extended risk covering not only the market price and financial risk, but also the risk associated with the subject of insurance, and in the case of life and endowment insurance the risk of mortality. Considering the extended risk means that the actuarial value of the reference portfolio is the following conditional expected value:

$$EV_t^{ULIP} = E(V_t(X(T)) | \mathcal{L}_t \wedge G_t \wedge \mathcal{H}_t)$$

where \mathcal{L}_t — is the knowledge of contract options,

G_t — interpreted as knowledge of the process of prices and value of a monetary unit,

 \mathcal{H}_t – is the knowledge of the mortality process.

Filtration L_t , interpreted as the risk of contract options determines the type of cash flows resulting from the concluded insurance policy. Filtration $\{G_t = \sigma\{(B_t, S_t), 0 \le t \le T\}$ means that information is obtained only from the observation of the price process and the process of the value of the monetary unit. H_t , in turn, is the knowledge obtained up to the moment of the mortality process, and information about it determines the future life of the insured; thus, it has been

assumed: $H_t = \{I(T \le t), 0 \le t \le T\}$. Assuming the independence of both risks, the valuation of the payout at the end of the insurance proceeds as follows:

$$\begin{split} EV_t^{ULIP} &= {}_{T-t}p_{x+t} \cdot E\left(\frac{v(T)}{v(t)} \bigg(G_\pi + \left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+\right) |G_t^{S \wedge B}\right) + \\ &+ \int_t^{T \wedge n} {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \\ &\quad \cdot E\left(\frac{v(T)}{v(t)} \cdot \left(G_\pi + \left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+\right) |G_t^{S \wedge B}\right) \cdot d\tau = \\ &= \frac{v(T)}{v(t)} \cdot {}_{T-t}p_{x+t} \cdot E\left(G_\pi |G_t^{S \wedge B}\right) + {}_{T-t}p_{x+t} \\ &\quad \cdot \frac{v(T)}{v(t)} E\left(\left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+ |G_t^{S \wedge B}\right) + \\ &\quad + \int_t^{T \wedge n} \frac{v(T)}{v(t)} \cdot {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(G_\pi |G_t^{S \wedge B}\right) d\tau + \\ &\quad + \int_t^{T \wedge n} \frac{v(T)}{v(t)} \cdot {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(\left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+ |G_t^{S \wedge B}\right) d\tau \end{split}$$

From the definition of the European call option price at time t, concerning assets with a price described by the process $\{S_t\}_{t\geq 0}$ and with a maturity T, the value $E\left[e^{-\delta(T-t)}\cdot h(S_T)|G_t\right]$ is called arbitrage pricing of the European instrument $h(S_t)$ (Milevsky and Salisbury, 2006). Accordingly, adopting designation of call option price $C_t(S_T,G_\pi)$ the following result is received:

$$\begin{split} EV_t^{ULIP} &= {}_{T-t}p_{x+t} \cdot e^{-\delta(T-t)} \cdot E\left(G_{\pi}|G_t^{S \wedge B}\right) + {}_{T-t}p_{x+t} \cdot C_t(S_T, G_{\pi}) \\ &+ \int_t^{T \vee n} e^{-\delta(\tau-t)} \cdot {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E(G_{\pi}|G_t^{S \wedge B}) d\tau \\ &+ \int_t^{T \vee n} {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot C_t(S_\tau, G_\pi) d\tau \end{split}$$

Even more complete information will be obtained by analysing not only the actuarial value of the unit-linked insurance portfolio, but also the probability with which it may occur, i.e., the distribution of the ULIP reference portfolio.

4. Monte Carlo Method

In the case of the valuation of unit-linked insurance contracts and determining the value of the portfolio, the issue of the correct valuation of the financial instrument, which is the option, becomes crucial. Depending on the insurance variant, these are a European or American call option respectively (Glasserman, 2010). The European option can only be executed at the time of expiration of T, while the American option can be performed at any time. In the classic approach, i.e., the Black-Scholes model, the price of the European option is expressed in an analytical formula, while the valuation of the American option is no longer simple and requires the use of simulation methods to determine its price.

Therefore, this paper focuses on simulation methods and uses the Monte Carlo method (MC) to determine their price. In this method, the distribution of the value of the underlying instrument on the option's expiration date is determined by a certain stochastic process. Knowing this process and using the Monte Carlo method, through multiple simulations one obtains a distribution of the final values of the primary instrument to which the option is exposed. The paper adopts the simplest model of price evolution, i.e., the geometric Brownian motion.

Thus, the price of a unit of the stock fund S_t is described by a geometric Brownian motion, with an appropriate drift coefficient, which can be done using the Euler scheme (Perna and Sibillo, 2008). Using such a mathematical apparatus, a formula that simulates the future value of the underlying instrument (option price) is obtained:

$$S_{t_k}^i = S_{t_{k-1}}^i \exp\left[\left(r - \frac{\sigma^2}{2}\right)(t_k - t_{k-1}) + \sigma\sqrt{t_k - t_{k-1}}\varepsilon_k^i\right]$$

where ε_k^i are the independent values generated from the normal distribution, r is the risk-free interest rate, while σ determines the instrument's price volatility. The stock price process is simulated in a finite number of time points: t_k . It is also assumed that the pay-out function depends only on the stock prices at these points. For a single trajectory of stock prices, the discounted price of any option is:

$$C_0 = \frac{v(T)}{v(t_0)} (S_0, S_{t_1}, \dots, S_{t_1})$$

Thus, the option price can be determined by simulating the discounted value of the pay-out profile, followed by the average after all executions.

5. Results of ULIP Portfolio Value Distribution Simulation

As an illustration, a unit-linked life futures contract has been analysed according to which, if the insured dies at the $t_i \in \Theta/\{t_0\}$ time, the insurer will pay them the sum insured resulting from the value of the reference portfolio:

$$V\left(B_{t_{j}}, t\right) = \frac{v(t_{j})}{v(t)} G_{\Pi} + \frac{v(t_{j})}{v(t)} \left(\Pi \sum_{i=0}^{t_{j}-1} \frac{S_{t_{j}}}{S_{t_{i}}} - 1\right)^{+}$$

It has been assumed that the insured pays premiums of a fixed amount at times $t_i \in \tilde{T}$, when $\tilde{T} = \{0 = t_0 < ... < t_n = T\}$, and the investment part of each of them is equal to Π , which corresponds to the % of the total gross premium (100% has been assumed). Furthermore, a risk-free rate of 5% and continuous capitalisation in line with the scenario have been considered:

$$\frac{v(t)}{v(t_0)} = e^{-t \ln(1,05)}$$

On the other hand, to determine the probability of survival and death, mortality tables based on Makeham's law [Biffis 2005] have been used, according to which: $l_x = 1000401,71 \cdot 0,99949255^x \cdot 0,99959845^{1,10291509^2}$

Moreover, it has been assumed that the insured can implement various investment strategies, thus differentiating the profits resulting from the variable value of the portfolio. The analysis covered insurance capital funds with such investment strategies as well as both classic and specialized portfolios. Thus, the following classic (C) insurance portfolios based on insurance capital funds with the following investment strategies have been included:

- F_EU_C European stocks,
- F_AA_C mixed foreign active allocation,
- F_DU_C global debt universal

as well as specialized ones whose managers used market-timing (MT) strategies:

- F_EU_MT European stocks,
- F AA MT mixed foreign active allocation,
- F_DU_MT global debt universal

Using the described price evolution model and the Monte Carlo method, a distribution of the final values of the primary instrument to which the option is exposed has been obtained. The starting point of the valuation is to generate possible

prices of the underlying instrument on the expiration date of the option, which is followed by determining its pay-out function. Then, considering the model of the mortality process, the basic characteristics of the portfolio's value have been calculated depending on the investment strategy.

Thus, the basic amount used by insurers to value life insurance, i.e., the actuarial value of the portfolio, has been examined. This value for the ULIP insurance portfolio with indicated investment strategies, with capitalization in accordance with the adopted scenario for a 20, 30, 40, 50-year UFK contract, depending on the adopted investment strategy, is presented in the table below.

Table 1. Actuarial value of the reference portfolio with different strategies and

period of insurance

Portfel		n=20	n=30	n=40	n=50
F_EU	C	33,16	181,85	966,00	3693,90
	MT	59,22	182,93	966,06	3702,40
F_AA	С	33,17	182,16	966,37	3755,50
	MT	33,20	182,09	965,84	3755,70
F_DU	С	33,27	182,63	967,67	3761,70
	MT	33,26	182,64	967,55	3761,70

Source: Own study.

Based on the above data on the actuarial value of ULIP insurance, it can be assumed that the investment strategy adopted by the insured as well as the fact whether managers use MT strategies does not significantly determine the value of the portfolio, and only the insurance period is an important factor here. Since the expected value does not provide complete information, the investment risk of the analysed portfolios has also been examined.

Table 2. Risk of the reference portfolio with different investment strategies and insurance period

		n=20			n=30			n=40			n=50		
Portfel		\mathbf{S}	λ_s	K	S	λ_s	K	\mathbf{S}	$\mathbf{l}_{\mathbf{s}}$	K	\mathbf{S}	λ_s	K
F_EU	С	10,30	0,54	0,51	39,17	0,65	0,74	241,45	0,77	1,11	1038,6	0,85	1,28
	MT	5,82	0,52	0,47	31,80	0,53	0,50	193,92	0,61	0,67	837,5	0,71	1,00
F_AA	C	3,81	0,34	0,18	25,60	0,43	0,33	157,39	0,49	0,44	685,4	0,55	0,53
	MT	2,92	0,26	0,14	18,70	0,30	0,16	114,84	0,36	0,24	498,2	0,39	0,27
F_DU	С	3,00	0,27	0,12	20,19	0,33	0,22	123,57	0,38	0,24	537,7	0,44	0,40
	MT	2,79	0,25	0,09	19,74	0,33	0,20	120,46	0,37	0,25	523,1	0,42	0,31

Source: Own study.

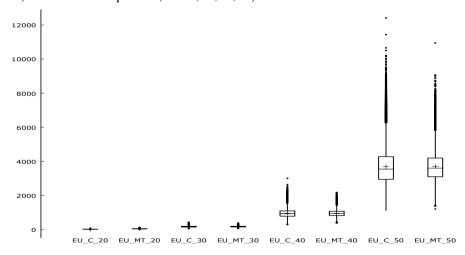
The results presented above indicate that the investment strategy adopted by the insured, regardless of the established insurance period, significantly determines the risk of the insurance portfolio. At the same time, these results confirm that the

increase in the risk of the investment strategy causes an increase in all measures indicating differentiation, asymmetry and flattening of the distribution, and the use of MT strategies affects not only the reduction of investment risk but also reduces asymmetry and kurtosis, thus making the distribution of rates of return closer to the normal distribution.

Hence, the weakest asymmetry and the lowest kurtosis are characteristic of the reference portfolio associated with the investment in debt securities. Investing in stocks increases the risk, thus increasing the asymmetry and kurtosis of the distribution. Kurtosis in all cases takes a positive value, which means that the distribution is leptokurtic (more slender than normal), and its value also increases along with the risk.

The presented results indicate that in the case of life insurance with an insurance capital fund, the composition of the reference portfolio is much more important than the insurance period itself. Therefore, this type of insurance should not only be considered in the long term, but above all the strategy should be adjusted to the current market situation, and it is undoubtedly beneficial to choose MT portfolios. These properties are confirmed by the box charts presented in Figures 1-3 below (blue chart for classical funds, purple for MT funds).

Figure 1. Distribution of F_EU portfolio values with different strategies (classic and MT) and insurance period (T=20,30,40,50)

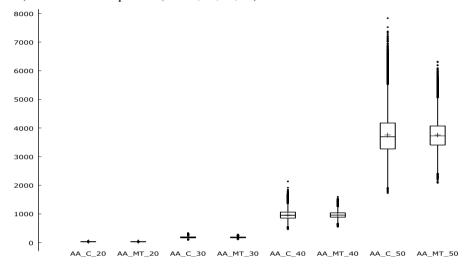


Source: Own study.

On the basis of the above charts, it can be concluded that the insurance period significantly determines the diversity of the distribution, while the adopted investment strategy in the field of fund specialization, i.e., the use of the fund's protection strategy, reduces the investment risk as well as the asymmetry and

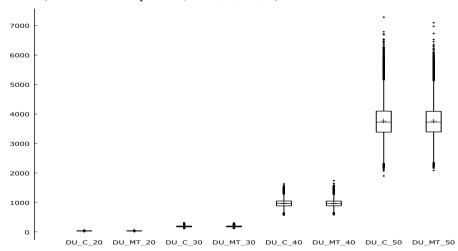
kurtosis of the distribution of the value of the insurance portfolio, making it more similar to the normal distribution.

Figure 2. Distribution of F_AA portfolio values with different strategies (classic and MT) and insurance period (T=20,30,40,50)



Source: Own study.

Figure 3. Distribution of F_DU portfolio values with different strategies (classic and MT) and insurance period (T=20,30,40,50)



Source: Own study.

Therefore, changes occurring during the insurance period have also been examined, which is demonstrated in the chart below on the example of the AKZ_EU portfolio characterized by the greatest risk. Therefore, the results presented above indicate that the insured can control the value of the portfolio, and thus the amount of the pay-out for unit-linked insurance by choosing the right risk aversion strategy, determining the part of the premium that builds the value of the investment portfolio depending on their expectations.

6. Conclusion

The analysis conducted shows that in the case of life insurance with an insurance capital fund in Poland, contrary to classic insurance, the composition of the reference portfolio is much more important than the insurance period. Therefore, this type of insurance should not only be considered in the long term, but above all the strategy should be adjusted to market realities. This paper demonstrates that:

- 1. The use of MT strategies in the case of aggressive portfolios (F_EU) significantly increases the value of the portfolio while reducing its risk at the same time.
- 2. For portfolios with a mixed investment strategy (F_AA), regardless of the capitalisation time, it shall guarantee the same rate of return with a lower risk.
- 3. Low-risk cash portfolios (F_DU) are characterised by the fact that the use of MT strategies does not significantly change the characteristics of the portfolio's value distribution.

Furthermore, the analysis of the distribution of the portfolio value considering hedging strategies confirms the importance of policyholders' awareness is in this type of insurance, because it is the insured who, by choosing the right strategy for the duration of the insurance, can maximize the value of the portfolio while minimizing its risk.

This becomes even more important because in Poland UFK contracts are not contracts with a guaranteed sum insured, so only this way can the insured ensure a proper amount of payment in the occurrence of an event covered by the contract.

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