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Inversions Distribution and Testing Correlation Changes for Rates of Return

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Abstract:

Purpose: *The paper presents the application of a test based on the inversions number distribution. The purpose of the work is to examine the stability of dependence measures between rates of return for financial instruments.*
Design/Methodology/Approach: *The issue plays a vital role in portfolio analysis because dependence measures are an essential factor affecting the portfolio risk of financial instruments. Direct application of the classic measure of dependence, such as Pearson's correlation coefficient, requires several additional assumptions regarding the existence of moments or the normality of the distribution of random variables analyzed. Therefore, the choice falls on the tau Kendall coefficient.*
Findings: *First of all, it is suitable for testing distributions for which variance does not exist (e.g., stable distributions). Secondly, we present a new way of testing the hypothesis about the equality of the Kendall coefficient over two separate periods. A miniature sample version was introduced in the article. An essential feature of the proposed test is the ability to calculate its power analytically.*

Keywords: *Inversions, risk, Kendall tau, portfolio analysis, stable distribution, rate of returns, power of the test.*

JEL Codes: *C12, C46, G11.*

Paper Type: *Research study.*

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1. Introduction

We say for the sequence of observations $(X_1, \ldots, X_n)$ and for i and j the inversion occurs, if and only if $i > j$, but $X_i < X_j$. The Kendall correlation coefficient [11] of the sample can be calculated from the formula.

$$\tau = \frac{\binom{n}{2} - 2I_n}{\binom{n}{2}}, \tag{1}$$

which means, that $I_n = \frac{1}{2}\binom{n}{2}(1-\tau)$. The mutual correspondence between the number of inversions and the tau coefficient allows the use of inversions as objects easier to analyze.

Many papers show the distribution of the number of inversions assuming that for each n, the additional number of inversions has a uniform distribution. In this article, we intend to use elements of the permutation theory based on the number of inversions. Thanks to the applied method, it is possible to detect a significant change in correlation between selected assets quickly. This is the primary purpose of the paper. The presented method was previously used to evaluate fashion experts. The new way of using it results from the fact that the methods used so far required certain assumptions.

These assumptions were either not met in practice, or it was impossible to verify them. The speculation about the normality of the distribution or the existence of moments is problematic. The problem of fat tails has been solved using the copula concept. This usually requires some form of multivariate distribution to be assumed. The solution we propose does not require any assumptions regarding the distribution of rates of return. In addition, it is a tool that allows someone to detect changes in correlation quickly.

The purpose of the article described above is not a way to build a portfolio. Relevant theories from Markowitz's (1952) view, through Sharp and Jobson *et al.* (1981), and copula concepts presented by Patton (2012), approach appropriate for the construction of a portfolio. There is a problem in studying the relationship between assets to estimate their risk in each of these theories. The proposed method is an essential step in achieving this goal.

The known sequence A008302 well describes the distribution of the number of inversions. Other calculation methods are presented in Czekala *et al.* (2017) and Margolius (2001). All these methods use explicitly the recursive dependencies given below for $0 \le k \le \binom{n}{2}$

$$I_{n+1}(k) = \sum_{i=1}^{n+1} I_n(k-i+1) \tag{2}$$

In addition, it is assumed that $I_n(k) = 0 \quad$ for $k < 0$ or $k > \binom{n}{2}$.

The number of inversions can be determined by a recursive formula $I_{n+1} = I_n + Y_{n+1}$.

In this case, the assumptions about the additional number of inversions are key. The case under the assumption of uniformity is well known in the case of any n. The central limit theorem is also well known (Feller, 1971). In the latter case $E(I_n) = \frac{n(n-1)}{4}$ and $var(I_n) = \frac{2n^3+3n^2-5n}{72}$.

An alternative distribution was proposed in works Czekala *et al*. (2017; 2018) for an assumption with a uniform distribution of an additional number of inversions. Its main advantage is maintaining the distribution class for the number of inversions. According to the mentioned proposal:

$$P(Y_{n+1} = i) = q^{n-i} p^i \pi_{n+1}, \tag{3}$$

for $i = 0,1,2, \dots n,$ where

$$\pi_{n+1} = \left(\sum_{j=0}^{n} q^{n-j} p^j\right)^{-1}. \tag{4}$$

The above probabilities could be performed by one parameter substituting $s = p/q$. Then the result is as follows:

$$P(Y_{n+1} = i) = \frac{s^i}{\sum_{j=0}^{n} s^j}\,. \tag{5}$$

The probability of inversion p could be an alternative to the Kendall coefficient, which measures dependence between variables. This concerns only the proposed distribution of the variable Y_(n+1). Many further conclusions can be drawn. The expected value is an increasing inversion probability function (or equivalently to the variable s). This property gives a way to establish the correspondence between the observed number of inversions, parameter τ oraz parameter s (or equivalently p) is an increasing function of a variable.

$$EI_n(s) = \sum_{k=1}^{n+1} EY_k(s) = H_n(s) \tag{6}$$

Expected value $EY_n(s)$ can be calculated by multiplying the relevant elements from the table below:

Table 1. *$EY_n(s)$ Calculation.*

Y_n	0	1	2	…	n-1
$Pr(Y_n = i)$	$1/\sum_{j=0}^{n-1} s^j$	$s/\sum_{j=0}^{n-1} s^j$	$s^2/\sum_{j=0}^{n-1} s^j$		$s^{n-1}/\sum_{j=0}^{n-1} s^j$

Source: *Own creation.*

Because individual probabilities in the proposed distribution of an additional number of inversions form a geometric sequence, it is possible to calculate moments also for the number of inversions. These moments were calculated in the work of *Czekala et al*. (2018). It turns out that they can be presented as functions of the ratio s=p⁄(q.)

$$EY_n(s) = EY_n = \frac{s - ns^n + (n-1)s^{n+1}}{1 - s - s^n + s^{n+1}} \tag{7}$$

and

$$var\big(Y_n(s)\big) = \frac{s - n^2 s^n + 2(n^2-1)s^{n+1} - n^2 s^{n+2} + s^{2n+1}}{(1-s)^2(1-s^n)^2} \tag{8}$$

Random variables Y_n are independent, therefore:

$$E(I_n) = m_n = \sum_{i=1}^{n} EY_n(s) = H_n(s) \tag{9}$$

And

$$var(I_n) = s_n^2 = \sum_{i=1}^{n} var\big(Y_n(s)\big) \tag{10}$$

In the work of Czekala *et al*. (2021), it was shown that the sequence of random variables. Y satisfies the Lindeberg condition, which means that the central limit theorem is satisfied. The expected value and variance of the number of inversions are expressed by the formulas (3) and (4).

In this work, the method of measuring dependence related to the Kendall coefficient will be considered. Formula (1) gives a direct relationship between the number of inversions and the Kendall correlation coefficient. On the other hand, formula (2) describes the dependence of the number of inversions on the parameter p. Because p = s / (s + 1), there is a mutual correspondence between the probability p and the expected value of the number of inversions.

However, this time cannot be expressed as simply as in the case of (1). Since the expected value is a monotonic function of the variable s, knowing the expected value of the number of inversions is equivalent to knowing the probability p. Considering the relation (1), it can be argued that there is also an exact correspondence between the p (and s) parameter and the expected value of the Kendall coefficient from the sample. Thus, testing critical hypotheses related to s parameters is key to investigating the relationship between variables. In a large selection, the tool for testing hypotheses regarding the parameter s is the central limit theorem for the number of inversions using formulas (3) and (4).

To estimate the parameter s (and hence p), it is desirable to use the inverse function to H. In the work of Czekala *et al.*(2018), an analogous estimation of the parameter p is proposed.

For the designations used in this work, it should be assumed that:

$$\hat{s} = H^{-1}(I_n) \text{ and } \hat{p} = \frac{\hat{s}}{1+\hat{s}} \ . \quad (11)$$

It follows from equation (3) that the estimator $\hat{s}$ is an unbiased estimator.

2. Literature Review

Rates of return is an exciting and vast topic. Rates of return perception are essential during the investment process. In literature exists theories of rates of return. Ideas could be divided into groups. The First could be Classical Theories, which generally connect the price level with money supply (Marshall, 1920) and some economic and political elements even sooner wherein many concepts (Say, Sidgwick). Then, another author made the development of rates of return, rates of return were important for monetary policy (Koch, 1929). Next, the speeds of return research development were even faster (Fisher, 1930). The analysis focused on short and long-term rates of return.

The research was continued, and new theories were proposed by Keynes (1930). Subsequently, other approaches besides classical can be in a group of leading ideas of the terminal structure of interest rates like Expectation Theory, Liquidity Premium Theory, and Market Segmentation Theory (Gruszczyńska-Brożbar, 1997). A good representative of expectation theory is Lutz (1940). The main assumptions concern, short-term interest rates, the costs of investment are not existing, possibility to postpone the duration of the investment.

The Liquidity Premium Theory complements Expectation Theory. Research has shown that the interests rate level of equilibrium doesn't exist (Tobin, 1958). Expectation Theory was a background to Market Segmentation Theory. Many elements have an impact on the change of interests rates structure (Culberston, 1957). Then, another researcher developed the theories (Modigiliani and Sutch, 1966), and the authors kept chosen assumptions (Elliot and Echols, 1976).

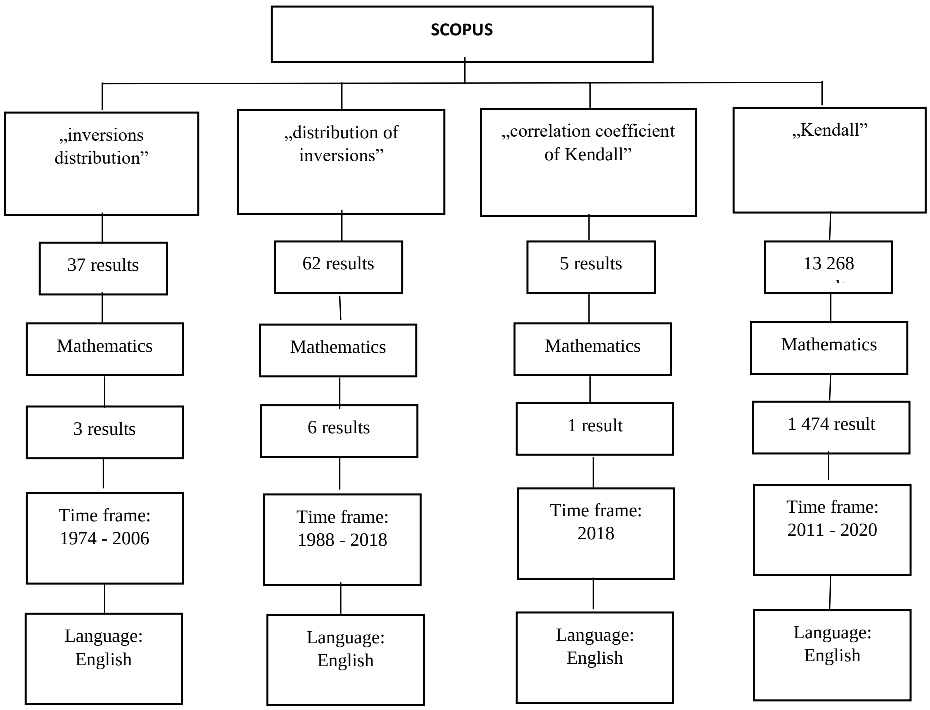
Availability of capital is critical, but risk metrics and predictions of the firm's possibilities in the future should also be underlined. This is the fact that rates of return of financial instruments are connected with the market hypothesis. This is the theoretical path from an adequate market hypothesis (Fama, 1970) to a fractal market hypothesis (Peters, 1991) in chaos theory. All elements are linked with return rates and risk levels.

The risk metrics, rates of return of financial instruments, and a lot of data are available to predict what will happen with recovery rates in the future using different methods. One of these methods could be the Distribution of Inversions. Rates of return distribution show what should be done when the rates of return distribution have a long tail or have not got variance. Kendall's regression is a perfect measure that has not been used frequently in university researches. Kendall's regression measures the dependence between two random variables (Derumigny and Fermanian, 2020).

Besides, in literature, we can find the research form 2020, extremal Markov sequences connected with the Kendall convolution (Jasiulis-Gołdyn *et al.*, 2020). The research in the article is an entirely new approach: distribution of inversions to check the interdependence between variables. The background is the research in the article, "Testing for Changes in Kendall's tau" (Dehling *et al.*, 2017).

Kendall's distribution has been recognized in literaturę (Kendall, 1938; Van Doorn and Ly *et al.*, 2018). The research is connected with departure from linearity, and in the literature is shown the way to the distribution of parameters. It can be recognized that this is a challenge to analyze nonparametric data analysis of data distribution. The background to the research of nonparametric distribution is research papers presented earlier (Serfling 1980; Johnson 2005). In the research papers, not many articles connected that kind of distribution with financial instruments' rate of return. The literature background was searched in September 2020 in Scopus. Results of statistics of the literature are presented below (Figure 1). The Scopus base was selected due to its well-recognized base worldwide. The searching process is focused on the mathematics part.

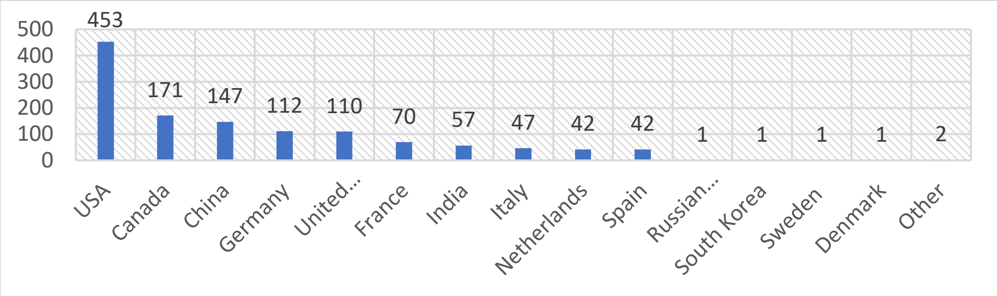
Figure 1. *Articles selection procedure*



Source: *Own analysis on basis on Scopus.*

The researcher's distribution per country was prepared to check where the main topic of article was well recognised in research papers (Figure 2).

Figure 2. *Researchers distribution per country*



Source: *Own analysis on basis on Scopus.*

3. Research Methodology

Inversion distribution and testing correlation rates of return is a very vast and exciting subject. The main reason for the research paper is financial instruments' rates of return level of changes. Nowadays, the worlds' pandemia situation could impact not only society's health but also could affect financial instruments' rates of return. Because of this aspect, the research methodology has been prepared. That is the background to check on the scientific field the possible impact of changes rates of return. The period before pandemia (3 years) has been compared to the pandemia period (1 year).

The research was conducted on selected financial instruments like global stock indexes, currencies, and others (golda21, crude oil, BTC/USD). The first part of the research indicates which financial instruments' rates of return have changed the most. The conclusion after analysis of differences in tau correlation has been made. In the next step, research was conducted comparing BTC/USD (the most significant volatility) to other financial instruments using a scientific methodology (tested hypothesis for the pairs).

The aim is to identify changes in the level of correlation between the selected financial instruments' rates of return before (3 years) and during the pandemic (1-year, monthly data). The research is based on three aspects:

- global stock indexes (DIJA, NASDAQ, NIKKEI 225, S&P500)
- currencies (EUR/USD, USD/JPY, GBP/EUR, USD/GBP)
- others (GOLDa21, CRUDEOIL, BTC/USD)

We propose a new tool that could test the hypothesis H0: tau=tau0. The alternative hypothesis H1 is tau < or > tau0. The correlation tau Kendall is calculated between chosen global stock indexes, currencies, and others before pandemic (Figure 3) and during pandemic (Figure 4). Then the differences are calculated to show the impact of the pandemic situation on chosen elements.

Table 2. *Correlation Kendall's tau. The determined correlation coefficients are significant with p <0.05000. Before pandemic situation*

Variable	DJIA	NASDAQ	GOLDa21	BTC-USD	EURUSD	USDJPY	CRUDE OIL	NIKKEI 225	S&P 500	USDGBP	GBPEUR
DJIA	1.0000	0.5650	-0.088	0.0444	0.2031	0.1174	0.4285	0.5428	0.7746	-0.333	0.193
NASDAQ	0.5650	1.0000	0.0095	0.0158	0.1555	0.0000	0.4000	0.3746	0.7333	-0.196	0.139
GOLDa21	-0.088	0.0095	1.0000	0.2063	0.1682	-0.387	-0.031	-0.177	-0.085	-0.088	0.006
BTC-USD	0.044	0.0158	0.2063	1.0000	0.0793	0.0000	-0.019	0.0952	-0.009	0.1079	-0.21
EURUSD	0.2031	0.1555	0.1682	0.0793	1.0000	-0.203	0.1587	0.1206	0.1682	-0.463	-0.13
USDJPY	0.1174	0.0000	-0.387	0.0000	-0.203	1.0000	0.1365	0.3904	0.1015	-0.053	0.168
CRUDE OIL	0.4285	0.4000	-0.031	-0.019	0.1587	0.1365	1.0000	0.3777	0.4698	-0.307	0.142
NIKKEI225	0.5428	0.3746	-0.177	0.0952	0.1206	0.3904	0.3777	1.0000	0.4825	-0.314	0.244
S&P 500	0.7746	0.7333	-0.085	-0.009	0.1682	0.1015	0.4698	0.482	1.0000	-0.279	0.184
USDGBP	-0.333	-0.196	-0.088	0.107	-0.463	-0.053	-0.307	-0.3142	-0.279	1.000	-0.39
GBPEUR	0.1936	0.1396	0.0063	-0.215	-0.139	0.1682	0.1428	0.2444	0.1841	-0.396	1.000

Source: *Own creation.*

Table 3. *Correlation Kendall's tau. The determined correlation coefficients are significant with p <0.05000. During pandemic situation*

Variable	DJIA	NASDAQ	GOLDa21	BTC-USD	EURUSD	USDJPY	CRUDE OIL	NIKKEI225	S&P 500	USDGBP	GBPEUR
DJIA	1.000	0.7575	0.0303	0.424	0.2424	-0.060	0.424	0.7878	0.818	-0.363	0.2727
NASDAQ	0.757	1.0000	0.2121	0.303	0.3030	-0.242	0.242	0.5454	0.818	-0.363	0.2727
GOLDa21	0.030	0.2121	1.0000	0.000	0.3636	-0.484	-0.181	-0.1212	0.090	-0.181	-0.090
BTC-USD	0.424	0.303	0.0000	1.000	0.2727	-0.090	0.212	0.3333	0.424	-0.636	0.3636
EURUSD	0.242	0.3030	0.3636	0.272	1.0000	-0.212	0.272	0.1515	0.242	-0.454	-0.181
USDJPY	-0.060	-0.242	-0.484	-0.090	-0.212	1.0000	0.333	0.1515	-0.121	0.2727	-0.060
CRUDE OIL	0.424	0.2424	-0.181	0.212	0.2727	0.3333	1.000	0.5151	0.303	-0.151	0.0000
NIKKEI225	0.787	0.5454	-0.121	0.333	0.1515	0.1515	0.515	1.0000	0.666	-0.151	0.2424
S&P 500	0.818	0.8181	0.0909	0.424	0.2424	-0.121	0.303	0.66666	1.000	-0.424	0.4545
USDGBP	-0.363	-0.363	-0.181	-0.636	-0.454	0.272	-0.151	-0.151	-0.424	1.0000	-0.363
GBPEUR	0.272	0.2727	-0.090	0.3636	-0.181	-0.060	0.000	0.242424	0.4545	-0.363	1.0000

Source: *Own creation.*

Table 4. *Differences (Before pandemic and during pandemic situation) in Kendall's tau correlation*

Differences tau	DJIA	NASDAQ	GOLDa21	BTC-USD	EURUSD	USDJPY	CRUDE OIL	NIKKEI 225	S&P 500	USDGBP	GBPEUR
DJIA	0.000	-0.1925	-0.1192	**-0.3798**	-0.0392	0.1781	0.0043	**-0.2450**	-0.043	0.0303	-0.079
NASDAQ	**-0.19**	0.0000	**-0.2026**	**-0.2872**	-0.1475	**0.2424**	0.1576	-0.1709	-0.084	0.1668	-0.133
GOLDa21	-0.11	-0.2026	0.0000	**0.2063**	**-0.1954**	0.0975	0.1501	-0.0566	-0.176	0.0929	0.0973
BTC-USD	**-0.379**	**-0.2872**	**0.2063**	0.0000	**-0.1934**	0.0909	**-0.2312**	**-0.2381**	**-0.433**	**0.7443**	**-0.579**
EURUSD	-0.0392	-0.1475	-0.1954	-0.1934	0.0000	0.0089	-0.1140	-0.0309	-0.074	-0.0089	0.0421
USDJPY	0.1781	0.2424	0.0975	0.0909	0.0089	0.0000	**-0.1968**	**0.2390**	**0.2228**	**-0.3267**	**0.2289**
CRUDE OIL	0.0043	0.1576	0.1501	**-0.2312**	-0.1140	-0.1968	0.0000	-0.1374	0.1668	-0.1564	0.1429
NIKKEI225	**-0.2450**	-0.1709	-0.0566	**-0.2381**	-0.0309	0.2390	-0.1374	0.0000	**-0.1841**	-0.1628	0.0020
S&P 500	-0.0436	-0.0848	**-0.1766**	**-0.4338**	-0.0742	0.2228	0.1668	-0.1841	0.0000	0.1449	**-0.270**
USDGBP	0.0303	0.1668	0.0929	**0.7443**	-0.0089	-0.3267	-0.1564	-0.1628	0.1449	0.0000	-0.033
GBPEUR	-0.0791	-0.1330	0.0973	**-0.5795**	0.0421	0.2289	0.1429	0.0020	**-0.270**	-0.0332	0.0000

Source: *Own creation.*

The main conclusions after analysis of differences in tau correlation are:

- negative differences for stock indexes except for USD/JPY and USD/GBP
- negative differences EUR/USD except for GBP/EUR
- the positive difference for Nikkei25 only for USD/JPY and GBP/EUR
- the positive difference for BTC/USD and Gold and USD/GBP
- negative contrast of crude oil to currencies except for GBP/EUR.

Some of the pairs considered seem to show significant changes in correlation. The significance of changes in the Kendall correlation coefficient will be examined below. Therefore, the next step is to check the dependence of BTC/USD on other chosen elements.

The method presented below can be used to test the correlation changes for any pair. Since the distribution depends on the value of the tested parameter, it is different each time. In some situations, it is possible to use the most used Pearson's correlation coefficient. In this case, however, statistical inference requires either the normality of the distribution of rates of return or a large sample. But this article intended to present a method that works with a small piece. In addition, we wanted to introduce a tool that can be applied to fat-tailed distributions (so different than usual). In the case of cryptocurrencies, this assumption looks justified. Data from 36 months before pandemia (February 2017 – February 2020) calculates 36 rates of return (1 month was included before the three-year periods considered for the calculation of the 36th rate of return). On this basis, the chosen selected pairs parameter was estimated $s=p/q$, where p is an inversion probability.

Table 5. *Calculation results for the period before pandemic*

BTC-USD	DJIA	NASDAQ	GOLD	EURUSD	USDJPY	CRUDE OIL	NIKKEI225	S&P 500	USDGBP	GBPEUR
tau	0,044444	0,015873	0,206349	0,079365	0	-0,01905	0,095238	-0,00952	0,107937	-0,21587
In	301	310	250	290	315	321	285	318	281	383
s	0,98965	0,99630	0,95142	0,98153	1,0000	1,00446	0,97784	1,00223	0,97487	1,05365
p	0,497399	0,499072	0,487553	0,49534	0,5	0,501113	0,494397	0,50055	0,493638	0,513062
q	0,502601	0,500928	0,512447	0,50466	0,5	0,498887	0,505603	0,49944	0,506362	0,486938

Source: *Own creation.*

Table 6. *Calculation results for the period of pandemic*

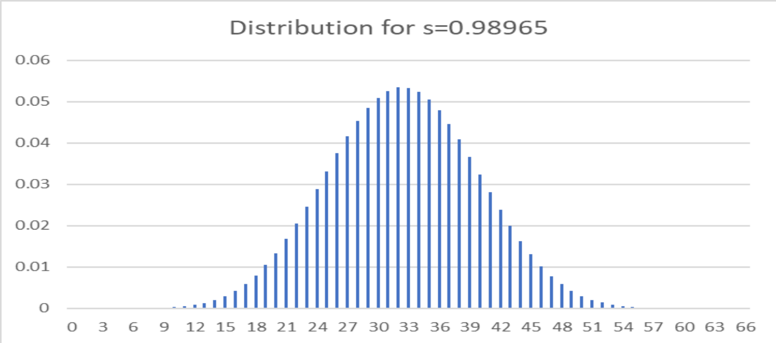
tau	0,424242	0,30303	0	0,272727	-0,09091	0,212121	0,333333	0,424242	-0,63636	0,363636
In	19	23	33	24	36	26	22	19	54	21
s	0,74284	0,81915	1,00000	0,83739	1,05835	0,87335	0,80064	0,74284	1,71871	0,78181
p	0,426223	0,450291	0,5	0,455749	0,514174	0,466197	0,444642	0,426223	0,632178	0,438772
q	0,573777	0,549709	0,5	0,544251	0,485826	0,533803	0,555358	0,573777	0,367822	0,561228

Source: *Own creation.*

The next step is a hypothesis test for the pairs. When formulating an alternative hypothesis, we are suggested by the trial results, specifying an alternative idea. In all the theories under consideration, the test statistic is the number of inversions. Economic comments have been added to each pair.

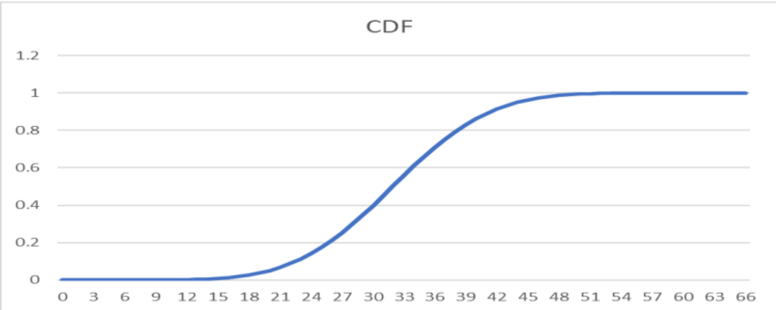
1. DJIA- tested hypothesis:
   $H_0: s = 0.98965$ versus $H_1: s < 0.98965$. The distribution of the number of inversions is presented below. According to formula (2):

Figure 3. *Distribution for s=0.98965*



Source: *Own creation.*

Figure 4. *CDF*



Source: *Own creation.*

Table 7. *Values of PDF for* $s = 0.98965$

0	1	2	3	4	5	6	7	8	9	10	11
2.93E-09	3.49E-08	2.22E-07	1.00E-06	3.60E-06	0.00001	0.00003	0.00007	0.00016	0.00032	0.00062	0.00114
	12	13	14	15	16	17	18	19	20	21	22
	0.00200	0.00335	0.00540	0.00841	0.01270	0.01863	0.02661	0.03707	0.05045	0.06719	0.08767
	23	24	25	26	27	28	29	30	31	32	33
	0.11221	0.14103	0.17420	0.21168	0.25325	0.29851	0.34693	0.39782	0.45040	0.50378	0.55707
	34	35	36	37	38	39	40	41	42	43	44
	0.60936	0.65979	0.70760	0.75215	0.79294	0.82963	0.86202	0.89011	0.91401	0.93394	0.95023
	45	46	47	48	49	50	51	52	53	54	55
	0.96327	0.97348	0.98130	0.98713	0.99138	0.99440	0.99647	0.99785	0.99874	0.99929	0.99962
	56	57	58	59	60	61	62	63	64	65	66
	0.99981	0.99991	0.99996	0.99998	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=19. Moreover P(I_12≤19)=0.03707<0.05. Therefore, the tested hypothesis should be rejected. As a result of the pandemic, the s parameter (and probability of inversion) decreased significantly. It also means that the correlation between the examined instruments has increased significantly. In a financial sense, this means that the most popular stock index and BTC are becoming more and more dependent. For the person who builds a portfolio with both listed securities, its possible change should reduce the sum of assets expressed in BTC or DJIA. No diversification benefit for a portfolio of these investments.

2. NASDAQ- tested hypothesis:
   $H_0: s = 0.99630$ versus $H_1: s < 0.99630$.
   The distribution of the number of inversions is presented below.

Table 8. *Values of PDF for* $s = 0.99630$

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
2.36E-09	2.82E-08	1.80E-07	8.20E-07	2.96E-06	0.00001	0.00002	0.00006	0.00013	0.00027	0.00053	0.00098
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.00173	0.00291	0.00472	0.00740	0.01124	0.01658	0.02381	0.03336	0.04566	0.06114	0.08022
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.10323	0.13043	0.16195	0.19781	0.23783	0.28172	0.32897	0.37898	0.43099	0.48415	0.53757
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.59034	0.64158	0.69049	0.73636	0.77865	0.81693	0.85097	0.88068	0.90612	0.92748	0.94506
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.95923	0.97040	0.97900	0.98547	0.99021	0.99360	0.99594	0.99751	0.99854	0.99917	0.99955
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.99977	0.99989	0.99995	0.99998	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=23. Moreover P(I_12≤23)=0.10323>0.05. Therefore, the tested hypothesis should not be rejected at a significance level of 0.05. During a pandemic, the s parameter changes insignificantly. It also means that the correlation between the examined instruments changes insignificantly. As NASDAQ includes high-tech stocks, the result does not necessarily support the conclusions of the DJIA analysis. The change happened in one direction but turned out to be insignificant. Further observation of these instruments seems desirable. GOLDa21- tested hypothesis:

3. $H_0: s = 0.95142$ versus $H_1: s > 0.95142$. The distribution of the number of inversions is presented below. Since we are analyzing the right-tailed test, the values of the right-tail of the distribution will be presented.

The observed value of the test statistic is I_12^*=33. Moreover:

P(I_12≥33)=P(I_12>32)=0.49622>0.05. Therefore, the tested hypothesis should not be rejected at a significance level of 0.05. During a pandemic, the s parameter changes insignificantly. It also means that the correlation between the examined instruments changes insignificantly. Gold is traditionally an instrument recommended in times of crisis. Both instruments are less correlated in the period of a pandemic than in the period preceding the crisis. The change is not statistically significant. Increasing the exposure to both instruments may be desirable when a significant decrease in correlation is identified.

Table 9. *Values of right tails of PDF for* $s = 0.95142$

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1.00	1.00	1.00	1.00	1.00	0.99999	0.99997	0.99993	0.99984	0.99968	0.99938	0.99886
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.99800	0.99665	0.99460	0.99159	0.98730	0.98137	0.97339	0.96293	0.94955	0.93281	0.91233
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.88779	0.85897	0.82580	0.78832	0.74675	0.70149	0.65307	0.60218	0.54960	0.49622	0.44293
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.39064	0.34021	0.29240	0.24785	0.20706	0.17037	0.13798	0.10989	0.08599	0.06606	0.04977
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.03673	0.02652	0.01870	0.01287	0.00862	0.00560	0.00353	0.00215	0.00126	0.00071	0.00038
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.00019	0.00009	0.00004	0.00002	0.00001	1.95E-06	5.33E-07	1.16E-07	1.79E-08	1.48E-09	1.48E-09

Source: *Own calculation.*

3. EURUSD- tested hypothesis: $H_0: s = 0.98153$ versus $H_1: s < 0.98153$. The distribution of the number of inversions is presented below.

Table 10. *Values of PDF for* $s = 0.98153$

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
0.0000	0.0000	0.0000	0.0000	0.0000	0.00001	0.00004	0.00009	0.00019	0.00039	0.00075	0.00137
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.00238	0.00396	0.00635	0.00982	0.01473	0.02145	0.03041	0.04208	0.05688	0.07524	0.09752
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.12400	0.15483	0.19004	0.22950	0.27289	0.31975	0.36947	0.42131	0.47442	0.52791	0.58086
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.63239	0.68168	0.72803	0.77087	0.80976	0.84446	0.87485	0.90098	0.92303	0.94126	0.95605
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.96779	0.97691	0.98383	0.98895	0.99265	0.99526	0.99703	0.99821	0.99896	0.99942	0.99969
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.99984	0.99993	0.99997	0.99999	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=24. Moreover P(I_12≤24)=0.15484>0.05. Therefore, the tested hypothesis should not be rejected at a significance level of 0.05. During a pandemic, the s parameter changes insignificantly. It also means that the correlation between the examined instruments changes insignificantly. The insignificant increase in the correlation suggests reduced exposure to the analyzed values when significant changes are observed. USDJPY-tested hypothesis:

4. $H_0: s = 1$ versus $H_1: s > 1$. The distribution of the number of inversions is presented below.
Since we are analyzing the right-tailed test, the values of the right-side tails of the distribution will be presented.

Table 11. *Values of right tails of PDF for $s = 1$*

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1.0000	1.0000	1.0000	1.0000	1.0000	0.99999	0.99998	0.99995	0.99988	0.99975	0.99951	0.99910
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.99841	0.99731	0.99562	0.99311	0.98951	0.98447	0.97763	0.96857	0.95684	0.94202	0.92370
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.90153	0.87521	0.84459	0.80965	0.77049	0.72740	0.68082	0.63135	0.57971	0.52672	0.47328
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.42029	0.36865	0.31918	0.27260	0.22951	0.19035	0.15541	0.12479	0.09847	0.07630	0.05798
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.04316	0.03143	0.02237	0.01553	0.01049	0.00689	0.00438	0.00269	0.00159	0.00090	0.00049
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.00025	0.00012	0.00005	0.00002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=36. Moreover P(I_12≥36)=P(I_12>35)=0.36865>0.05. Therefore, the tested hypothesis should not be rejected at a significance level of 0.05. During a pandemic, the s parameter changes insignificantly. It also means that the correlation between the examined instruments changes insignificantly. Also, in this case, an insignificant increase in the correlation suggests a reduction in exposure to the analyzed values when significant changes are observed.

4. CRUDE OIL- tested hypothesis:
$H_0: s = 1.00446$ versus $H_1: s < 1.00446$. The distribution of the number of inversions is presented below.

Table 12. *Values of PDF for $s = 1.00446$*

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1.80E-09	2.17E-08	1.40E-07	6.40E-07	2.33E-06	0.00001	0.00002	0.00005	0.00011	0.00022	0.00044	0.00081
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.00144	0.00245	0.00400	0.00631	0.00965	0.01434	0.02073	0.02925	0.04030	0.05434	0.07178
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.09298	0.11825	0.14778	0.18163	0.21974	0.26186	0.30759	0.35638	0.40754	0.46026	0.51367
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.56687	0.61894	0.66905	0.71644	0.76048	0.80068	0.83671	0.86842	0.89580	0.91898	0.93821
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.95383	0.96625	0.97589	0.98320	0.98861	0.99249	0.99521	0.99704	0.99825	0.99900	0.99946
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.99972	0.99987	0.99994	0.99998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=26. Moreover P(I_12≤26)=0.18163>0.05. Therefore, the tested hypothesis should not be rejected at a significance level of 0.05. During a pandemic, the s parameter changes insignificantly. It also means that the correlation between the examined instruments changes insignificantly.

Interestingly, in this case, a change in the direction of the correlation was observed. Here are investments related to the real economy and BTC, which are often assessed as speculative investments. The obtained result does not confirm this yet, but leaving the high exposure in both investments does not improve the portfolio risk. Both analyzed pairs are worth further observation.

5. Nikkei 225- tested hypothesis:
   $H_0: s = 0.97784$ versus $H_1: s < 0.97784$. The distribution of the number of inversions is presented below.

Table 13. *Values of PDF for* $s = 0.97784$

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
4.32E-09	5.07E-08	3.19E-07	1.42E-06	5.07E-06	0.00002	0.00004	0.00010	0.00021	0.00043	0.00082	0.00149
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.00258	0.00428	0.00683	0.01053	0.01574	0.02285	0.03230	0.04454	0.06003	0.07916	0.10229
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.12967	0.16144	0.19758	0.23792	0.28213	0.32969	0.37996	0.43217	0.48546	0.53893	0.59166
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.64279	0.69151	0.73715	0.77917	0.81718	0.85096	0.88044	0.90569	0.92691	0.94440	0.95853
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	`	0.97835	0.98488	0.98971	0.99318	0.99561	0.99726	0.99835	0.99904	0.99947	0.99972
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.99986	0.99993	0.99997	0.99999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=22. Moreover P(I_12≤22)=0.10229>0.05. Therefore, the tested hypothesis should not be rejected at a significance level of 0.05. During a pandemic, the s parameter changes insignificantly. It also means that the correlation between the examined instruments changes insignificantly. The increase in correlation, in this case, turned out to be insignificant. Nevertheless, a similar tendency can be observed as in the US indexes.

6. S&P500- tested hypothesis:
   $H_0: s = 1.00223$ versus $H_1: s < 1.00223$. The distribution of the number of inversions is presented below.

Table 14. *Values of PDF for* $s = 1.00223$

0	1	2	3	4	5	6	7	8	9	10	11
1.94E-09	2.33E-08	1.50E-07	6.85E-07	2.49E-06	7.67E-06	2.07E-05	0.0001	0.0001	0.0002	0.0005	0.0009
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.00151	0.00257	0.00418	0.00659	0.01006	0.01492	0.02154	0.03032	0.04171	0.05614	0.07401
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.09570	0.12149	0.15156	0.18596	0.22460	0.26720	0.31336	0.36250	0.41391	0.46677	0.52020
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.57330	0.62516	0.67496	0.72195	0.76551	0.80520	0.84069	0.87185	0.89869	0.92137	0.94014
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.95536	0.96742	0.97677	0.98385	0.98907	0.99281	0.99542	0.99718	0.99833	0.99905	0.99949
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.99974	0.99993	0.99997	0.99999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=19. Moreover P(I_12≤19)=0.03032<0.05. Therefore, the tested hypothesis should be rejected. As a result of the pandemic, the s parameter (and probability of inversion) decreased significantly. It also means that the correlation between the examined instruments has increased significantly. Contrary to the Nikkei 225, correlation increased significantly. The investment risk associated with pandemic makes stocks investments similar to speculative investments.

7. USDGBP- tested hypothesis:
   $H_0: s = 0.97487$ versus $H_1: s > 0.97487$. The distribution of the number of inversions is presented below. Since we are analyzing the right-tailed test, the values of the right-hand tails of the distribution will be analyzed.

Table 15. *Values of PDF for* $s = 0.97487$

0	1	2	3	4	5	6	7	8	9	10	11
4.75E-09	5.57E-08	3.49E-07	1.56E-06	5.52E-06	1.66E-05	4.37E-05	0.0001	0.0002	0.0005	0.0009	0.0016
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	0.00275	0.00455	0.00724	0.01114	0.01660	0.02403	0.03389	0.04661	0.06265	0.08242	0.10624
	23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	0.13436	0.16689	0.20377	0.24482	0.28967	0.33777	0.38846	0.44094	0.49435	0.54778	0.60031
	34	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
	0.65108	0.69932	0.74438	0.78573	0.82302	0.85606	0.88481	0.90936	0.92993	0.94683	0.96044
	45	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
	0.97118	0.97945	0.98569	0.99029	0.99358	0.99588	0.99744	0.99846	0.99911	0.99951	0.99974
	56	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
	0.99987	0.99993	0.99997	0.99999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=54. Moreover, P(I_12≤53)=0.99911. Thus P(I_12>53)=0.00089. Therefore, the tested hypothesis should be rejected. As a result of the pandemic, the s parameter (and probability of inversion) increased significantly. It also means that the correlation between the examined instruments has decreased significantly. In this case, analysis is more difficult. In addition to the pandemic crisis, perturbations related to Brexit have overlap. The result suggests that the relationship of short positions in USDGBP is clearly, significantly negative. The result may be surprising, but it should be remembered that the shock associated with Brexit has been discounted earlier.

8. GBPEUR- tested hypothesis:
   $H_0: s = 1.05364$ versus $H_1: s < 1.05364$. The distribution of the number of inversions is presented below.

Table 16. *Values of PDF for* $s = 1.05364$

0	1	2	3	4	5	6	7	8	9	10	11
3.46E-10	4.36E-09	2.93E-08	1.40E-07	5.34E-07	1.72E-06	4.87E-06	0.0000	0.0000	0.0001	0.0001	0.0003
	12	13	14	15	16	17	18	19	20	21	22
	0.00047	0.00083	0.00141	0.00232	0.00370	0.00573	0.00863	0.01269	0.01822	0.02558	0.03517
	23	24	25	26	27	28	29	30	31	32	33
	0.04740	0.06269	0.08144	0.10399	0.13061	0.16147	0.19662	0.23596	0.27923	0.32600	0.37571
	34	35	36	37	38	39	40	41	42	43	44
	0.42763	0.48096	0.53478	0.58818	0.64023	0.69006	0.73692	0.78018	0.81935	0.85414	0.88442
	45	46	47	48	49	50	51	52	53	54	55
	0.91022	0.93173	0.94926	0.96320	0.97400	0.98216	0.98813	0.99237	0.99528	0.99720	0.99841
	56	57	58	59	60	61	62	63	64	65	66
	0.99915	0.99993	0.99997	0.99999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: *Own calculation.*

The observed value of the test statistic is I_12^*=21. Moreover P(I_12≤19)=0.02558<0.05. Therefore, the tested hypothesis should be rejected. As a result of the pandemic, the s parameter (and probability of inversion) decreased significantly. It also means that the correlation between the examined instruments has increased significantly. It seems that the influence of Brexit is dominant here. The result suggests that the relationship of short positions in GBPEUR is significantly negative. The result may be surprising, but it should be remembered that the shock associated with Brexit has been discounted earlier. It seems that short positions in GBP appear to be healing for the portfolio.

4. Conclusions

The research has shown a new way of testing the hypothesis about the equality of the Kendall coefficient over two separate periods. The chosen instruments' correlations were tested and divided into two groups, before pandemia and during pandemia.

During pandemia, increases correlation between BTC/USD and stock indexes. In the case of DJIA and S&P500 increase proved to be statistically significant. In the case of NASDAQ and NIKKEI-225 that is worth observing the future correlation because the sample is small (p-value, about 0.10 and n=12). BTC/USD and stock indexes correlation increase. The strengthening of the correlation of BTC and stock indices may indicate the growing role of BTC. BTC is more and more often mentioned as an alternative to traditional investments. Many countries are considering the creation of local digital currencies like BTC. Financial markets are also speculating on the possibility of replacing the US dollar with a digital currency in commodity quotes. The pandemic period is conducive to changes in investment directions, and the order in which BTC is heading is worth further observation.

The most significant increase in absolute value was the increase for the pair USDGBP. The change was from a level of about 0.1 to about -0.6. That means that for the reverse pair, GBP/USD dependence from BTC-USD is positive. Investments in GBP and BTC have a positive correlation. During pandemia, there is a significant change in the direction and strength of correlation. The difference in the order of dependence for this pair may be related to Brexit. But the most recent adaptations of leadership came during the pandemic period when the effects of Brexit have likely already been discounted. The accompanying increase in uncertainty, however, supports the GBP.

The significant change of direction and strength correlation (positive) also concerns pair GBP/EUR. Investments in GBP and EUR are interested in investors investing in EUR. That process was started in pandemia.

The changes of correlation for other pairs are not significant. The most stable and not connected with pandemia is the pair USD/JPY. That shows that for JPY investors, USD and BTC are independent. Pandemia, nothing changes in this case.Pandemia influence on gold and crude oil is practically insignificant. Correlation with BTC-USD is low, and pandemia nothing changes.

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