THE DESIGN AND DEVELOPMENT OF

A RESOURCE PACK FOR AN OUTDOOR MATHEMATICS TRAIL

FOR YEAR 10

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L-Università ta' Malta

FACULTY/INSTITUTE/CENTRE/SCHOOL FACULTY OF EDUCATION

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ABSTRACT

The Design and Development of a Resource Pack for an Outdoor Mathematics Trail for Year 10

The aim of this dissertation is to design and develop an outdoor mathematics trail resource for students in Year 10 at SEC Level 3-3*(Ext). A mathematics trail is an organized route which gives students a first-hand experience to apply mathematical knowledge in a way that is real and hands on. The location chosen for this trail is Birgu as it offers several mathematically rich sites that are also safe for students to access. Other aspects, such as the historical element, give students a cross-curricular experience. The trail consists of several tasks covering various mathematical topics where students can investigate, conjecture, estimate, measure, and calculate. Teachers can use this resource to stimulate students' appreciation towards the utilitarian, aesthetic, and communication aspects of mathematics in an outdoor setup. In addition, the resource can also be used as one of the coursework assignments (or school-based assessment) in the new Mathematics SEC Syllabus 2025. The resource consists of two packs – a student's pack and a teacher's pack. The difference between these two packs is mainly that the teacher's pack also includes a teacher's guide, the expected working and answer of each task. It also includes additional information related to each station such as the Learning Outcomes, ideas for preparatory work, and difficulties students may encounter whilst carrying out the task. The instructional design of the resource is based on Gagne's theory of instruction, whereas Pettersson's and Smaldino's principles provide guidelines for the process activities and information design. Suggestions for further research, namely on digital mathematical trails and gamification of mathematical trails, are provided in the concluding chapter.

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Keywords:

Mathematics Trail	**Outdoor Education**	**Resource Pack**
Mathematics	**Coursework**	**Year 10**

STATEMENT OF AUTHENCITY

I, Celine Vassallo Grant, hereby declare that I am the author of this dissertation entitled:

The Design and Development of

a Resource Pack for an Outdoor Mathematics Trail

for Year 10

Being presented in part fulfilment of the requirements for the Master in Teaching and Learning (MTL) in Mathematics at the University of Malta, I declare that the material presented in this dissertation is my own work, and all the sources that were used and quoted have been acknowledged by full references.

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Celine Vassallo Grant

August 2023

DEDICATION

to my daughters Sophie and Emily

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LIST OF ABBREVIATIONS

DfES	Department for Education and Skills
DQSE	Directorate for Quality and Standards in Education
GPS	Global Positioning System
MATSEC	Matriculation and Secondary Education Certificate
MCM	MathCityMap
NCF	National Curriculum Framework
NCTM	National Council of Teachers of Mathematics
SEC	Secondary Education Certificate
ZPD	Zone of Proximal Development

Chapter 1

Introduction

Chapter 1: Introduction

1.0 Introduction

This project-based dissertation focuses on the design and development of a mathematics trail for Year 10 students. Considering the local trails that are available to the public on websites, as well as to the trails carried out by schools and then reported by the media, it seems that, locally, the mathematics trails are more popular at primary level than at secondary level. A possible reason for this is that teachers might associate hands-on experiences with younger students or students of lower ability. However, Cockcroft (1982)[1] discusses the importance of the three stages of mathematical development, i.e., passing from the handling of actual objects to the diagrammatic or pictorial representation and lastly to the symbolic representation. These modes of representation, coined by Bruner, helps students understand "any idea, or problem, or body of knowledge" (Bruner, 1966, p. 44). Cockcroft (1982) further explains that these stages are important for all students and is thus "a mistake to suppose that there is any particular age at which children no longer need to use practical materials or that such materials are needed only by those whose attainment is low" (p. 85).

This chapter begins by a recount of my background as a student and as a student teacher. It then proceeds to describe how the MTL course transformed my ideas on how mathematics should be learnt and taught. The subsequent section highlights the importance of mathematics as a subject. Section 1.3 explains what a mathematics trail entails, its origins, and its role in the curriculum as well as the Mathematics SEC syllabus. Section 1.4 outlines the rationale of this study, in particular the aims and choice of location. This chapter concludes with a brief description of the structure of the remaining chapters.

[1] The references in this dissertation follow the APA 7th edition referencing style.

1.1 Motives Influencing the Choice of the Research Question

My learning experience as a student of mathematics together with the knowledge I now have of how students should learn mathematics are the basis of my research question, as shall be explained in the following sections.

1.1.1 My Experience as a Student of Mathematics

During the primary and secondary years, my learning experiences as a student of mathematics, were very much teacher centred. Teachers normally used the 'talk and chalk' approach, and students were expected to 'sit and get'. Thus, in a lesson, the teacher typically stayed in front of the class, at times even on a platform, to explain the content of the lesson, and then use the chalk and blackboard, the two main resources used in the lessons, to work out several examples. Homework typically consisted of numerous similar problems from the textbook. I do not remember any moments where the teachers provided opportunities for students to appreciate mathematics as being practical, useful, and linked to what people need on an everyday basis. Neither did the teachers highlight the importance of mathematics in understanding the world around us, such as observing patterns in the environment, finding relationships between mathematical objects that have been derived from physical objects found in the space we observe, and reasoning logically in real-life situations. A classic example of this is the *pi* ratio. I learnt that *pi* is an irrational number approximately equal to 3.14 and used it in an endless amount of problems. However, I was never taught that it is the ratio of the circumference to the diameter of any circle, and that I could actually estimate it using real measurements.

On the other hand, in Geography (which I studied as an option subject), we were given the opportunity to go out on organized field trips (as part of the coursework) where we observed some of the content being taught in class, as well as measured and estimated aspects of the natural and human environment that were covered in the syllabus. For example, we used quadrats to estimate vegetation cover, or the tape measure to measure fissures in rocks, and then drew graphs of the collected data when back at home. Apart from being rich in content, these fieldtrips increased the enjoyment towards the subject and made it more relevant. Ultimately, it is what I remember the most.

1.1.2 My Experience as a Student Teacher of Mathematics

Twenty-five years have passed since I finished secondary school, and although chalk and blackboards have been replaced with markers and whiteboards, and most of the classrooms are nowadays equipped with interactive whiteboards, when going back to the secondary schools for my field placements, I noticed that little has changed. The traditional teaching method is still very prominent in mathematics classrooms and students are not as active in their learning as the curriculum promotes. My general feeling is that what is done in the classroom is still very much driven by what is assessed in the exams. Rich mathematical experiences where students can manipulate objects, carry out hands-on activities, discuss, create links between mathematical ideas and links with other subject areas, i.e. opportunities for students to enhance their learning and enjoyment towards the subject, are still not given their due importance.

1.1.3 My Transformation through the MTL Journey

Coming from an engineering background and also from all the life experiences in which a deep understanding of mathematics came in very useful, I started the MTL course with the mindset that mathematics is a 'toolkit' that all students should acquire in order to widen their opportunities in life and to become independent citizens.

Throughout this course I have strengthened this position, however, I have completely changed the way I thought about how teaching and learning should be carried out. Now I know that students should be active participants in the process of learning and that they learn mathematics through listening, observing, and doing, as well as reflecting on what they have learnt.

In addition, I now also believe that making use of a variety of resources is of utmost importance to help students achieve a relational understanding of the subject, rather than an instrumental one. From the short teaching experience I had with students, I could sense their desire to have more of these opportunities because they truly appreciated them. Some of the feedback I received from Year 7 and Year 8 students when asked about their favourite lesson during my five weeks of teaching practice includes:

"*My favourite lesson is the one where we measured our desk*" (the measurements were used to find the perimeter),

"*My favourite was tally*" (in this lesson we collected data together),

"*My favourite lesson was the first lesson of percentages*" (in this lesson we discussed the term 'percent' and where we find percentages, such as nutritional information on packages, exam marks, sales etc.),

"*I liked the lesson of volume the most where we filled up the cuboid with 1x1x1cm cubes"*,

"*My favourite lesson was when we found the circumference and diameter of different circular objects*" (the measurements were then used to find the *pi* constant),

"*I feel like the lessons were fun and interactive, as we didn't only do from the notes but we had many fun activities that personally helped me understand more*".

1.1.4 The Research Question

In my secondary years, teachers hardly allowed for any sufficient experiences where I could discuss and do mathematics with fellow students, or where doing mathematics was fun. Luckily for me, I still developed a love towards the subject and found it to be enjoyable. However, several of my fellow students did not share the same attitude towards mathematics. To the contrary, they found it boring, irrelevant, and frustrating. This attitude seems to be still shared amongst several students in secondary years.

I believe that one way how to help students discuss mathematics and increase their enjoyment towards the subject is to create resources that promote this. The first time I came across a mathematics trail was when I started the MTL course, and I was instantly attracted to the idea. Considering my positive experience with fieldworks in another subject, and my wish for students to 'get their hands dirty' and start enjoying mathematics, the desire to learn how to create such an activity for students developed naturally. In fact, the research question that guides this study is:

> "I wish to explore how I can design and develop an outdoor mathematics trail to help Year 10 students apply the mathematics they learn in the classroom."

The choice of the year group is such that students would have already covered all the mathematical content that is needed to carry out this trail. However, a maths trail can be designed for any year group. In view of the reasons stated above, I believe that a maths trail should be carried out at least once every year with every year group, irrespective of whether it is part of the coursework or not.

1.2 The Importance of Mathematics

Maths trails can help students appreciate the importance of mathematics namely the aesthetic, utilitarian, and communication aspects of mathematics. Such aspects will be discussed in the following subsections.

The importance of mathematics dates back to ancient times as several civilisations were driven by a profound curiosity to understand the world around them. Philosophers such as Plato recognized the importance of mathematics and its indispensable role in education, especially for those aspiring to become leaders (Burton, 2011). In fact, it is believed that Plato marked the entrance to his academy with the phrase "Let no one ignorant of geometry enter here" (Burton, 2011, p. 83; Shapiro, 2011, p. 3).

Nowadays, several courses at post-secondary and tertiary levels follow the lead of Plato's phrase at the entrance of his academy and consider mathematics as one of the entry requirements. Wake (2011) describes how mathematics is a performance measure not only of students, but also of schools ('league tables' judging schools by the grades students attain in high-stake exams) and nations ('international comparative studies' such as the Programme for International Student Assessment (PISA)).

In the curriculum of developed countries, mathematics is given more importance than non-compulsory subjects, in order to sustain the economy (Wake, 2011). Locally, this is reflected in the amount of curriculum time assigned to mathematics in both primary and secondary education. In primary years the minimum entitlement assigned to Mathematics is 15% of the curriculum time. This is the same amount of time that is allocated to Maltese, English, and Science & Technology. Similarly, in secondary years the subjects that are given the most importance include Mathematics (12.5%), Science & Technology (12.5%), and Maltese & English (30%), where 12.5% is the equivalent of at least 5 lessons per week and 30% equates to 12 lessons per week (Ministry of Education, 2012).

Following Cockcroft (1982), the reasons why mathematics should be taught in school, can be split into three views - the aesthetic aspect, the utilitarian aspect, and the communication aspect.

1.2.1 The Aesthetic Aspect of Mathematics

Mathematics is "a discipline worthy of study in its own right" (NCTM, 2000, p.264). It provides "a powerful universal language and intellectual toolkit for abstraction, generalization and synthesis" (Smith, 2004, p. 11). Halmos, a professional mathematician, interviewed by Albers, defines mathematics as:

> It is security. Certainty. Truth. Beauty. Insight. Structure. Architecture. I see mathematics, the part of human knowledge that I call mathematics, as one thing – one great glorious thing. (Albers, 1985, p. 127)

Mathematical purists consider mathematics to be "educationally valuable in the development of thought rather than for learning about any applications" and identify mathematics "more as an art than a science" (Westwell, 2011, p. 7).

The aesthetic aspects of mathematics emphasize the study of patterns, the logical reasoning, the abstract nature of the subject, the relationships between mathematical objects, amongst others. Thus, when learning mathematics for its aesthetic aspect, students are given a space for "intellectual challenge, excitement, satisfaction, and wonder" (MATSEC, 2022, p. 4).

1.2.2 The Utilitarian Aspect of Mathematics

Learning of mathematics for its usefulness can be divided into four justifications, according to Smith (2004, pp 11-13):

Mathematics for the citizen - Various aspects in everyday life are becoming "increasingly mathematical and technological" (NCTM, 2000, p.4). Thus, "the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations is important for all learners" (Ministry of Education, 2012, p. 35). Such situations can include budgeting and purchasing decisions (e.g., determining the best buy), measurement and conversion (e.g., cooking measurements), data interpretation (e.g., health-related data), and time management and scheduling (e.g., scheduling appointments). In addition, the Ministry of Education (2012) states that, numeracy, together with literacy and digital literacy, are the "foundations for further learning" (p. 51). Numeracy is the "least basic mathematical skill" (Smith, 2004, p.13), and its acquisition is important for individuals not to end up unemployable, or unable to move up the career ladder, or be socially excluded (Smith, 2004).

Mathematics for the workplace - Mathematics serves as a foundation for a wide range of careers. NCTM (2000) argue that "those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed" (p. 5). Hoachlander (1997) provides examples of how mathematical topics link with the needs of the workplace. Some of the examples include 'geometry' – routinely used by graphic designers, and 'trigonometry' – its principles are used by carpenters, surveyors, navigators, and architects.

Mathematics for science, technology, and engineering – Although several careers necessitate a basic understanding of mathematics, certain occupations place a higher emphasis on mathematical skills and require a more intensive application of mathematical knowledge (NCTM, 2000). These include professions in science, technology, and engineering. According to Smith (2004), people with expertise in these subjects play a crucial role in driving innovation and productivity within a country. This ultimately contributes to the "future strength and success" of the country (Smith, 2004, p. 12).

Mathematics for the knowledge economy – Apart from the scientific advancements and innovation, mathematics is fundamental for developing a skilled workforce in other fields within the knowledge economy, such as business and management studies, economics, banking, and geography (Cockcroft, 1982; MATSEC, 2022). According to Cockcroft (1982), mathematics "is essential to the operations of industry and commerce in both office and workshop" (p. 2). Hoachlander (1997) refers to how 'algebra' permeates the domains of "computing and business modeling, from everyday spreadsheets to sophisticated scheduling systems and financial planning strategies" (p. 135), and that 'statistics' is key in the roles of "economists, marketing experts, pharmaceutical companies, and political advisers" (p. 135).

1.2.3 The Communication Aspect of Mathematics

Several agree that "mathematics originated with the practical problems of counting and recording numbers" and "the earliest and most immediate technique for visibly expressing the idea of number is tallying" (Burton, 2011, pp. 1-2). As mathematics developed over the years, mathematicians created new words, notations, and conventional styles of argument (Morgan, 2011) to provide "a means of communication which is powerful, concise and unambiguous" (Cockcroft, 1982, p. 1).

Mathematics, being a language with its own vocabulary, syntax, and semantics, is at times "compared with learning a foreign language" (Morgan, 2011, p.146), and it can be very confusing for those who are still struggling on how to use it (Morgan, 2011). Cockcroft (1982) explains how the same symbolic notation that enables mathematics to be useful as a means of communication, "can also make mathematics difficult to understand and to use" (pp. 3-4).

"Mathematics can be used to present information in many ways, not only by means of figures and letters but also through the use of tables, charts and diagrams as well as of graphs and geometrical or technical drawings" (Cockcroft, 1982, p. 1). Some examples include 'x' to denote a variable, 'π' to denote the ratio between the circumference and diameter of a circle, '3' to denote a quantity of three, and a distance-time graph to represent a journey.

Thus, teachers can utilize maths trails to emphasize all these three aspects of mathematics as the trail:

- showcases the beauty of mathematics by observing the environment, for example analysing patterns and symmetry – 'aesthetic aspect'.

- engages students in activities that highlight the practical utility of mathematics such as measuring, estimating, calculating, tallying, and interpreting data – 'utilitarian aspect'.
- encourages participants to communicate their mathematical thinking and to articulate their thoughts clearly. It also promotes discussion as part of a group – 'communication aspect'.

1.3 An Outdoor Mathematics Trail

1.3.1 Background of the Outdoor Mathematics Trail

The idea of a mathematics trail developed in the early 1980s in England, with the first publications being those of Lumb in 1980 (Mathematics Trails in Newcastle) and Blair et al. in 1983 (From St Philip's to St Martin's: A Birmingham Maths Trail). The aim of these trails was to generate meaningful mathematics for students, aged between 8 to 12, using the environment (Blair et al., 1983; Lumb, 1980). Then, in 1984, educators Dudley Blane and Doug Clarke developed the first mathematics trail that was intended for families, not just children, during a week of school holiday, around the centre of Melbourne, Australia (Cahyono, 2018; Shoaf-Grubbs et al., 2004). The aim was of popularizing mathematics in a period where a large proportion of the population had a negative attitude towards this subject possibly because of their past experiences at school (Blane, 1989).

The philosophy behind Melbourne's mathematics trail was:

- to "use the environment to simulate the learning of mathematics",
- to "develop an appreciation and enjoyment towards mathematics",
- to extend the work carried out by students in the classroom,

in a way that is non-competitive, "non-threatening" and "above all intended to be fun" (Blane, 1989, p. 126).

Melbourne's trail was a huge success and "caught the imagination of education and the wider community" (Blane, 1989, p. 126). Many others have adapted it and created their own mathematics trails. Many of those creating the trail, referred to as 'trailblazers', are often educators working in the school environment (Shoaf-Grubbs et al., 2004).

1.3.2 Definition of an Outdoor Mathematics Trail

A mathematics trail is a pre-determined trail or route that students follow while carrying out a series of tasks or activities, such as measuring, calculating, estimating, and describing (Cross, 1997). This offers a concrete opportunity for students to "use and apply mathematics already learnt" (Selinger & Baker, 1991, p. 4). The mathematics trail offers "guide points to places where walkers formulate, discuss, and solve interesting mathematical problems" (Shoaf-Grubbs et al., 2004, p. 6), and hence it provides "a variety of contexts in which mathematical ideas can occur" (Selinger & Baker, 1991, p. 4).

1.3.3 Characteristics of an Outdoor Mathematics Trail

An outdoor mathematics trail encourages students:

- to "seek out the mathematics in their environment" and to "see the relevance of mathematics outside the classroom and to appreciate its aesthetic value" (Selinger & Baker, 1991, p. 1). According to the Department for Education and Skills (DfES, 2006), outdoor educational experiences "build bridges between theory and reality" (p. 3) and this can improve students' motivation, involvement, and creativity, amongst other benefits. Mathematics trails are "an excellent opportunity to integrate an outdoor lesson into the mathematics curriculum" (Zender et al., 2020, p. 1).
- to be more autonomous and less dependent on the teacher's prompting and feedback. A local study by Cefai (2007), suggests that students still have "a very limited say in the decisions made in the classroom" and they are "still heavily dependent on their teachers in their learning, with limited opportunities for independent and self-reliant learning" (p. 129). During the trail, students cannot be reliant on their teacher as if they were in the classroom. They need to "make decisions" (Zender et al., 2020, p. 4) such as how to reach the task location, and "follow instructions" (Selinger & Baker, 1991, p. 2) such as how to carry out the task.
- to "gather information and data" and "estimate measurements" (Selinger & Baker, 1991, p. 2) of real-life objects. The questions posed can thus be more "authentic" (Zender et al., 2020, p. 4).

- to use resources such as the navigating compass, tape measure, protractor, and stopwatch, in a way that is real and hands-on. Such resources are normally referred to in textbooks or teachers' explanations, however students are rarely provided with opportunities for practical measurement sessions during lessons. The mathematics trail "complements the classroom treatment of the syllabus" (Blair, 1983, p. 15).
- to connect the products of mathematics i.e., facts, skills and techniques, and concepts, to the processes of mathematics, i.e., solving, investigating, reasoning, and communicating strategies. Tasks in which students need to spot patterns, make conjectures, record data, interpret and discuss results - amongst other processes of mathematics - help them to develop a relational understanding rather than just being limited to an instrumental understanding (Skemp, 2006).
- to "work together" (Selinger & Baker, 1991, p. 3), and hence promote collaboration and teamwork. Groupwork encourages students to listen, talk, describe, and discuss, thus helping them to develop their communicating strategies.
- to appreciate "the cross curricular nature of mathematics" (Selinger & Baker, 1991, p. 3). When mathematics is combined with subjects such as history, geography, physics, art and design, students are given an experience that is enriching and enjoyable.
- to appreciate that "Mathematics is not an arbitrary collection of disconnected items, but has a coherent structure in which the various parts are inter-related" (HMI, 1987, p. 3). Mathematics trails can link topics from various strands in the syllabus.

1.3.4 Mathematics Trail in the Local Curriculum and Syllabus

In the NCF, the Ministry of Education (2012) advocates for teaching pedagogies that are "student-centred" (p. 63). The Mathematics SEC Syllabus 2025[2], which is based on the principles outlined in this curriculum, thus provides a list of approaches that SEC teachers should adopt in order to ensure that "all teaching and learning activities should be student-centred rather than teacher-centred" (MATSEC, 2022, p. 4). Some of these include encouraging students to:

- "Develop an ability to work independently and collaboratively",

[2] This dissertation is based on the Mathematics SEC Syllabus 2025 as was available in April 2022.

- “Develop a positive attitude towards Mathematics”,
- “Apply mathematical knowledge and understanding to solve several real-life ‘applied mathematics’ problems arising from situations in their own lives as well as in society”,
- “Appreciate the interdependence of the different strands of Mathematics”,
- “Develop the ability to use Mathematics across the curriculum”, and
- “Think and communicate mathematically - precisely, logically, and creatively”.

These strategies are in line with the characteristics of a mathematics trail, as discussed in the previous section. Thus, a mathematics trail is a medium that can be used by teachers to adopt these approaches. As Barbosa and Vale (2018) state, “the learning of mathematics is heavily dependent on the teacher and the tasks proposed to students” (p. 183). Thus, teachers are expected to include more than just routine tasks, and therefore a “math trail is an example of a great resource to engage students in a meaningful activity” (Barbosa & Vale, 2018, p. 184).

At secondary level, the Mathematics SEC Syllabus 2025 is based on “conceptual structures” that are grouped in four strands, namely “the ‘Number’, ‘Algebra’, ‘Shape, Space & Measures’ and ‘Data Handling & Chance’ strands” (MATSEC, 2022, p. 2). Very often topics are compartmentalized when taught to students, giving the false impression that they are discrete and unrelated. As already indicated in the previous section, the mathematics trail can link topics from these various strands in the syllabus, and it thus gives students the opportunity to see that “mathematics is whole and not a disembodied collection of topics like ratio, trigonometry and algebra” (Selinger & Baker, 1991, p. 4).

Furthermore, according to the Mathematics SEC Syllabus 2025[3] the scheme of assessment for school candidates sitting for the SEC exam consists of two components - the ‘controlled assessment’ and the ‘coursework’. The ‘controlled assessment’ is a two-hour written exam and amounts to 70% of the total mark, whereas the ‘coursework’ is made up of three assignments carried out between Year 9 and Year 11 and amounts to 30% of the total mark (MATSEC, 2022). One of the coursework assignments being suggested is that of a mathematics trail. The trail at MQF Level 1-2-3 needs to have 30% of the content at Level 1,

[3] The Mathematics SEC Syllabus 2025 was revised in May 2023, and the coursework was replaced with school-based assessment. Nonetheless, one of the assessment methods chosen by the school can still be a mathematics trail.

30% at Level 2 and 40% at Level 3. In addition, the trail needs to include tasks that are to be carried out "on site", as well as tasks that students need to carry out "individually, using the data gathered on site to complete the task" (MATSEC, 2022, p. 94). Each student is to submit the completed trail on an individual basis for assessment purposes.

1.4 Rationale of the Study

1.4.1 Aim

The aim of this dissertation is to design and develop an outdoor mathematics trail resource pack for students in Year 10 at SEC Level 3-3*(Ext). The resource will consist of two packs – one aimed for students and another one for teachers. The teachers' pack will be similar to the students' pack however it will also include expected answers, guidelines of how to use the pack, instructions for students, and any other important information that teachers need to know about the trail.

The idea to develop this resource pack came from the fact that, unfortunately, "mathematics is a subject that polarizes students" (Gurjanow et al., 2020, p. 103), with many having a negative attitude towards mathematics because they associate it with "despair", "anxiety", "depression", "lack of emotions and discussions" (Kollosche, 2018, as cited in Gurjanow et al., 2020, p. 103) and perceive the subject as "too difficult", "boring" and "not useful in life" (Brown et al., 2008, p. 6). The aim is thus to create a resource that can help students increase their appreciation towards the utilitarian (practical), aesthetic (beauty), and communication aspects of mathematics; consolidate and give meaning to the mathematics taught in the classroom; and foster the idea that mathematics can be fun.

Additionally, the resource aims at helping students to feel connected with mathematics even after completing the trail. As the Mathematics Association (1974), cited in Blair et al. (1983), aptly puts it "after an interesting experience there is an urge to talk about it and this can deepen the interest and also help to communicate it" (p.14).

1.4.2 The Mathematics Trail Location

Birgu is one of the oldest localities in Malta and is hence rich in various architectural features. It thus provided the opportunity to create a trail that encompasses several mathematical topics as well as other subjects in the curriculum such as History and Physics, thereby giving students a cross-curricular experience.

Generally, it seems that the most popular site for a mathematics trail for students in secondary years is Mdina, possibly because people are more familiar with this city rather than with the gem that Birgu is. Mdina may be deemed as more suitable due to its central location and due to it being considered safer for students because of a somewhat limited vehicle access into the city. However, Birgu, having been built on a narrow peninsula and being surrounded by bastions is also very self-contained, away from busy main roads and the chaos normally associated with a city. In planning out this trail, efforts were made to find the safest spaces in Birgu for students to work in, and the city lends itself adequately for this purpose providing a good number of pedestrian zones.

Another advantage was that the distance between the various stations was relatively short, making the total walking distance of the trail reasonable and comfortable for students to withstand. In addition, the regeneration of Birgu over the last two decades, such as that of Birgu ditch and Couvre Porte Belvedere, provided suitable stop points for students to carry out the trail tasks, including surfaces that could be used as seating areas.

Therefore, taking the above reasons into consideration together with the fact that this location is not such a preferred option for a mathematics trail as yet, resulted in Birgu being the most attractive choice.

1.5 Outline of the Remaining Chapters

The following is the outline of the remaining chapters:

Chapter 2 focuses on the theories underpinning the development of the Resource Pack. Theories of experiential learning by Dewey, Bruner and Dale will be discussed in the first part of this chapter. Subsequently, the focus will be on Piaget’s and Vygotksy’s perspectives on constructivism. Cooperative learning, outdoor education and the purposes of mathematics trails will be also reviewed. Finally, Chapter 2 considers Gagne's principles on how best to approach teaching and learning, and also the design principles for material consisting of text and images.

Chapter 3 outlines the methodology in the design and development of the Resource Pack. The first part of this chapter will focus on the development phase in terms of choice of location, selection of tasks, and ideas from other existing trails. The second part of this chapter will discuss how the instruction and information design principles identified in the literature review were used to develop the Resource Pack. To conclude, this chapter will outline the type of activities developed in Resource Pack.

Chapter 4 discusses the strengths and limitations of this project. It also offers suggestions for further research that can be carried out in line with this research project.

Chapter 2

Literature Review

Chapter 2: Literature Review

2.0 Introduction

As discussed in the previous chapter, mathematics is important for its aesthetic, utilitarian and communication aspects. In order to foster students' appreciation towards these three dimensions, it is necessary to provide them with the right experiences, as will be discussed in this chapter.

The sections in this chapter look into the theories that inspired and illuminated the work in this dissertation. Section 2.1 gives an overview of the learning theories on hands-on experiences in education. Therefore, it focuses on the main elements of an educative experience, the three stages for an effective learning experience, and the progression of the learning experiences. Section 2.2 then looks at how students construct knowledge and the importance of social experiences. Section 2.3 discusses the characteristics, advantages, and challenges of outdoor educational experiences. Section 2.4 elaborates on experiential learning in mathematics, focusing mostly on outdoor mathematics trails. Following the underlying theories of experiential learning, the second part of this chapter focuses on the principles that underpin the design of instructional material. Section 2.5 focuses on both the instructional design and the visual design of the trail as a Resource Pack. Finally, Section 2.6 provides a conclusion of the literature review.

2.1 Learning Theories on Experience in Education

Various prominent educational theorists have discussed the role of 'experience' in education. Of particular interest in relation to this dissertation include those by Dewey, Bruner, and Dale.

2.1.1 Dewey's Perspective

John Dewey, a philosopher and an educational reformer, believes that experiences play an important role in the learning process and that experiences are educative if they are "purposeful and holistic" and "developed with forethought and planning" (Garrett, 1997, p. 130).

2.1.1.1 Education and Miseducation

In his book, "Experience and Education", Dewey (2015) states that there is an "organic connection between education and personal experience" (p. 25). However, he also argues that not all experiences are educative:

> The belief that all genuine education comes about through experience does not mean that all experiences are genuinely or equally educative. Experience and education cannot be directly equated to each other. For some experiences are miseducative. Any experience is miseducative that has the effect of arresting or distorting the growth of further experience. (Dewey, 2015, p. 25)

Examples of miseducative learning experiences include students being blocked from sharing ideas, students feeling demotivated because of the way they are learning, and students learning with an attitude of "automatic demand" thereby limiting their capability to adapt and respond effectively to new situations or challenges (Dewey, 2015, p. 37). On the other hand, an educative experience "arouses curiosity", "strengthens initiative" and "is a moving force" (Dewey, 2015, p. 38), thereby creating continuity.

2.1.1.2 The Experiential Continuum

According to Dewey (2015), it is not the experience that counts but the "quality of experience" (p. 27). Dewey (2015) argues that such quality has two important aspects – its immediate aspect and its influence on future experiences. Dewey (2015) describes this as "the experiential continuum" (p. 33) as every experience "takes up something from those which have gone before and modifies in some way the quality of those which come after" (p. 35).

2.1.1.3 The Meaningful Experience

In addition to the quality of experience, Dewey's perspective is also about the 'meaning' integrally linked to the experience. For Dewey, the action of 'doing' without any thoughtful reflection is not an experience. As Dewey (2012) states "no experience having a meaning is possible without having some element of thought" (p. 155). Thinking about and reflecting upon the experience can be done, according to Schon (1987), at the end of the activity - reflection "on-action" ("thinking back"), or while still immersed in the experience - "reflection-in-action" ("action present") (p. 26). Schon (1987) suggests that at times it is more preferable and effective to "reflect-in-action" (p. 26) as it allows for an active reshaping of the situation at hand.

2.1.1.4 Dewey's Perspective - Implications for Practice

As Dewey points out, every experience influences, to some extent, for better or worse, attitudes and decisions about further experiences. This has strong implications on the type of experiences teachers choose for their students. Selecting experiences that ensure that students have "experiences that live fruitfully and creatively in subsequent experiences" (Dewey, 2015, p. 28) is critical. For example, a mathematics trail, is an educative experience: if the assigned tasks are at the appropriate difficulty level; if the tasks are varied and engaging; and, if adequate preparation is carried out prior to the trail so that students have a clear picture of what is expected of them. Otherwise, if students feel helpless, demotivated, or overwhelmed, the trail results in a miseducative experience.

Furthermore, 'doing' the activity itself does not equate to an educational experience. Students need to understand what they are doing by thinking and reflecting. "For Dewey thought and action are not separate entities but are unified in experience" (Ord & Leather, 2011, p. 19).

2.1.2 Bruner's Perspective

Jerome Bruner, a psychologist, influenced by Dewey's perspective on 'experience' in education, added another dimension to the teaching and learning through the formulation of the theory of cognitive growth (Schunk, 2012).

2.1.2.1 Theory of Cognitive Growth

Bruner (1966) advocates three ways by which the "domain of knowledge" can be represented, and these include (pp. 44-45):

- **Enactive representation** – actions on objects or objects that can be manipulated.
- **Iconic representation** – visuals, pictures, graphics, or icons.
- **Symbolic representation** – words, numbers, or symbols.

These modes of representations can be viewed as a sequential movement from the enactive to the iconic representation and finally to the symbolic representation. Figure 1 shows these modes of representation, using two fifths as an example.

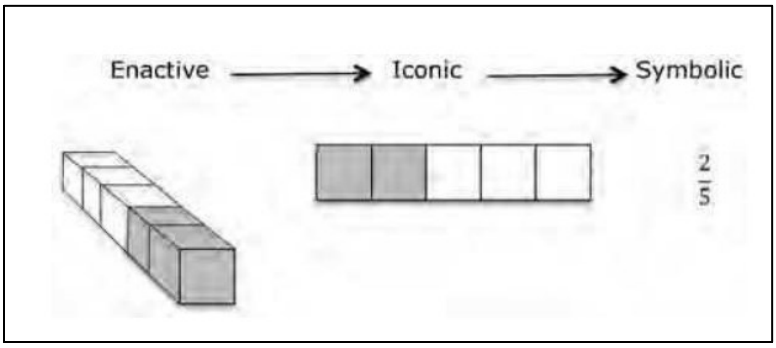


Figure 1: Bruner's modes of representation [4]

Bruner promotes teaching in which students get to think mathematically in all of these stages rather than producing "little living libraries on that subject" (Bruner, 1966, p. 72). Imparting knowledge should not be the purpose of education but rather a means that facilitates students' thinking and problem-solving skills, and that develops symbolic thinking (Bruner, 1966).

2.1.2.2 Bruner's Perspective - Implications for Practice

Gurganus (2017) explains how students who are not given an "adequate experience" (p. 109) in the enactive and iconic stages, usually have difficulties to understand new mathematical concepts and procedures, because such stages act as bridges to the demands of the abstract stage. An adequate experience comprises also having the right amount of time at each stage, for example by not moving too quickly from the enactive mode into the iconic and symbolic modes, or by not allowing the use of concrete manipulatives far longer than required (Gurganus, 2017, p. 110). This highlights the importance of using this enactive-iconic-symbolic

[4] *Source: Gurganus, 2017, p. 109.*

sequence when teaching new mathematics concepts and skills, "even with higher-level topics such as algebra and trigonometry" (Gurganus, 2017, p. 110).

According to Zender and Ludwig (2019), mathematics trails are strongly supported by Bruner's theory of cognitive growth. They explain how, typically, at school, the lessons are mostly at the symbolic level, supported by illustrations on textbooks and whiteboards at the iconic level. When students go out on a mathematics trail, "in addition to the iconic and symbolic level they have experienced in the classroom, they can get their hands-on mathematics in an enactive way" (Zender & Ludwig, 2019, para. 4), for example by seeing and measuring real objects. Hence, one way of including the enactive element in mathematics is the trail.

2.1.3 Dale's Perspective

2.1.3.1 The Fourfold Organic Process

Edgar Dale, a philosopher and educator, expands on Dewey's ideas regarding the role of experience in education. Dale (1969) identifies four elements that need to be considered when thinking about a meaningful educational experience, referred to as the "fourfold organic process" (p. 42), and these include:

- **Needs** - refers to the learning goals;
- **Experiences** - refers to the students' backgrounds and developmental stages;
- **Incorporation** of the experience - to use previous knowledge to assimilate and accommodate new knowledge; and
- **Use** - to have opportunities to try out and practise the new knowledge.

2.1.3.2 The Cone of Experience

In order to explain the different ways experiential learning can be included in the curriculum (Garrett, 1997), as well as to represent the relationship between the different types of learning experiences, Dale developed a visual representation named the 'Cone of Experience'.

Dale's Cone, as shown in Figure 2, illustrates the "progression of learning experiences, from direct, firsthand participation to pictorial representation and on to purely abstract, symbolic expression" (Dale, 1969, p. 108), i.e., the progression (a continuum) from the concrete to the abstract. It progresses from broad to narrow, in order to represent the increasing level of abstraction. Dale (1969) categorized these learning experiences into Bruner's Theory of Instruction, i.e., 'enactive' (learning by doing), 'iconic' (learning by observation) and 'symbolic' (learning by abstraction).

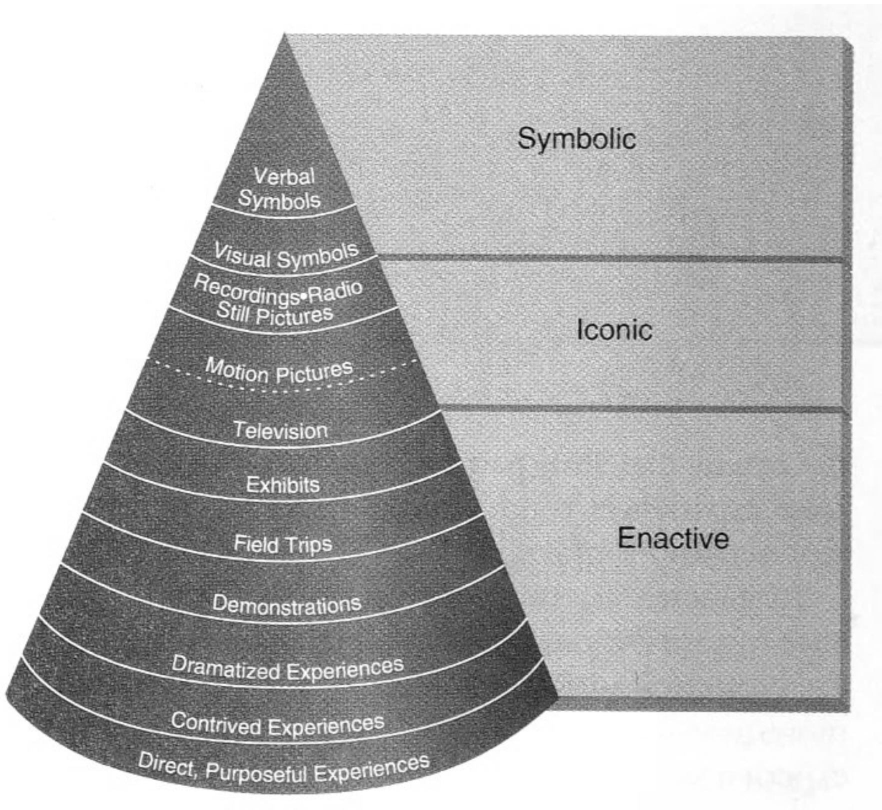


Figure 2: Bruner's theory of instruction superimposed on Dale's Cone of Experience [5]

In the enactive mode, the broad base of the Cone, the experiences are direct, purposeful, real, concrete, and are "the foundation of their [learners] permanent learning" (Lee & Reeves, 2007, p. 57). Dale (1972) explains how "to experience is to test, to try out. It means be a concerned participant, not a half-attentive observer" (p. 4). Learners in this mode are thus normally active participants. Such experiences can include hands-on activities, field trips, and experiments. A mathematics trail is thereby an example of a concrete learning experience as it incorporates all these elements.

According to Dale's perspective, the concrete experiences will help learners give meaning to the instructional experiences at the top part of the Cone, where the verbal and visual symbols are highly abstract (for example symbols representing numbers or words), as long as "they

[5] *Source: Smaldino et al., 2005, p. 12.*

had built up a stock of more concrete experiences to give meaning to the more abstract representations of reality” (Smaldino et al., 2005, p. 11).

2.1.3.3 Dale’s Perspective - Implications for Practice

All experiences in Dale’s Cone, a concrete-to-abstract continuum, are important in the learning process, and are “fluid, extensive, and continually interacting” (Dale, 1969, p. 110). Thus, Dale argues that too much reliance on one type of experience should be avoided, even in the case of concrete experiences, as they can be the costliest in terms of money, time, and effort. In addition, too much of them “may actually obstruct the process of meaningful generalization” (Dale, 1969, p. 130). Smaldino et al. (2005) describe the decision between the concrete experiences and time constraints as a “trade-off”, and teachers need to continually make these “trade-off” decisions (p. 10). Providing learners with a balanced combination of concrete, iconic and abstract learning experiences is hence key.

2.2 Constructivism and Cooperative Learning

2.2.1 Constructivism

Constructivism is an epistemology, i.e., a theory of how humans acquire knowledge. The basic principle of constructivism is that learners actively construct their own knowledge, rather than passively receive or accept information from others (Cahyono & Ludwig, 2018; McLeod, 2023b).

For Dewey, teaching and learning is a continuous process of reconstruction of experience, where experience, involves the “continuity and interaction between the learner and what is learned” (Dewey, 2015, p. 10). Even though Dewey’s approach to education intersects with constructivist thinking, the two major central ideas of constructivism are ‘cognitive constructivism’ based on the work of Piaget, and ‘social constructivism’ based on the work of Vygotsky.

2.2.1.1 Cognitive Constructivism – Piaget's Perspective

Jean Piaget, a psychologist, concludes that our thinking process passes through a fixed sequence of levels of thinking, from birth to maturity, referred to as the "stages of cognitive development" (Schunk, 2012, p. 237). These stages involve the "sensorimotor" stage (0 to 2 years), the "pre-operational" stage (2 to 7 years), the "concrete operational" stage (7 years to adolescence) and the "formal operations" stage (adolescence to adulthood) (Schunk, 2012, p. 237), as illustrated in Table 1. The order of this sequence is invariant, according to Piaget, however, the ages are just an indication of how a child of a certain age thinks, since it varies from person to person (Schunk, 2012).

STAGE	APPROXIMATE AGE	CHARACTERISTICS
Sensorimotor	0–2 years	Learns through reflexes, senses, and movement—actions on the environment. Begins to imitate others and remember events; shifts to symbolic thinking. Comes to understand that objects do not cease to exist when they are out of sight—object permanence. Moves from reflexive actions to intentional activity.
Preoperational	Begins about the time the child starts talking, to about 7 years old	Develops language and begins to use symbols to represent objects. Has difficulty with past and future—thinks in the present. Can think through operations logically in one direction. Has difficulties understanding the point of view of another person.
Concrete Operational	Begins about first grade, to early adolescence, around 11 years old	Can think logically about concrete (hands-on) problems. Understands conservation and organizes things into categories and in series. Can reverse thinking to mentally "undo" actions. Understands past, present, and future.
Formal Operational	Adolescence to adulthood	Can think hypothetically and deductively. Thinking becomes more scientific. Solves abstract problems in logical fashion. Can consider multiple perspectives and develops concerns about social issues, personal identity, and justice.

Table 1: Piaget's stages of cognitive development [6]

He also identifies four factors that, interact with, and hence influence, the thinking process, and these include the "biological maturation, activity, social experiences, and equilibration" (Woolfolk, 2016, p. 71). Equilibration is the "motivating force behind cognitive development"

[6] *Source: Woolfolk, 2016, p. 73.*

and "it coordinates the actions of the other three factors" (Schunk, 2012, p. 236). Equilibration is the process by which one tries to solve a 'cognitive conflict' or 'disequilibrium' either by assimilation, i.e., altering new information to fit it in existing schemes (thinking blocks) or by accommodation, i.e., developing new schemes to fit in new information (Woolfolk, 2016).

The central idea of cognitive constructivism is that "learners are active in constructing their own knowledge" (Woolfolk, 2016, p. 399). According to Piaget knowledge is constructed whenever there is a 'cognitive conflict'. This implies that conflict "should not be too great" (Schunk, 2012, p. 238) as it will not trigger the process of equilibration, meaning that no new knowledge will be assimilated or accommodated (Schunk, 2012). In addition, Piaget believes that "the impetus for developmental change is internal" and "environmental factors are extrinsic" on development (Schunk, 2012, p. 238). This implies that although teachers can cause cognitive conflict, they have no control of how the child will resolve the conflict (Schunk, 2012).

2.2.1.2 Piaget's Perspective – Implications for Practice

Piaget's perspective has strong implications for practice. As Schunk (2012, pp. 239-240) suggests, teachers need to:

- **Understand cognitive development** by finding out the stages of development so that they can pitch their teaching accordingly.
- **Keep students active** by creating "rich environments that allow for active exploration and hands-on activities".
- **Create incongruity** by providing material that is not "readily assimilated but not too difficult to preclude accommodation".
- **Provide social interaction** such that it creates disequilibrium.

According to James and Williams (2017) when students "are engaged in constructing their own knowledge through experience" this "has the effect of bringing learning alive or rekindling a love of learning" (p. 65). A mathematics trail is one example of how students can be actively engaged in hands-on activities. The trail, however, needs to be pitched at the right level, such that it creates a cognitive conflict that students can resolve via equilibration. Hence, a trail needs to be designed in a way that the development stage of the targeted audience is always taken into consideration.

2.2.1.3 Social Constructivism – Vygotsky's Perspective

Piaget's theory of learning is very individualistic as it overlooks the influence of cultural and social groups on cognitive development (Woolfolk, 2016). On the other hand, Vygotsky, a psychologist and a constructivist, emphasizes that development and learning is shaped by the interaction of the "social, cultural-historical, and individual factors" (Schunk, 2012, p. 242). Of particular importance from these three influences is social interaction.

Vygotsky believes that social interactions are "more than simple influences on cognitive development – they actually create our cognitive structures and thinking processes" (Woolfolk, 2016, p. 83). He assumes that knowledge is first co-constructed between two learners or more, and then the co-constructed process is internalized by the learner (Schunk, 2012; Woolfolk, 2016). The central idea of social constructivism is that social interactions and active engagement are important in the process of knowledge construction and learning (Woolfolk, 2016).

Vygotsky also believes that through cultural tools, including technical tools (e.g. mobile devices, assistive technologies etc.) and psychological tools (signs and symbols like the number system, language etc.), children develop a "cultural tool kit" that gives them not only the tools to "make sense of and learn about their world" but also to "construct their own representations, symbols, patterns and understandings" (Woolfolk, 2016, pp. 84-85). In Vygotsky's theory the most critical tool in this 'tool kit' is language as it "provides a way to express ideas and ask questions, the categories and concepts for thinking, and the links between the past and the future" (Woolfolk, 2016, p. 85).

Another key concept in Vygotsky's theory is the 'Zone of Proximal Development (ZPD)' and is defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). The ZPD, thus, represents the area where cognitive change, and hence learning, may occur (Woolfolk, 2016).

2.2.1.4 Vygotsky's Perspective – Implications for Practice

The "interaction between students and their teachers and environment plays an important role in cognitive development and varies across cultures" (Cahyono, 2018, p. 18). Since in such interactions language is a key factor, verbalization of "rules, procedures and strategies" (Schunk, 2012, p. 249) needs to be encouraged as it can improve learning, and "talking about mathematics helps to bring it to life and to build confidence in one's abilities" (Shoaf-Grubbs et al., 2004, p. 6).

Hence, teachers have an important role in the learning process and should provide support ('scaffolding') to learners to reach the ZPD, "by organizing activities and providing artefacts that can mediate the process of knowing" (Cahyono & Ludwig, 2018, p. 2). Sometimes, however, "the best teacher is another student who has just figured out how to solve the problem, because this student is probably operating in the learner's ZPD" (Woolfolk, 2016, p. 91). The implication of this is the importance of group work as "when peers work on tasks cooperatively, the shared social interactions can serve an instructional function" (Schunk, 2012, p. 246).

Constructivists believe that students should be presented with "complex, realistic and relevant learning environments" that "provide for social negotiation and shared responsibility as part of learning" (Woolfolk, 2016, p. 403). The NCF promotes such learning programmes where the emphasis is on the "active co-construction of meaning rather the mere acquisition of content" (Ministry of Education, 2012, p. 31). A mathematics trail is a way of adopting this social constructivist approach.

2.2.2 Cooperative Learning

As discussed in the previous section, group work is important in the learning process. One must bear in mind that it is "so much more than a seating arrangement – just because students are seated together does not mean they are working together effectively" (Jacobs & Renandya, 2019, p. 9). In cooperative learning a small group of learners "work together as a team to solve a problem, complete a task, or accomplish a common goal" (Artzt & Newman, 1990, p. 2), with a belief in the “power of cooperation, i.e., that two heads are better than one” (Jacobs & Renandya, 2019, p. 7).

According to Artzt and Newman (1990) for students to work cooperatively it must be ensured that the group members:

- appreciate that they are part of a team and as a team they have a common goal;
- comprehend that the task assigned is to be carried out as a group, and the success or failure of the group will be shared by all of them;
- understand that in order to accomplish the task assigned they need to communicate verbally with each other and be involved in a ‘verbal conflict’ (“conflict over ideas”, Artzt & Newman 1990, p. 16);
- recognise that the success or failure of the group depends on the input of each individual in the group.

Thus, when in a group, students working on the task individually or students letting one person do all the work is not cooperative learning (Artzt & Newman, 1990). In order to avoid having group members receding in the background because the group is too large, or members who are vocal taking over, Artzt and Newman (1990) recommend that the size of the group is of three to five members. According to Leikin & Zaslavsky (1999) four members is the optimal number. Artzt and Newman (1990) also recommend a heterogeneous group in terms of ability and personal characteristics. This implies that the teacher’s role is “one crucial point of any cooperative-learning setting” (Leikin & Zaslavsky, 1999, p. 245) and this requires a lot of preparation in terms of the formation of the group, as well as the learning material to be presented to the students.

From a Piagetian perspective, the interactions within a group can create the cognitive conflict that leads to the learner to question his understanding and try out new ideas. On the other hand, Vygotsky’s theory suggests that such social interactions help the individual to develop the higher-order processes such as reasoning and critical thinking skills, which are then internalized by the learner (Woolfolk, 2016).

Activities such as a mathematics trail set a scenario that promote teamwork, where students “can exchange ideas”, “can use their peers as a learning resource” and have the opportunity to verbalize their thinking (Vella, 2005, pp. 24-25).

2.3 Outdoor Education

2.3.1 Definition of Outdoor Education

There are several definitions of 'outdoor education'. One of the early concise definitions of outdoor education is "education *in*, *for* and *about* the outdoors" (Bunting, 2006 p. 4; Cahyono, 2018, p. 27). Bunting (2006) added a fourth addition to this definition – *through*.

In essence, this means that education is taking place *in* the outdoor environment, *for* the future benefit of the environment; education is *about* the relationships within the natural environment and between nature and humans; education is *through* activities in the natural environment (Bunting, 2006 p. 4; Cahyono, 2018, p. 27).

A revised definition is that by Priest:

> Outdoor education: (1) is a method for learning; (2) is experiential; (3) takes place primarily in the outdoors; (4) requires the use of all senses and domains; (5) is based upon interdisciplinary curriculum matter; and (6) is a matter of relationships involving people and natural resources. (Priest, 1986, p. 13)

This definition highlights the importance of outdoor education as a method for learning that offers meaningful experiences. Outdoor education, according to this definition, also supports the connection between various disciplines in the curriculum, as well relationships between humans and nature.

2.3.2 Characteristics of Outdoor Education

Outdoor learning compliments the learning carried out in the classroom – it scaffolds on "prior well-designed classroom learning" and builds on "field-obtained information once back in the classroom" (James & Williams, 2017, p. 60). Similarly, DfES (2006), describe learning outside the classroom as "not an end in itself, rather, we see it as a vehicle to develop the capacity to learn" (p. 3), and Bunting (2006) refers to this as an extension of "structured learning activities beyond the classroom into the community, natural environment, and other locations of topics being studied" (p. 4).

Outdoor education provides opportunities for students to be actively engaged in structured hands-on activities in a real-life context. Such activities provide an enactive representation of the material typically presented at the iconic and symbolic level in class, thereby facilitating students' learning and enabling deeper understanding. The outdoor hands-on learning experiences also "build bridges between theory and reality" (DfES, 2006, p. 4).

Outdoor environments such as the school playground or the shopping centre contain several "math-rich examples" and thus "provide endless opportunities for teaching and learning mathematics" that can "empower children's understanding of mathematics" (English et al., 2010, p. 403). The tasks assigned can be more authentic than those given in class. In addition, the so-important interdisciplinary connections are readily available (Bunting, 2006).

The hands-on experiences, however, must be well prepared and well-structured for students to be fully engaged and take full benefits of such activities. Outdoor learning is not "a matter of taking students into an outdoor environment and letting them play" (Bunting, 2006, p. 14). As discussed in previous section, lack of preparation in such activities can lead to the undesired miseducative experiences.

2.3.3 Benefits of Outdoor Education

The advantages of outdoor education are multifaceted, some of which include:

> **Effective learning** - Research suggests that applying the educational content learnt in the classroom in real-life context situations, results in a deeper understanding of concepts (Bunting, 2006; DfES, 2006; James & Williams, 2017), "which are frequently difficult to teach effectively using classroom methods alone" (DfES, 2006, p. 2). Learning outside the classroom allows for knowledge to be transmitted in a way that encourages exploration and involvement (Bunting, 2006).
>
> **Motivation** - This learning of "in and from the environment appears to increase motivation for the student" (Blane, 1989, p. 130), as well as resulting in "memorable, comprehensive, and long-term learning" (James & Williams, 2017, p. 59).

Cooperation and Communication – In outdoor education students normally work in groups and thus they need to develop “skills and attitudes necessary to understand and appreciate the interrelatedness among people and their individual differences, along with the ability to communicate effectively” (Bunting, 2006, p. 22).

A recreative experience - Another positive aspect associated with outdoor education is the fun element. A student in James and Williams’s study states that “it is a wonderful way to learn because you get fresh air and you’re not cooped up in a classroom” (James & Williams, 2017, p. 64).

Appreciation for the environment – James and Williams (2017) refer that children are spending less time outdoors and more time indoors doing technology-related activities. On the other hand, outdoor education gives students the opportunity to develop “respect and appreciation for the environment” (Barlow, 2015, p. 53).

2.3.4 Concerns Related to Outdoor Education

Despite all the benefits related to learning beyond the classroom, it seems that teachers are still not heading outdoors that very often. According to Hepburn (2014), some of the reasons why this is happening include budget cuts, teachers lacking a genuine understanding of outdoor education, and the focus on exam results. In Taranto et al. (2021) teachers raised concerns regarding the supervision of students when they are outdoors.

On the issue of exam results, James and Williams (2017) argue that for the past twenty years, there has been an increased emphasis on improving standardized test scores, and “this has led to a narrowed curriculum where active, experiential, in-context learning has been de-emphasized or eliminated” (p. 68). Hepburn (2014) suggests that “outdoor learning remains the cherry on top rather than an essential ingredient” (p.1).

According to Bunting (2006), outdoor education can be intimidating for teachers accustomed to teaching indoors. However, given good class organisation skills, disposition to be flexible, and allowing enough time for students to get used to learning in an outdoor class, outdoor learning is not only possible but “it can enliven the teaching and learning experience” (Bunting, 2006, pp. 14-15).

2.4 Experiential Learning in Mathematics

2.4.1 Background

According to Cockcroft (1982) “for most children practical work provides the most effective means by which understanding of mathematics can develop” (p. 84). When students are not given opportunities to put the theoretical knowledge into real-life practice, and hence appreciate mathematics for its utilitarian aspect, they often end up finding the subject too abstract and extraneous. In addition, instead of considering the learning of mathematics as being worthwhile and as an important tool that widens their possibilities in life, students end up asking questions such as “but where am I going to use this is my life?”, as well as thinking that the last time they would ever need to do mathematics is the end of year exam in Year 11 or the SEC exam. As Cockcroft (1982) argues “children cannot be expected to be able to make use of their mathematics in everyday situations unless they have opportunity to experience these situations for themselves” (p.86).

The NCF also advocates for students to be engaged in experiences that are related to the “everyday mathematics” and sets this as one of the two main aims related to Mathematics in secondary education (Ministry of Education, 2012, p. 61). Tangible experiences are, thus, an essential part of the learning process, and this is a view shared by many philosophers and educators (Garrett, 1997). A mathematics trail is one way of providing this type of experiential learning as shall be discussed in the subsequent section.

2.4.2 The Outdoor Mathematics Trail

Following what has been discussed in previous sections, an outdoor mathematics trail can be thus considered to be a resource that can be used to:

- illustrate that “mathematics is more than just sums” (Selinger & Baker, 1991, p. 4). “While completing activities on the trail, children use mathematics concepts they learned in the classroom and discover the varied uses of mathematics in everyday life” (Richardson, 2004, p. 8);

- offer a different context to the classroom. “The use of similar mathematics within different contexts gives students an appreciation of the power of mathematics and its generality.” (NCTM, 2000, p. 202);
- show that mathematics is everywhere. “Any public place that allows safe walking is ripe with math problems for an imaginative trailblazer” (Shoaf-Grubbs et al., 2004, p. 10). Trails are “becoming increasingly popular at different historic and natural sites” (Crack, 2022, para. 1);
- carry out learning outdoors. English et al. (2010) describe how nine-year old students in Australia had positive sentiments towards mathematics after completing a mathematics trail – “I really like doing math when it’s outdoors” and “my favourite bit this week was the math trail because it’s outside and you get to do more things” (p. 403);
- experience both the products and processes of mathematics. The NCTM, in the spirit of “talking about and doing mathematics” (Shoaf-Grubbs et al., 2004, p. 9), emphasizes on the processes of “Problem Solving, Reasoning and Proof, Communication, Connections, and Representation” (NCTM, 2000, p. 29);
- encourage mathematical talk. “Managed in the right way, trails present a golden opportunity to enrich mathematical talk. This can be done at all stages of the trail - in the classroom preparation beforehand, doing the trail itself and during follow-up work” (Crack, 2022, para. 2);
- connect enactive representations to the iconic and symbolic representations. This is considered valuable for learning mathematics outdoors (Zender et al., 2020);
- encourage students develop “interest in and respect for the community in which they live” (Druken & Frazin, 2018, p. 45).

Such a resource is an ‘experience’ for all students. Hence, all students benefit from a mathematics trail, it is “not just for math lovers or A-students!” (Shoaf-Grubbs et al., 2004, p. 9).

2.5 Design of the Resource Pack

There are different types of design principles that deal with the design of ‘messages’. (Pettersson, 2023a). Of particular interest to this dissertation are ‘Instructional Design’ and ‘Information Design’.

2.5.1 Definitions

A ‘message’ is “information content conveyed from a sender to a receiver in a single context at one occasion” (Pettersson, 2023a, p. 44). In the case of this Resource Pack, the sender, also referred to as the “information provider” (Pettersson, 2023a, p. 117), is the teacher, whereas the receivers are the students.

‘Design’ is “the identification of a problem and the intellectual process of an originator, manifesting itself in plans and specifications to solve the problem” (Pettersson, 2023a, p. 260).

2.5.2 Instructional Design

‘Instruction’ is “the deliberate arrangement of events in the learner’s environment for the purpose of making learning happen, but also to make it effective” (Gagne, 1985, p. 244).

2.5.2.1 Gagne’s Theory of Instructional Design

Robert Gagne, a psychologist, formulated his ‘instructional theory’ based on cognitive principles (Schunk, 2012). Gagne’s instructional theory offers “advice on how the teacher should manage individual lessons” (McLeod, 2023a, para. 2). In Gagne’s own words it is “the relationship between instructional events, their effects on learning processes, and the learning outcomes that are produced as a result of these processes”, where instructional events are “those external events that are designed to help learning occur” (Gagne, 1985, p. 244).

Gagne's theory provides a framework for planning and organizing instructional events to promote effective learning. While originally developed for classroom instruction, its principles can be adapted and applied to various learning contexts, including designing tasks for a mathematics trail.

2.5.2.2 Gagne's Nine Events of Instruction

Gagne (1985) offers a sequence of nine instructional events that the teacher should use to help students learn, i.e., teaching steps that characterize a good instruction, and these include:

1. **Gaining attention** – Teachers need to gain students' attention in the introductory phase of the instruction by introducing a "rapid stimulus" (Gagne, 1985, p. 246) so that students are alert and ready to participate actively in the instruction.
2. **Informing learners of the objective** – Learners need to know what the overall learning objectives of the instruction will be, as this will help them to organize their thoughts about what they are about to do.
3. **Stimulating recall of prior learning** – Teachers need to help students associate the current knowledge with previously learnt knowledge because this will facilitate the learning process, increase the willingness to learn, and create a sense that learning is continuous.
4. **Presenting the stimulus** – In the initial phase of learning teachers need to present an "essential stimulus" (Gagne, 1985, p. 251). This depends on what is to be learnt. Thus, teachers need to use different presentation formats, teaching approaches and activities, and present new knowledge in a logical and easy-to-understand manner.
5. **Providing 'learning guidance'** – Teachers need to make "the stimulus as meaningful as possible" (Gagne, 1985, p. 252). Meaningfulness can be enhanced by providing learners with clear and specific instructions that are both written and spoken, by preparing explanations that include examples and non-examples, and concrete examples to explain abstract concepts, and by connecting new ideas with those already in memory.
6. **Eliciting performance** – Teachers need to provide opportunities for learners to put the new knowledge into practice, either as individual work or group work, by asking questions that help learners think and reflect on what they are doing.

7. **Providing feedback** – Teachers need to provide clear and meaningful feedback to learners "about the correctness and the degree of correctness of the performance" (Gagne, 1985, p. 254). Teachers need also to promote reflective thinking and help students look back on what they have learnt through an effective concluding plenary phase at the end of instruction.
8. **Assessing performance** – Teachers need to assess whether learning has taken place and "has a reasonable stability" (Gagne, 1985, p. 255). This can take various forms such as special homework, topic tests and examinations.
9. **Enhancing retention and transfer** – Teachers need to find ways to ensure that the new knowledge that has just been learnt is retained over a long period of time by providing practice that is frequent and "spaced" (Gagne, 1985, p. 255). Preferably this is done using a variety of activities, including cross-curricular activities so that students are not bored.

According to Gagne not all of the 'nine events of instruction' need to be included in every learning episode because as learners gain experience "they are increasingly able to practice self-instruction" (Gagne, 1985, p. 256). However, he highly recommends the use of the entire set with young students, even if not used exactly in this order.

2.5.3 Information Design

'Information Design' is a means of "communication by words, pictures, charts, graphs, maps, pictograms, and cartoons, whether by conventional or electronic means" (Passini, 1999, p. 84). Pettersson (2023a) highlights that the main goal in 'information design' should always be "clarity of communication" (p. 57) as ultimately the message needs to be understood by most of the receivers.

2.5.3.1 The Process Activities

An 'information and message design model' created by Pettersson (2023a) is made up of the following four process activities:

Analysis and synopsis - In this introductory planning phase it is important to spend enough time analysing the ability and experiences of the receivers, as well as defining the objectives, and the purpose of the intended message. It is also recommended to develop an outline of the material ("synopsis") and decide on the structure of the material so that receivers can have "a good reading value" (Pettersson, 2023a, pp. 101-102).

Production of draft - Following the production of the first draft, it is necessary to establish a document numbering system to manage the various versions of the document, to decide on the text design that will be used throughout the entire project, to add explanations to drawings and photographs, to organize the layout between text and pictures, and to carry out a pedagogical review to ensure that the content is "well structured and comprehensible" and has "a high reading value" (Pettersson, 2023a, p. 105).

Production of script - In this phase all the necessary work related to text, pictures and photographs resulting from the review carried out in the previous phase is to be completed in order to create a final version. In addition, the material needs to be checked for "copyright clearance" (Pettersson, 2023a, p. 106).

Production of original and master - Before confirming the original as the master material, it is key to ensure that in cases where the material is made up of various documents, only the final versions are used for the originals. As part of this overall final check, it is also important to check the quality of the text, pictures, and photographs, and to correct any errors that there might be.

2.5.3.2 The Design Principles

Pettersson suggests a set of principles for message design (Pettersson, 2023a), as well as another set of guidelines to "enhance readability of text and pictures in information design" (Pettersson, 2022b, p. 180). Smaldino et al. (2005) also propose design guidelines when creating visual displays with "pictorial and text elements" (p. 88). The design principles relevant to this dissertation, from these lists, were combined and summarized into a single list as follows:

Legibility - Legibility is “how easy it is to read” (Pettersson, 2023a, p. 120) the text and pictures of a message, in accordance with the medium of presentation – in this case printed on paper. The use of “unusual typefaces” or fonts that are “too small or too large” should be avoided (Pettersson, 2023a, p. 120). The content needs to clearly stand out from the background. Smaldino et al. (2005) refer to this contrast as the “Figure-Ground Contrast” (p. 90).

Readability – Readability is “how well the contents and the presentation of the contents are adapted to the readers” (Pettersson, 2023a, p. 121). “Buzzwords, grandiose and pompous words, jargon, and slang” should be avoided (Pettersson 2022b, p. 187). Instead, ordinary words should be used, especially to explain new terms and terminology. Words should be carefully chosen in order to limit the chances of misunderstanding, thus contracted forms, such as ‘it’s’, should be avoided. Instead expanded forms like ‘it is’ are to be used. In addition, only one idea is to be expressed per sentence (Pettersson, 2022b).

Consistency - The material needs to have “an overall togetherness” (Pettersson, 2023a, p. 122), i.e., material that is consistent in terms of layout, style, and terminology, as otherwise receivers might get confused.

Structure – The content needs to be developed in a structured sequence that makes sense to the receiver. In case of reference material, a list of contents at the beginning of the material, distinct headings for each level within the structure, and page numbers need to be included (Pettersson, 2023a).

Alignment – Elements within a display should be positioned in a way that creates “a clear visual relationship” (Smaldino et al., 2005, p. 92). This will help the receivers concentrate on the message being conveyed. Such relationship can be established using alignment, i.e. elements “aligned on the same imaginary horizontal or vertical line” (Smaldino et al., 2005, p. 92).

Focus Attention – Important text should be emphasized to “attract, direct, and to keep attention” (Pettersson 2022b, p. 186), and this can be done by using contrasts such as bold face, colour, and italics. According to Smaldino et al. (2005), in instructional material, red and orange (warm colours) are associated with “an active, dynamic, feeling", whereas blue, green, and violet (cool colours) are associated with "a contemplative, thoughtful, cool feeling" (p. 95). Thus, they suggest to "take advantage of this effect by highlighting important cues in red or orange, helping them leap out at the viewer the way a red STOP sign stands out even in a cluttered urban landscape" (Smaldino et al., 2005, p.95).

Headings – Headings are aimed to attract the attention of readers, to provide reference points, and to show the hierarchical structure of the text. To attract attention, headings can be set in bold in black or in colour, and key words are set in a different colour or italics (Pettersson, 2022b). To achieve a clear structure, a numbering system can be used (e.g. 1, 1.1, 1.2; 2, 2.1, 2.2; 3, 3.1, 3.2 etc.) (Pettersson 2023c). Headings should never end with a full stop (.) but can end with a question mark (?) or an exclamation mark (!) (Pettersson 2022b).

Captions – When pictures are used to provide information, captions help the reader focus on the intended content within the picture, especially content that might otherwise be overlooked. "A picture without a caption has limited informational value" (Pettersson 2022b, p. 189). Thus, captions need to be simple, informative, and brief.

Proximity – Elements that are related should be placed close to each other, and unrelated elements need to be moved far apart. The is referred to as the "principle of proximity" (Smaldino et al., 2005, p. 96).

Balance and Eye Flow – The layout choices need to support the eye movement through the material. There also needs to be a good balance between pictures, photographs, and text. Images and pictures help to attract and maintain attention, as well as illustrate, facilitate, and clarify information (Pettersson, 2023b).

2.6 Conclusion

The literature review of learning theories on experience in education by Dewey, Bruner, and Dale, gave an insight on the importance of developing a trail that aims for students to have an educative experience. The need for students to have enough experiences in the enactive stage in order to understand the concepts in the iconic and symbolic stages, was also highlighted. This idea tied in with Piaget's constructivism, where providing environments that allow hands on activities to construct knowledge, is also given importance. Vygotsky's constructivism shed light on the importance of students interacting with other students when constructing knowledge. This influenced the decision to develop a trail to be carried out as group work rather than individual work. Following the literature review on outdoor education aspects, such as appreciation of the environment, prompted the idea to include historical information in the trail. Gagne’s theory of instructional design provided guidance on how to plan and organize the instructional material presented to students as in the case of the tasks in the trail, in particular those related to students’ attention, learning objectives, and learning guidance. With regards to the process of activities involved when developing material such as the Resource Pack, Pettersson offered a guiding framework from the planning phase to the final phase. Visual design principles by Pettersson and Smaldino provided guidance on how to create the Resource Pack in terms of layout, structure, legibility, readability, and attention, as will be shown in the next chapter.

Chapter 3

Methodology –

Design and Development

Chapter 3: Methodology - Design and Development

3.0 Introduction

The various aspects researched in the literature review phase provided a framework for this next phase of the dissertation, i.e., the design and development of an outdoor mathematics trail resource pack.

The Resource Pack is actually made up of two packs – the teacher's pack and the student's pack. The two packs are similar to each other. However, the teacher's pack also includes a teacher's guide, the expected working and answer of each task, additional information related to each station such as the Learning Outcomes, ideas for preparatory work, and difficulties students might encounter whilst carrying out the task. The Learning Outcomes are taken from the Mathematics SEC Syllabus 2025 as available in 2022. For the sake of clarity, reference to the Resource Pack in this write-up refers to both packs.

This chapter starts by explaining the various factors influencing the choice of location. In section 3.2 examples of and ideas taken from existing trails are discussed. Subsequently, the application of the theory underlying the instructional design and the information design principles, is described in sections 3.3 and 3.4 respectively. Both sets of principles are required as the principles of information design are applicable to any material, not necessarily instructional material, and are mostly related to visual elements. On the other hand, the instructional design principles focus on how best to approach the design of the material from a teaching and learning perspective. Section 3.5 aims at highlighting the variety of tasks developed in the trail as well as ideas of how the tasks can be extended after the trail. Finally, section 3.6 concludes the chapter.

3.1 Developing the Mathematics Trail

The first step in developing the mathematics trail was to select the location (Richardson, 2004). Several factors such as safety, history, distance, duration, and any other aspects that might affect the interest of the participants (Cahyono, 2018) were considered when choosing Birgu as the trail location, as shall be discussed in the following subsections.

3.1.1 Safety Concerns

One of the primary considerations, if not the main one, was that of safety. Thus, it was ensured that:

- The location is a quiet area that is not negatively impacted by busy roads and crowds.
- Students need to cross the least possible amount of roads.
- The area where the stations are located is safe from cars and away from edges such as low bastions and staircases. Thus, students can fully concentrate on the activities without worrying that they are on the edge of a pavement, for example, or that any of their tools accidentally fall over the bastions.
- In all stations, except for one, students have space where to sit down. Considering the length of the trail it was important to make provision for such spaces to students. These are also useful for placing bags and other belongings when students are using tools to accomplish tasks.

3.1.2 Selection of Tasks

Following the issue of safety, another major consideration, when choosing Birgu as the site of the trail, was the number of areas “rich in mathematics” (Richardson, 2004, p. 9) in order to create a “variety of activities” (Selinger & Baker, 1991, p. 8) that cover various mathematical topics.

Hence, before creating the preliminary trail, several scouting visits to Birgu were carried out, each time looking out for “patterns, shapes, and things to measure, count or graph … or other things that use numbers” (Richardson, 2004, p. 9), amongst others, whilst taking plenty of photographs, drawing sketches, and taking notes. The photographs helped to observe mathematical elements that were not as noticeable whilst on site.

To develop the trail, six stations were chosen, as Richardson (2004) recommends. There were several other tasks that were identified, but eventually were not used in the “final lot” in order to ensure a “good balance” (Shoaf-Grubbs et al., 2004, p. 10) and to avoid overemphasizing any one topic such as geometry (Shoaf-Grubbs et al., 2004, p. 12).

3.1.3 Length and Duration

The estimated time to complete the trail is of three hours. The average total time to carry out the tasks at each station is of twenty minutes and the average travel time from one station to the next is of about ten minutes. The time between tasks was kept to a minimum as according to Shoaf-Grubbs et al. (2004, p. 11), "spacing consecutive trail stops more than 10 minutes apart risks losing a walker's attention". Literature suggests having a break halfway through a trail, however considering the limited time set by the school hours, it was envisaged that the break is taken at the end of the trail.

The actual time spent at each station depends on the ability of the students, and whether students were given enough preparation and practice for the trail by their teacher (for example, learning how to use equipment such as the clinometer or compass, how to follow the directions given on a map etc.), and the level of the adult support available at each station to handle any queries students might have (considering that they might not be mathematics teachers). Depending on these factors, the teacher can decide whether students should carry out all the tasks designed for each station or some of them only. Another aspect is the topics the teacher wishes to cover in the trail. This could be another reason to do away with some of the tasks. The tasks were designed independent of each other so the teacher can easily omit some of them either because of time constraints or the content being assessed.

The total walking distance, as calculated using Google Earth, is of around 1.75km (equivalent to 1.09 miles). This is a reasonable distance as "one mile will do for many people" (Shoaf-Grubbs et al., 2004, p. 11).

3.2 Examples of Mathematics Trails

Some of the mathematical ideas in Blane and Clarke's first mathematics trail include discovering the invariance of π by investigating a pattern of concentric circles in a shopping mall, approximating the height of a cathedral spire by using its reflection in a pool, counting the number of windows in one of the sides of a skyscraper by estimation and calculation, analysing the height of the river and its effect on the surrounding towns, and using the co-ordinates system to locate the position of the private mail boxes inside the general post office (Blane, 1989). Refer to Appendix A for Blane and Clarke's trail extracts.

Blane and Clarke's layout makes use of answer boxes at the right-hand side of each question, as well as hints in some of the questions, as shown in Figure 3 below. These interesting ideas were used in the mathematical trail included in this dissertation. The hints idea was then further developed into an information guide giving students the necessary information to work out the task.

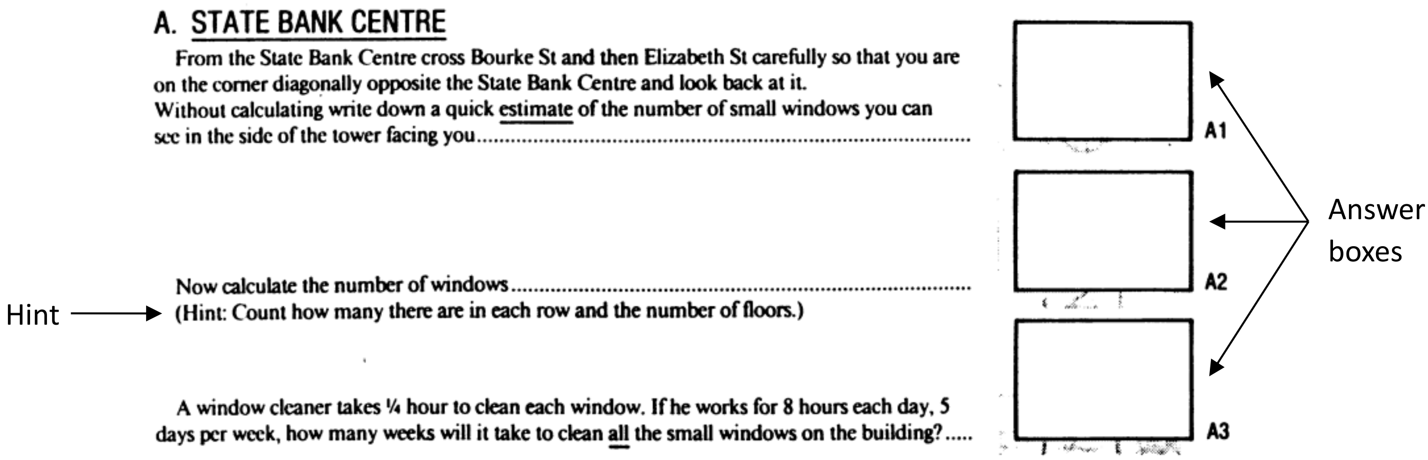


Figure 3: An activity example in Blane and Clarke's trail [7]

Some of the trails designed in the local context, include those of San Anton Gardens, Chinese Garden, Mdina, and Valletta in Malta, and Villa Rundle Gardens and Cittadella in Gozo[8]. The trails carried out in the gardens are typically targeted for students in primary years, whereas the trails around the cities are normally designed for students in secondary years.

Mathematical ideas for the trails set at primary level include, amongst others:

- Measurement, e.g., the circumference of a tree or the length of a bench.
- Estimation, e.g., the amount of people that can fit on a number of benches.
- Shapes and symmetry, e.g., to identify the shape of a particular tile or to indicate the line of symmetry of a door.
- Analogue and digital time, e.g., writing the time of arrival and the time taken to get to a particular station in the trail.
- Numerical calculations, e.g., working through situations that involve addition, subtraction, multiplication, or division.

[7] *Source: Blane, 1989, p. 131.*
[8] Web links to access these maths trails can be found in Appendix C.

- Fractions e.g., representing the number of vowels and consonants in words written on a tablet or sign as fractions.

The trails designed for secondary years focus on mathematical ideas such as:

- Measurement, e.g., the diameter and height of a cylindrical-shaped letter box in order to find its volume and surface area.
- Estimation, e.g., the percentage area of a façade occupied by windows.
- Trigonometry, e.g., using the angle of elevation to determine whether a ramp can be installed on a flight of stairs, or to find the height of a statue.
- The number system, e.g., expressing numbers on plaques in standard form, a product of three factors, or a product of prime numbers.
- Statistics, e.g., recording data in a frequency table and using this data to find the mean, median, and mode.
- Geometry, e.g., using Pythagoras' Theorem to calculate the slanting edge of a triangular-shaped monument.

Following the literature review on outdoor education, the idea to include some historical facts along the trail was developed. The same idea had already been used in two independent Valletta trails prepared by the Primary Maths Support Team within the Directorate for Learning and Assessment[9], and the Heads of Department for Mathematics within the Secretariat for Catholic Education[10]. The trail prepared by the latter presented another interesting idea – that of using icons next to tasks to indicate what the question was about. In the case of the trail presented in this dissertation the icons were used to indicate added guidance such as which resource needs to be used by students during the task, whether the task needs to be carried out on site or at home, whether the result is an 'aha' moment, or whether students need to calculate, record data, or think and explain.

[9] Same as previous footnote.
[10] This trail was kindly provided by a teacher in a church school as listed in Appendix C.

3.3 Task Design – Instructional Design

One of the main aims of this trail is for students to have a meaningful learning experience. Thus, Gagne's 'nine events of instruction', as discussed in the previous chapter, were used to ensure correct instruction, when preparing the material.

1. **Gaining attention** – The six different stations act as a 'rapid stimulus' as students change the physical location and are presented with a new piece of historical information.

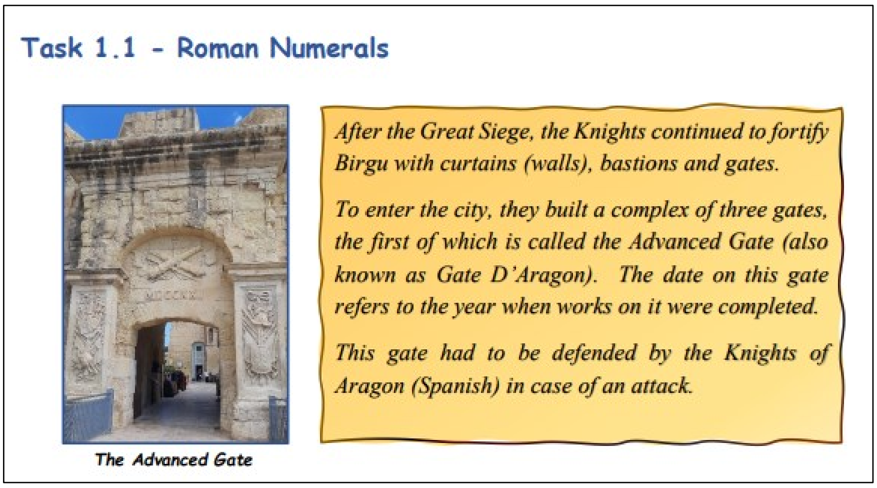


Figure 4: Using change of location and historical information as a 'rapid stimulus'

2. **Informing learners of the objective** – In the first page of each station students are given a list of the learning objectives of the tasks within that particular station.

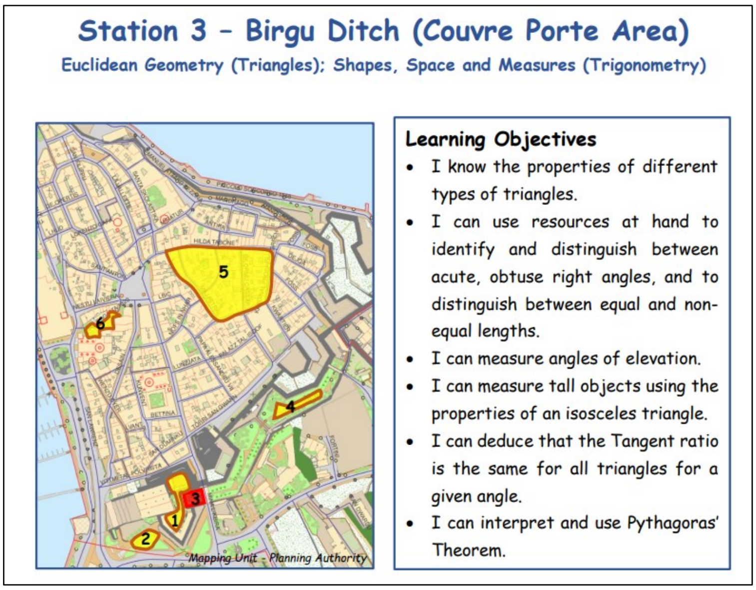


Figure 5: Informing students about the Learning Objectives of each station

3. **Stimulating recall of prior learning** – The tasks were designed in a way that students are engaged, are able to make connections with the content they would have learnt in class (for example, trigonometry, Pythagoras' Theorem, the notion of *pi*), and are able to actually appreciate it in the real-life context. This is because mathematics concepts "become much more clear to students when they can tie them to something that is familiar and meaningful to them" (Classen, 2002, p. 30). Important information that students need in order to work out the task is given to them in the 'information box' within the task, or as a formula in the 'list of formulae'.

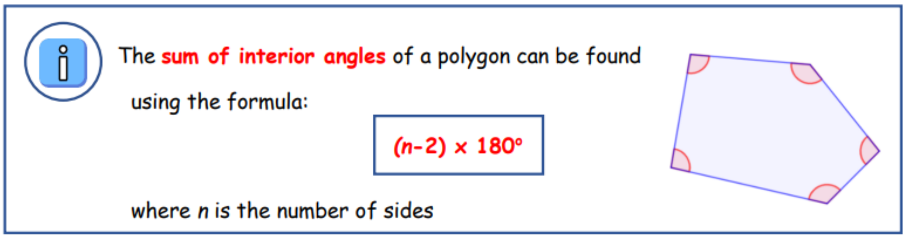


Figure 6: Using the 'Information Box' to stimulate recall of prior learning

4. **Presenting the stimulus** – The tasks are independent of one another, thus at each stop students feel "encouraged by a fresh problem" Shoaf-Grubbs et al. (2004, p. 12). The pictures associated with each task act as an 'essential stimulus'.

Figure 7: Using pictures as an 'essential stimulus' for each new task

5. **Providing 'learning guidance'** – The tasks start with easy questions, so that all students can answer them. The level of difficulty increases, normally leading to either an 'aha' moment or else an open question that promotes discussion, "but not so much so as to frustrate" (Selinger & Baker, 1991, p. 13). Shoaf-Grubbs et al. (2004) also discuss this and suggest that "posing the problems at different levels and with different mathematical focuses will help to achieve a good variety in the overall trail" (p. 12).

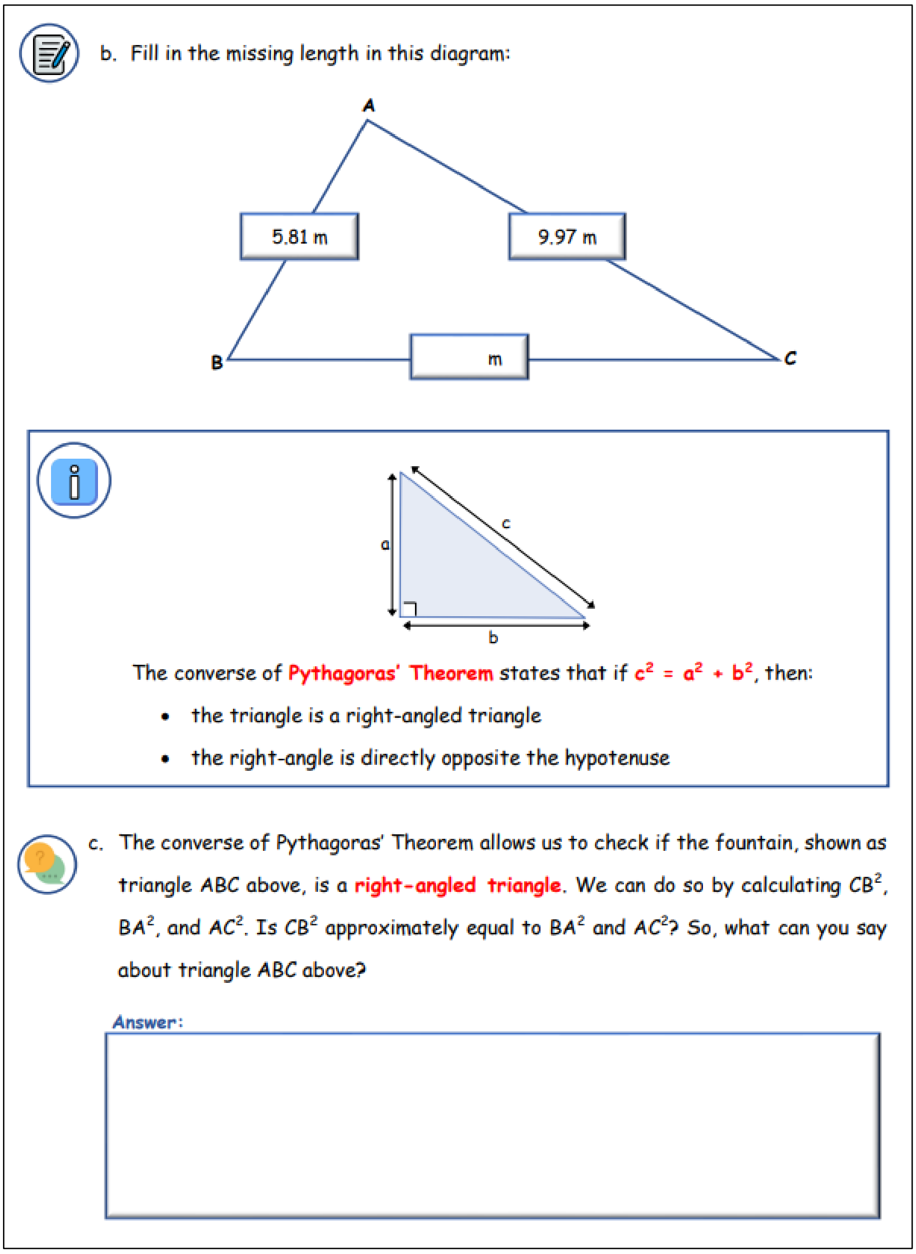


Figure 8: Providing learning guidance in the task questions

6. **Eliciting performance** – Throughout the trail students are encouraged to participate actively, and to apply their mathematical skills and knowledge, so as to complete the tasks. This participation is to be carried out as groupwork and everyone should “feel the accomplishment of contributing to the problem solving” (Shoaf-Grubbs et al., 2004, p. 9).

Task 3.2 - Measuring Heights using Isosceles Triangles

a. Select **one student** from your group. The student needs to measure the **angle of elevation** using the clinometer as follows:

- Student needs to stand and face the vertical strip highlighted in yellow.
- With the help of the group, the student needs to move slowly backwards till the angle of elevation from the ditch to the top of the bridge is **45°**.

b. The student **needs to remain** in the position where the angle of elevation is 45°. The rest of the group needs to use the tape measure to find:

(i) the **horizontal distance** between the student and the bridge (BC). Give your answer correct to the **nearest cm**.

Note: The ground level gives you the most accurate horizontal measurement.

Answer:

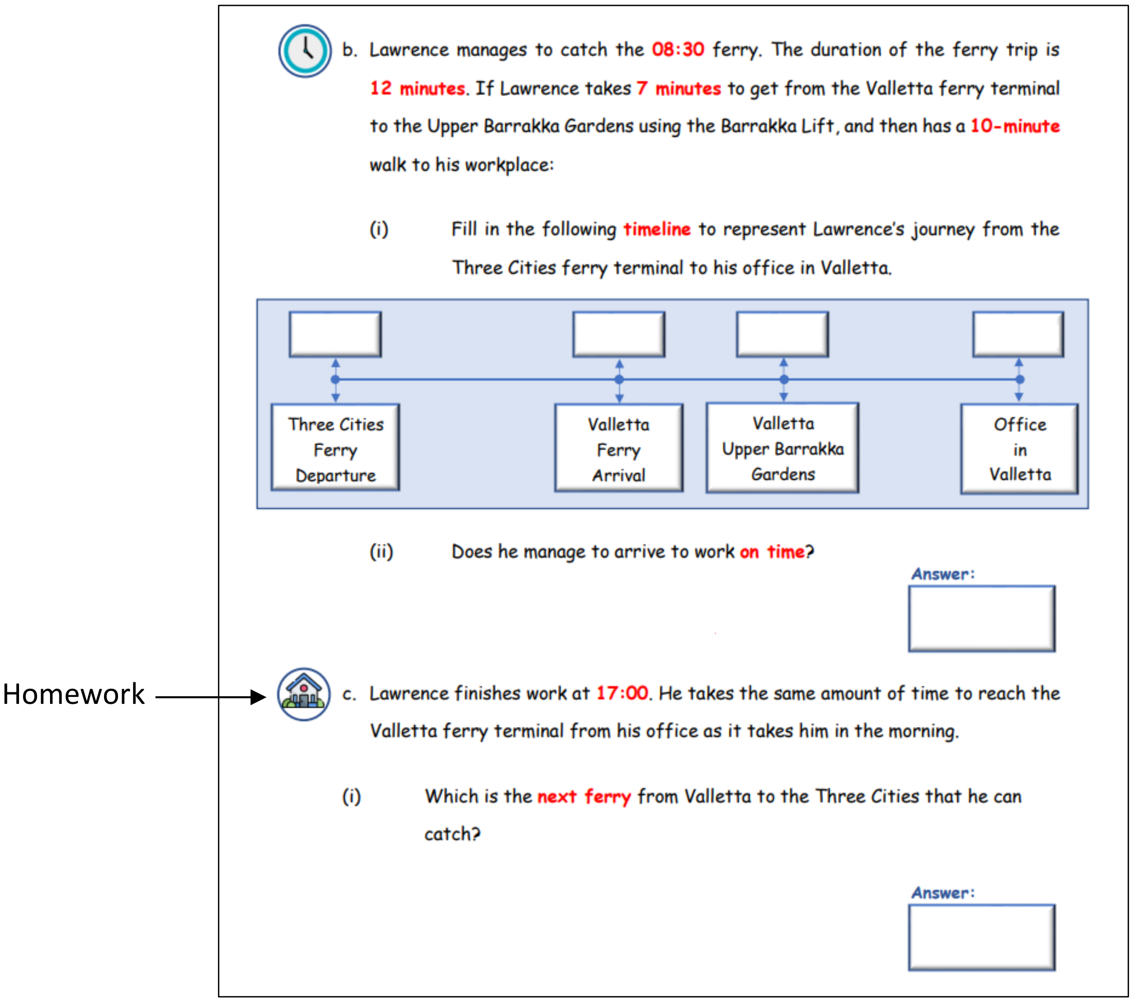
cm

Figure 9: An extract illustrating how students need to cooperate to accomplish the task

7. **Providing feedback** – Feedback about the correctness of the performance can be given by peers whilst on site. It can also be given by the teacher whilst checking out on students during the trail. Definitely, after the trail, the teacher needs to dedicate time with students to discuss and reflect on all the important aspects of the trail.

8. **Assessing performance** – The tasks were also designed so that part of the task would be “solved on the spot” and another part would be “extended at a later date in the classroom or at home” (Selinger & Baker, 1991, p. 8). The reason for this is to allow

space for class discussion, reflection, as well as independent work in accordance with the coursework guidelines stipulated in the Mathematics SEC Syllabus 2025. Also related to the assessed coursework is the fact that the tasks were designed in a way to facilitate the setting up of a marking scheme and the correction process.


b. Lawrence manages to catch the 08:30 ferry. The duration of the ferry trip is 12 minutes. If Lawrence takes 7 minutes to get from the Valletta ferry terminal to the Upper Barrakka Gardens using the Barrakka Lift, and then has a 10-minute walk to his workplace:

(i) Fill in the following timeline to represent Lawrence's journey from the Three Cities ferry terminal to his office in Valletta.

(ii) Does he manage to arrive to work on time?

Answer:

c. Lawrence finishes work at 17:00. He takes the same amount of time to reach the Valletta ferry terminal from his office as it takes him in the morning.

(i) Which is the next ferry from Valletta to the Three Cities that he can catch?

Answer:

Figure 10: Using icons to distinguish between on-site and off-site tasks

9. **Enhancing retention and transfer** – As much as possible tasks were designed to be different from what students would normally solve in class. Shoaf-Grubbs et al. (2004) suggest including a mathematics user such as an engineer when designing the trail in order to “prevent a school-like tone to the problems” (p. 14). Such tasks have the potential of transferring learning beyond the trail and for students to reflect on the trail experience.

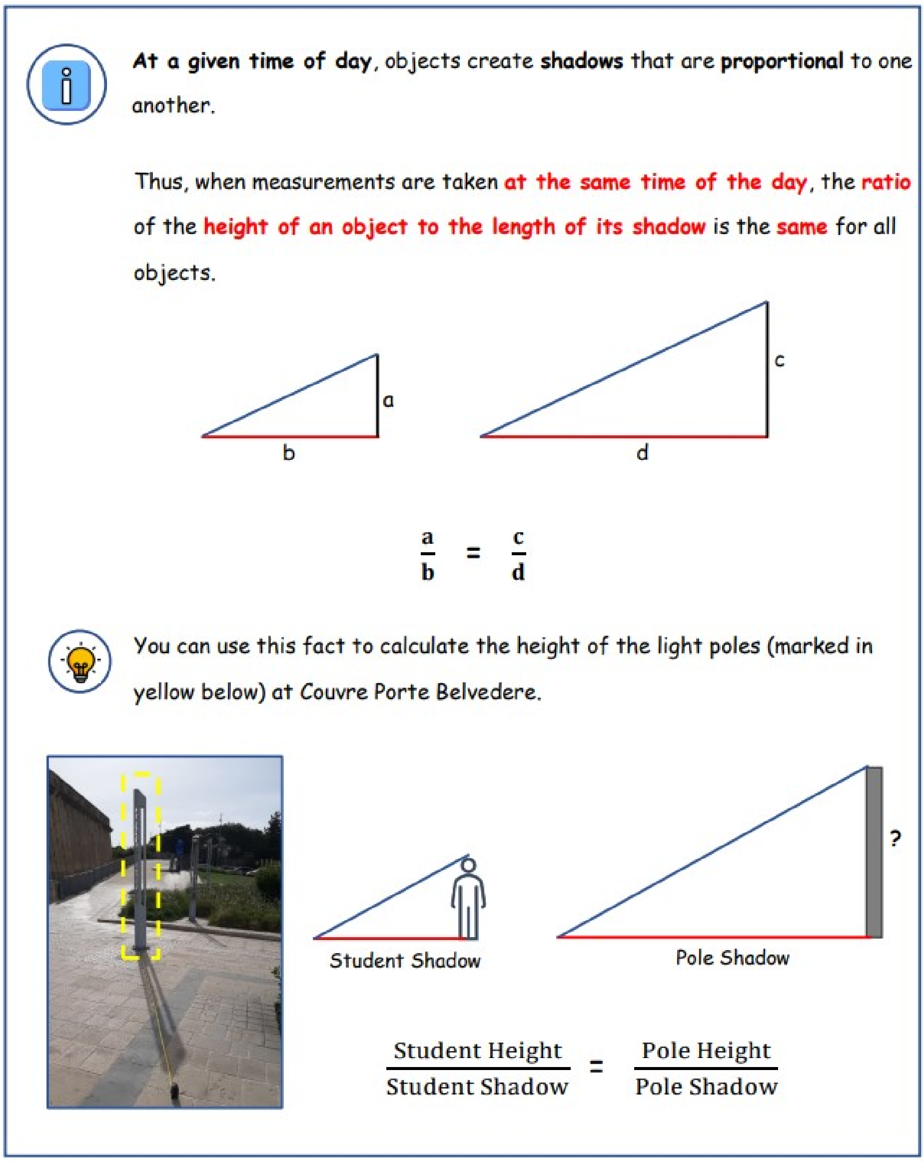


Figure 11: Non-textbook question to enhance knowledge retention and transfer

3.4 Task Design – Information Design

Considering that the Resource Pack is intended as instructional material, a design that ensures good communication between the teacher and the students was necessary. Hence, the pack was developed according to the design guidelines described in the previous chapter, namely:

Legibility – The trail aims at having a good balance between pictures, photographs, and text. Images and pictures help to attract and maintain attention, as well as illustrate, facilitate, and clarify information (Pettersson, 2023b). The various elements within the trail were grouped by using sections and also enclosed within borders.

Enough space was left for students to write their working and answers (Selinger & Baker, 1991; Shoaf-Grubbs et al., 2004).

Readability – The wording was kept concise as much as possible, including the elements related to history. It was also written at the right level of the audience, i.e., Year 10 students. The readability of images and pictures was also taken into consideration in order to ensure "a high level of picture quality" (Pettersson, 2022a, p. 27).

Consistency – It was ensured that throughout the Resource Pack there was consistency in terms of typography (style, layout, typefaces, line spacing, captions etc.), visuals (maps, icons, labelling etc.), numbering system (task number, page number etc.), terminologies, and active words (e.g. tick the correct answer).

Structure – Headings were set in a large font size and in colour to show the structure of the trail and attract students' attention. Page numbers and a table of contents were included so that students can find the information in the document easily.

Alignment – The various elements, for example headings, text, pictures, answer boxes, etc., were aligned in relation to the page margin.

Focus Attention – The most important information was presented in a way to emphasize its importance, either by using larger font size, or using bold, or by highlighting the text. The red colour was used such that important words "leap out" and students give them their due attention (Smaldino et al., 2005, p. 95).

Headings – In order to "achieve a clear structure" (Pettersson, 2023c, p. 190) a numbering system as recommended by Pettersson was used, i.e., 1.1, 1.2, 1.3, 1.4, 2.1, 2.2, 2.3 etc. Headings were also set in bold and in colour.

Captions – Captions were used for all the pictures that required additional information. Captions were placed beneath the pictures as according to Pettersson (2023c) that is where readers expect to find them.

Proximity – Labels and captions were placed close to the picture elements to reflect the relationship between them. Similarly, picture elements were placed close to the related text. Headings were placed "above and close to the following text" as the placement of the heading "enhances the hierarchic structure" and the heading "belongs with its own text" (Petterson, 2023c, p. 192).

Balance and Eye Flow – The Resource Pack was designed to have a good balance between photographs, general images (including icons), maps, diagrams, and tables. Students are given a clear starting point, and the layout supports their eye movement through the material.

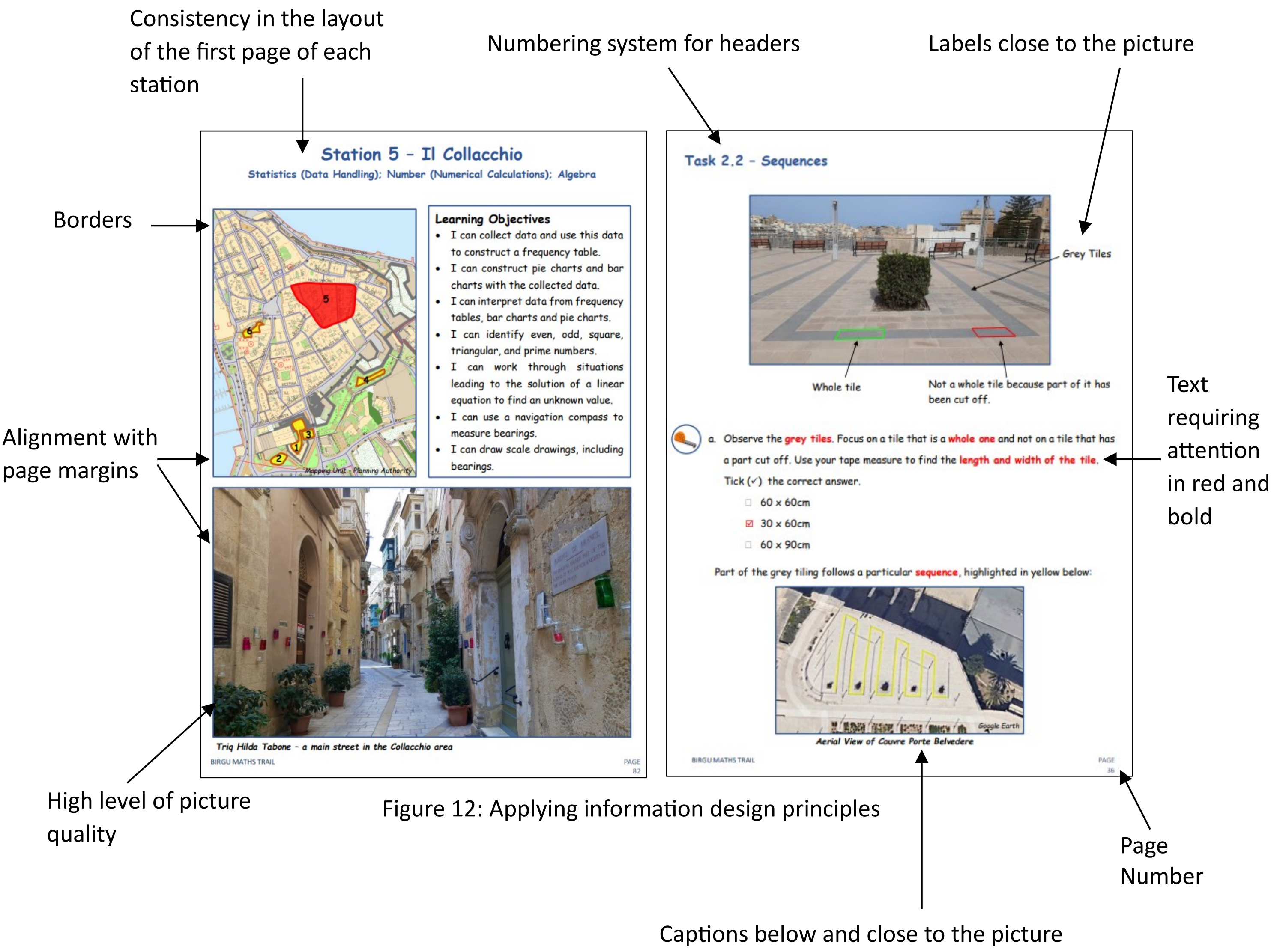


Figure 12: Applying information design principles

3.5 Type of Activities included in the Resource Pack

As Classen (2002) states "it is not always easy to show the relevance or practical application of the math we teach. This is why it is so important that we grab the opportunity to do so whenever possible" (p. 30). Hence the type of activities chosen for this trail include both those which are an extension of what students learn in the classroom in accordance with the Learning Outcomes set in the Mathematics SEC Syllabus 2025, as well as others that either promote the processes of mathematics (as discussed in the first chapter) or aim to generate interest and curiosity in the subject that goes beyond the content of the syllabus. If students are expected to make use of certain skills such as measuring skills, carry out conjectures, calculations, or estimations, develop a sense of curiosity etc. in everyday situations, then they need to be given the opportunity to experience the application of these skills first.

3.5.1 Measuring

There are various measuring tasks throughout the trail using different tools and involving various mathematical concepts. Students are not asked to measure something just for the sake of measuring it. As NCTM (2000) refers, "the value of a mathematical task is not dependent on whether it has a real-world context but rather on whether it addresses important mathematics, is intellectually engaging, and is solvable using tools the learner has or can draw on." (p. 201). Furthermore, in cases where similar type of measurements are needed in order to reach the learning objective, such as in Tasks 2.2 and 3.3, some of these measurements are provided to students, as otherwise the measuring activity might be repetitive and boring.

Measuring lengths - Students have four tools to measure lengths namely a ruler, a sewing tape measure, a tape measure, and a disc tape measure. Even though a sewing tape measure and a tape measure might seem very similar in reality one might be more suitable to use than the other. For example, in Task 1.2 it is best to use the sewing tape measure to find the circumference due to its flexibility. On the other hand, in order to find the diameter, it is best to use the tape measure as the 'end hook' provides a secure anchor point whilst trying to find the longest point on the rim. In Task 2.1 it is best to use the tape measure to measure shadows as it is longer and sturdier than the sewing tape measure. In Task 4.3, it is better to use the tool which has the longest length considering the distance to be measured. On the other hand, for the scale drawing in Task 5.4, it is best to use the ruler. It is important for students to experience these different situations and be able to compare between the tools they have available and choose the one that meets their needs the most.

Measuring angles – Students measure various angles in various ways during the trail. The tools being used are the clinometer, navigating compass, and protractor. The clinometer is used in Tasks 3.2 and 6.1 to measure the angle of elevation. The navigating compass is used in Task 5.4 to measure bearings. The protractor is used in Task 4.2 to measure the angles at a point. Angles are a difficult concept for students to develop because they are abstract (not tangible). Keiser (2004) describes angles as a "multifaceted concept" (p. 289) and mentions three classifications to describe an angle: "A measure of the turning of a ray about a point from one position to another (dynamic), the union of two rays with a common endpoint, and the region contained between the two rays (both static conceptions)" (p. 288). In her study, Keiser (2004) finds that the majority of sixth-grade students "thought of angles as quantities" which "could be measured in some way" and they thought that "the longer the rays, the greater the measure of the angle" (p. 292). This is because textbooks "typically provide a static definition for angle" rather than a multiple representation (Keiser, 2004, p. 303). Thus, the more experiences students have to measure different angles, the more are the chances of success in understanding the multifaceted nature of angles.

Measuring time – Students use a stopwatch (either a physical one or an app on the mobile phone) to measure time in Task 4.3. One of the main challenges when measuring time is the reaction time to start and stop the stopwatch – the faster a student is to perceive an event and reacting to it by pressing the stopwatch button, the more precise the measurement is. Another challenge is the resolution of the timer and what the digits shown on the timer represent, i.e. whether it is to the nearest second (minutes:seconds) or 1/100th of a second (2 digits after the seconds) or 1/1000th of a second (3 digits after the seconds). Knowledge about this decimal representation of seconds is important for students especially when they need to round to the nearest second as otherwise it can be mistakenly assumed that the rounding is to base-60 rather than base-10. These are knowledge gaps that typically come to light when students are physically handling tools.

3.5.2 Estimation

Estimation is a very important skill to learn as it "serves as an important companion to computation. It provides a tool for judging the reasonableness of calculator, mental, and paper-and-pencil computations" (NCTM, 2000, p.155).

In Task 3.1 the resource at hand is an A4 paper. Students need to use the corner of the A4 paper to estimate if the angle of the triangles they are measuring is less than or more than 90^{o}. They also need to use the sides of the same A4 paper to estimate if there are any equal sides in the same triangle. Students then have to measure the angles and sides of the triangle using the appropriate tools. In Task 6.1 students need to measure the height of one stone course to estimate the height of a pilaster, and then use trigonometry to find the actual height of the pilaster.

3.5.3 Data Collection

When learning about the different ways of grouping and illustrating data, very often students are provided with such data. However, when students use a set of information that they have collected themselves it increases their "interest and motivation" (Rangecroft, 1996).

In Task 5.1 students collect data about door numbers, colours, and knockers in Triq Hilda Tabone - one of the prominent streets in Birgu. In Tasks 5.2 and 5.3 students make use of this data for illustration purposes. The representation of this data is more meaningful than the data that is presented in textbooks and typically irrelevant to students. A student in James and Williams's study describes this as "we didn't just get data from some worksheet; we saw how the data was collected, and that makes it much more meaningful" (James & Williams, 2017, p. 64).

Apart from experiencing one of the several ways of collecting data, the activity also encourages interest in the above-mentioned features which are unique to the traditional Maltese doors. Such doors, together with the facades, are full of interesting features which are part of the Maltese identity and tradition. Other attractive aspects, apart from those already mentioned, can include balconies, religious house plaques, and house names. It is important for students to be exposed to and learn about the environment that surrounds them as this helps to increase their cultural knowledge as well as their sense of responsibility, appreciation, and respect towards it.

3.5.4 Directed Discovery

In Tasks 1.2 and 3.3 students collect data, process it as per directions given as the activity progresses, and summarize the results to identify common properties (Directorate for Quality and Standards in Education [DQSE], 2015). In the case of Task 1.2 students analyse the ratio of the circumference to the diameter of various round-shaped pots to identify 'pi', and in the case of Task 3.3 students analyse the ratio of the opposite side to the adjacent side of similar triangles to identify the 'tangent ratio'. Considering the limited time students are allocated to work at each station as well as given the fact that students need to work on-site without the teacher's help, it was preferred to create directed discovery tasks rather than tasks by guided discovery or exploration.

3.5.5 Activities using GeoGebra

In Task 4.2 students are asked to take a photo of the tiling and to import the photo into GeoGebra when back at home to measure the interior angles of a polygon formed by the tiling itself. This feature was used to calculate the angle of sector, the radius and the parallel lines of the trapezia when designing Task 1.3. GeoGebra is a software that has great potential to improve the quality of teaching and learning in mathematics. It offers several opportunities to visualise, discover, investigate, and explore mathematical concepts. Thus, both students and teachers should use every opportunity to incorporate GeoGebra in their work. In addition, frequent use of this software helps teachers and students use it more efficiently.

3.5.6 Non-Syllabus Activities

Roman Numerals (Task 1.1) – Even though the Roman number system has been replaced by the Hindu Arabic numerical system, it is still being used in different scenarios such as on clock faces, in books, or on facades of buildings to commemorate significant dates. Thus, even though Roman numerals are not included in the syllabus, it is still beneficial for students to have a basic understanding of how to decipher such numbers. This task was developed with this idea in mind, as it seems that "the importance of maintaining a functional understanding of Roman numerals may depend simply on how much someone personally values this somewhat niche expertise" (Yousuf, 2020).

Porter (1960) considers the teaching of Roman numerals advantageous as it can "emphasize the vital importance of the concept of place value and the zero symbol in a way which cannot be duplicated by any other mathematics topic" (p. 97). He suggests teaching Roman numerals by capitalizing on student's "acquaintance with the familiar decimal number system" (Porter, 1960, p. 98) as presented in the table below, rather than using the 'rules' that many textbooks use to try to define the Roman number system.

	hundred-thousands'	ten-thousands'	thousands'	hundreds'	tens'	ones'	
(1)	$\overline{\mathrm{C}}$	$\overline{\mathrm{X}}$	M	C	X	I	(1)
(2)	$\overline{\mathrm{CC}}$	$\overline{\mathrm{XX}}$	MM	CC	XX	II	(2)
(3)	$\overline{\mathrm{CCC}}$	$\overline{\mathrm{XXX}}$	MMM	CCC	XXX	III	(3)
(4)	$\overline{\mathrm{CD}}$	$\overline{\mathrm{XL}}$	$\mathrm{M}\overline{\mathrm{V}}$	CD	XL	IV	(4)
(5)	$\overline{\mathrm{D}}$	$\overline{\mathrm{L}}$	$\overline{\mathrm{V}}$	D	L	V	(5)
(6)	$\overline{\mathrm{DC}}$	$\overline{\mathrm{LX}}$	$\overline{\mathrm{V}}\mathrm{M}$	DC	LX	VI	(6)
(7)	$\overline{\mathrm{DCC}}$	$\overline{\mathrm{LXX}}$	$\overline{\mathrm{V}}\mathrm{MM}$	DCC	LXX	VII	(7)
(8)	$\overline{\mathrm{DCCC}}$	$\overline{\mathrm{LXXX}}$	$\overline{\mathrm{V}}\mathrm{MMM}$	DCCC	LXXX	VIII	(8)
(9)	$\overline{\mathrm{CM}}$	$\overline{\mathrm{XC}}$	$\mathrm{M}\overline{\mathrm{X}}$	CM	XC	IX	(9)

Table 2: Roman numerals grouped according to the place value in the decimal system [11]

In this table, the seven symbols of the Roman number system are grouped such that when combined they can represent all numbers "between one and 999,999" (Porter, 1960, p. 98). The 'bar' represents one thousand times the value of the symbol.

It was felt that using the 'rules' set out in textbooks would be more practical for the aim of this task. However, this table can be used in class as a follow-up activity so that students can "appreciate more the importance and the value of the decimal system" (Porter, 1960, p. 98).

Arches (Task 1.3) – In this task students are introduced to the idea that in architecture there are different types of arches. The teacher can then expand on this concept when back in class. Even though arch types are not part of the syllabus as such, there is a lot of mathematics that

[11] *Source: Porter, 1960, p. 98.*

can be discussed, and this would help students to appreciate the beauty (the 'aesthetic' aspect) and relevance (the 'utilitarian' aspect) of mathematics.

One way how to classify and differentiate between the different arch types is based on the number of centre points from which the arc is created. For example, a Roman arch (also known as a 'semi-circular' arch) is formed by one centre, whereas a Gothic arch (also referred to as a 'pointed' arch) is composed of two centres (Murdock, 2012a, n.d.). On the other hand, "the most standard basket-handle arch is made up of three circumferential arches" (Alcayde et al., 2019, p. 3), i.e. a large central arc of radius (R) and two smaller lateral arcs of radius (r), as shown in Figure 13 below.

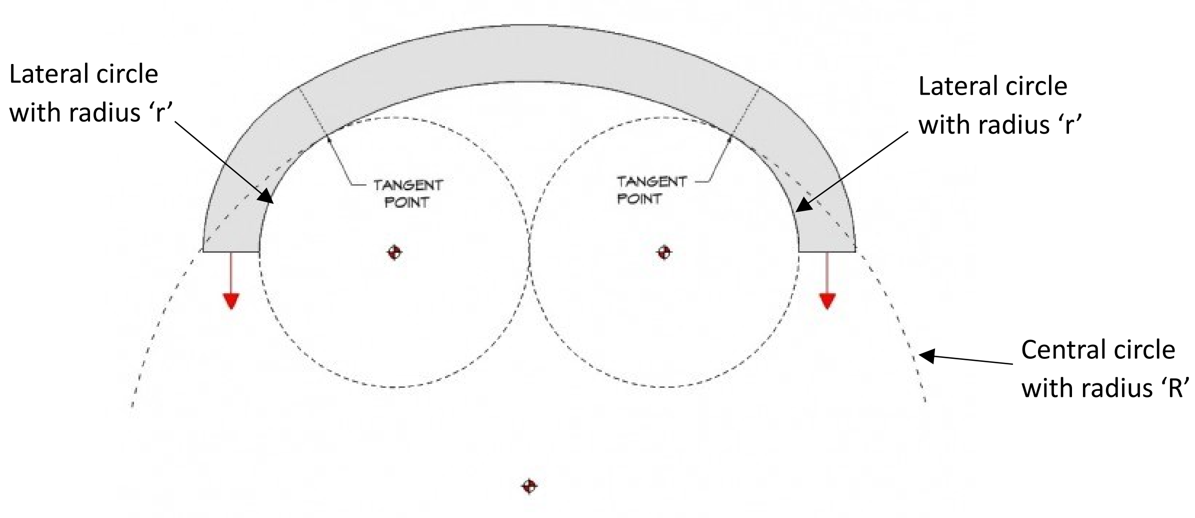


Figure 13: An example of a three-centred arch [12]

3.5.7 Historical Information

As Cahyono (2018) states "it is important for the user to recognize the surrounding environment while learning mathematics" (pp. 66-67). Thus, in addition to a generic historical information about Birgu in the introduction page, brief notes in different sections of the Resource Pack related to the area where the task is located were prepared so that students could make a connection with the site whilst doing and learning mathematics. This includes information about the unique and complex gateway structure at Birgu's entrance, the Ditch, Dockyard creek, 'il Collacchio', the door knockers, the Norman House, St. Lawrence Church,

[12] *Source: Murdock, 2012b, n.d.*

and the Oratory of St. Joseph. Such integration of historical elements provides a more holistic learning experience for students.

3.6 Conclusion

This chapter gave an overview of how the framework identified in the literature review was used to design and develop the Resource Pack. Importance was given to creating a trail that is truly an educative experience that, according to Dewey, creates continuity and does not block or arrest students. This was reflected in the level of the tasks, the setup of the questions, and the facts given to students in the information guide. Bruner's and Dale's perspectives, emphasising on enactive learning, were used to create a trail that encompasses hands-on activities as much as possible. Importance was also given to Gagne's theory of instructional design, and this was manifested in the way the stations were designed by using photos, diagrams, and historical information to attract students' attention, defining the learning objectives so that students can organize their thoughts about what they are about to do, and giving them the necessary guidance to recall information and complete the task. Pettersson's and Smaldino's principles on text and image design provided valuable guidance when creating the Resource Pack such as the importance of white space, the position of captions in diagrams and photos, the use of the red colour, the addition of page numbers and list of contents, amongst others. Additionally, literature review on cooperative work, outdoor education and mathematics trails influenced the decisions taken when designing the trail. These include the tasks that promote groupwork, the length and duration of the trail, the type of activities carried out in each task and the recommendations set in the teacher's guide in the teacher's resource pack.

Chapter 4

Conclusion

Chapter 4: Conclusion

4.0 Introduction

This study aimed to explore different perspectives in order to design and develop an outdoor mathematics trail that engages and motivates students in a meaningful learning experience. Significant effort was invested to create tasks in which students can apply the content learnt in the classroom in the real-life context, in a way that is enriching. Hopefully, this will create the desire to extend the activity at home or in class. As Classen (2002) states "it is much easier to motivate students when they see the value of the tasks they are being asked to perform", and "a rich learning task is one that sparks curiosity of students and encourages them to ask questions and extend problems" (p. 30).

This concluding chapter starts by discussing the strengths and limitations of this research study in section 4.1. The subsequent section, section 4.2, discusses the suggestions for further research while section 4.3 concludes this dissertation. My final thoughts will be shared in this last section.

4.1 Limitations of this Research Study

Ideally, an extensive evaluation should be carried out before the Resource Pack is used with students. This evaluation would include the following steps:

- Step 1: Evaluation Phase 1 - Select 1 or 2 mathematics teachers to give feedback on the Resource Pack and carry out a pilot study with 2 or 3 groups of 4 students each. Carry out the trail with each group. Monitor the exact timing for each task to be completed, any instructions which students might find unclear, any task in which students do not seem particularly motivated or in which students are not able to answer the questions, and any feedback students might give while carrying out the trail.
- Step 2: Following the pilot study, carry out any amendments (if needed) to the Resource Pack.
- Step 3: Evaluation Phase 2 - Select 2 to 4 mathematics teachers to evaluate the pack and to highlight its strengths and weaknesses. Preferably these teachers carry out the

trail with the students too. Select 2 or 3 classes of 20 to 25 students each to carry out the trail. Give the participants a questionnaire to fill in after the trail. This can be used to collect information regarding the quality of the design of the Resource Pack. In addition, carry out a semi-structured interview with all participants, in order to probe into their thoughts about the tasks in the trail.

- Step 4: Analyse the data gathered from questionnaires and interviews. Check, if for example, there is any text that needs to be simplified or any information that is missing.
- Step 5: Following the evaluation and data analysis, amend the Resource Pack accordingly.

In the case of this project, such an evaluation was beyond the aim of this dissertation, but nonetheless, it was felt that some feedback on the trail was important to improve the Resource Pack further. Hence, an initial assessment to get a feel on the quality of the trail was carried out with a Mathematics and Physics teacher (B.Ed. (Hons.)) who has more than fifteen years of experience in the field.

The feedback received whilst on site was:

Safety - the trail is well planned in terms of safety. All station locations and the paths from one station to the other feel safe. Students' safety was also the primary consideration for this teacher.

Variety of activities – the trail covers different aspects of the syllabus and not just topics typically associated with similar fieldtrips like shapes, measures, and estimation.

Interesting activities – some of the activities are particularly interesting, namely measuring height using shadows, identifying a linear sequence in the tile layout of Couvre Porte Belvedere, using the concept of nested triangles to find the tangent ratio, measuring heights using the properties of an isosceles triangle, measuring the interior angles of the polygons formed by the tiling in parts of the Ditch, and using the list of names on the plaque at the side of St. Lawrence Church for a task on probability. Also interesting are the activities related to aspects that go beyond what is covered in the syllabus such as that of the arches.

Information guide and formulae – this information helps students not to be penalized or discouraged, whilst on site, because they cannot remember the formula or the definition. It also provides information about interesting aspects, such as the door knockers, that gives students an enriching experience.

Maps – the maps are of good quality, easy to follow, and the directions given on them are clear. The maps highlighting the active station create a conducive learning environment as they reduce confusion and promote independence.

Following what has been discussed on site, some minor adjustments related to proximity and readability were carried out – in Task 1.3 an additional diagram was positioned close to the question, and in Task 3.3 the addition of the word ‘nested’ (nested triangles) was added to give students a better understanding of what was being described in the task.

However, as already indicated, this was just a preliminary informal evaluation of the trail - it was neither an evaluation of the trail, nor a pilot study. Thus, important aspects such as

- if the allotted time per station is actually sufficient for students to complete the tasks,
- how well the instructions are understood by students, and
- the type of difficulties that students might encounter (apart from those that were already taken into consideration)

could not be assessed.

4.2 Suggestions for Further Research

Whilst carrying out research, two interesting aspects that could enhance this trail and make it even more appealing to students were identified. These include a digital mathematics trail and gamification of a mathematics trail.

4.2.1 A Digital Mathematics Trail

A digital mathematics trail still has “the math trail concept at its core”, however it utilizes “the advantages of the latest technology” to offer students meaningful activities (Cahyono, 2018, p.4).

An example of such a digital tool is the MathCityMap (MCM). When using MCM, teachers can create the tasks as they would normally do in the 'traditional' maths trail, but then input the information in a database by means of a web portal. The portal also offers a digital map of the world, and the tasks can be "pinned" to this digital map (Cahyono, 2018, p. 49). Users (teachers) can also access trails created by other 'trailblazers' on the web portal as the portal aims "to support creation of maths trails by standardizing and simplifying the creation process" (Gurjanow & Ludwig, 2020, p. 269). Students can access the digital trail using the MCM application on their GPS-enabled mobile device. Refer to Figure 14.

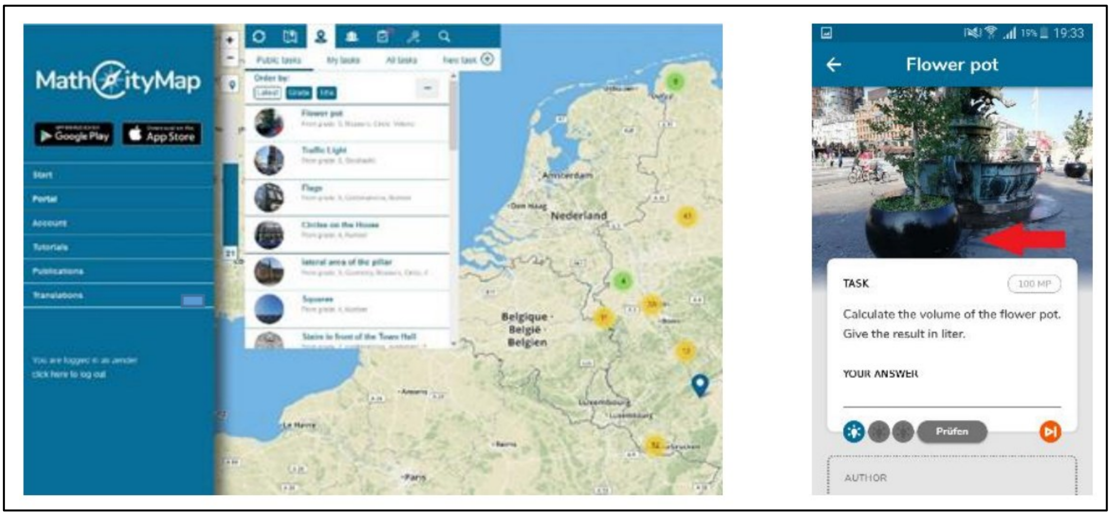


Figure 14: MathCityMap portal (left) and smartphone application (right) [13]

Similar to the 'traditional' trail, each task would still include the "question, brief information about the object, the tools needed to solve the problem" (Cahyono, 2018, p. 49). However, in a digital trail:

- hint(s) would be given as "stepped hints" and "can be opened or used only if needed by users" (Cahyono, 2018, pp. 47-48).
- students would receive real-time feedback on the correctness of the answers directly from the system.
- teachers can create a digital classroom, communicate with students, and monitor their progress, hint retrievals and location (via GPS).

[13] Zender et al., 2020, p. 3.

Given the potential benefits a digital mathematics trail offers to both teachers and students, it is an area that merits further research.

4.2.2 Gamification of a Mathematics Trail

Gamification is “the use of game design elements in non-game contexts” (Deterding et al., 2011, p. 9). In education, gamification refers to “the introduction of game design elements and gameful experiences in the design of learning processes” (Dichev & Dicheva, 2017, p. 2).

Toda et al., 2019, propose five gamification dimensions, namely:

Performance / measurement – These are elements that give extrinsic feedback to the learner to praise a specific set of actions, such as badges, medals, progress bars and points.

Ecological – These are extrinsic concepts based on chance (luck or fortune), imposed choice or explicit decisions, limited or exclusive resources to stimulate the learner through a specific goal, and time pressure such as the use of countdown timers.

Social – The elements in this dimension are related to competition, for example using scoreboards or leader boards, cooperation (teamwork), reputation and peer pressure.

Personal – This dimension provides meaning for the learner. It includes novelty in order to avoid disengagement, objectives also referred to as milestones or missions, cognitive challenges such as puzzles, renovation (boosts or extra life) so that learners can have a second chance if they fail a task, and sensation - using the learner’s senses (e.g., sound or visual stimulation) to improve the experience.

Fictional – This is a mixed dimension that is related to the user (Narrative) and the environment (Storytelling). Narrative is “the order of events as they happen in the game, through the user experience. This experience is influenced by implicit choices made by the user” (Toda et al., 2019, p. 8). Storytelling is “the way the story of the environment is told (as a script). It is told through text, voice, or sensorial resources” (Toda et al., 2019, p. 8).

The idea of gamification in mathematics trails has been present since the early trails. Gurjanow et al. (2019) describe how "the first element of Gamification reported is storytelling" (p. 66). In his paper, Blane (1989) refers to a trail at Sovereign Hill, Australia (an old gold mining town), where the storyline immerses the users in a role-playing experience of a gold miner.

```
"You've decided to try your luck at gold
 panning in the creek. But before you have
 a go, you will need to buy a Goldmining
 License. Go to the post office:
    a. How much does a license cost? (old money)
    b. How much would this be, when compared to
       todays costs? That is, would the license
       have cost the miners a lot of money?
```

Figure 15: A gamification element in the maths trail at Sovereign Hill [14]

Also, in the 1990s Muller created two trails ("Niagara Falls Math Trail" and "Welland Canal Math Trail") where three characters, Mathise Phun, Geo Metry, and Trig, discuss specific topics of interest and ask users to perform certain calculations along the trail.

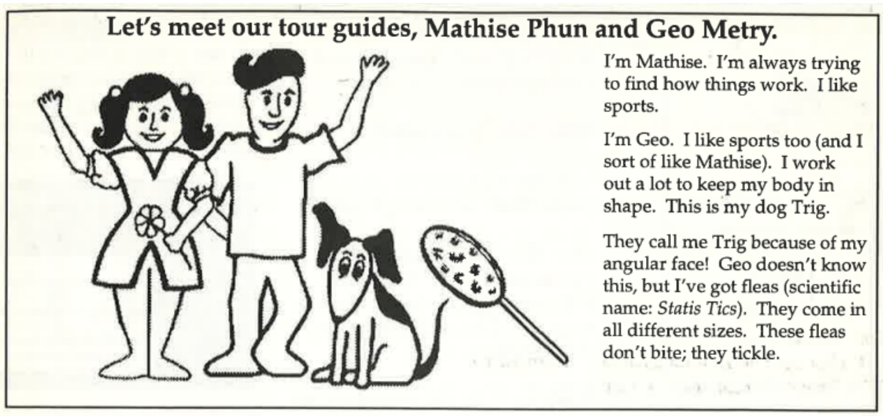


Figure 16: A gamification element in Muller's (1990s) maths trails [15]

According to Gurjanow et al. (2019) the main goal of gamification is for the users to feel intrinsically engaged, and as a result their "intrinsic motivation and commitment" (p. 66) is increased. Given that one of the main aims of a mathematics trail is also to increase motivation in students, the gamification of mathematics trails is thus an area worth exploring.

[14] *Source: Blane, 1989, p.127.*
[15] *Source: Muller, n.d., p. 3.*

4.3 Conclusion

Designing a mathematics trail demands quite an amount of time to prepare; "it can take as much time and energy as you have available" (Shoaf-Grubbs et al., 2004, p. 10). However, creating the trail is a "good math challenge" (Shoaf-Grubbs et al., 2004, p. 12), and the amount of mathematics that one can find in buildings and landscapes is actually surprising (Selinger & Baker, 1991). From my end, creating the trail took a lot of time, more than anticipated. Nonetheless it was time well spent as, apart from being an enjoyable experience, it was also a learning experience.

One of the main takeaways, which I will carry with me as a teacher of Mathematics, is that this project helped me to develop a "mathematical eye" (Barbosa & Vale, 2016, p. 70) to start observing, discovering, and exploring the mathematics "which may previously have been ignored or missed" (Blane, 1989, p. 130). Prior to this project, even though I believed in the importance of hands-on experiences, my 'mathematical eye' was quite limited. In fact, I struggled to come up with interesting tasks that cover the various topics of mathematics during my first visit to observe possible activites at Birgu. However, now I am able see the mathematics in the objects around me more easily and can use these as resources for students in the classroom. For example, grocery bills can be used to practise estimation in a way that engages students, and carton packaging of most items can be used to demonstrate various nets of cubes and cuboids. Now I know that there is a variety of tasks that students can do hands-on, and as a result appreciate and understand mathematics better.

Another important aspect is that I learnt how to create instructional material, and the importance of legibility, readability and focusing attention when designing such material. Before learning about this I thought that the more content I fit in one page the better. I also did not give much importance to font size, font colour and white space. Furthermore, I also learnt about the principle of proximity - the importance of placing related elements close to each other – and the need to have a good balance between images and text. Now I know that the language used needs to be clear and self-explanatory, the amount of content in one page needs to be limited, and the colour scheme needs to be such that important information stands out. I am now also aware that images help to attract and maintain attention, and that any labelling or reference to such images needs to be in close proximity. Thus, the experience gained by developing the Resource Pack will help me, as a teacher, to prepare instructional material that attracts and motivates students, and that also captures and engages their attention.

Another positive aspect of planning this trail is that it has given me the opportunity to discover various new aspects of Birgu so much, that, I will never see this place in the same light again.

In conclusion, designing this trail has been an enriching experience from which I gained new insights, both about teaching and learning mathematics, as well as Birgu as a heritage site. I hope that students carrying out this trail will be inspired in the same way too!

References[16]

[16] References used in the Resource Pack are included in the 'References' section in the Teacher's Resource Pack.

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Appendix A

Extracts from Blane and Clarke's Maths Trail

APPENDIX A

[17]

A. STATE BANK CENTRE

From the State Bank Centre cross Bourke St and then Elizabeth St carefully so that you are on the corner diagonally opposite the State Bank Centre and look back at it.
Without calculating write down a quick estimate of the number of small windows you can see in the side of the tower facing you.. A1

Now calculate the number of windows.. A2
(Hint: Count how many there are in each row and the number of floors.)

A window cleaner takes ¼ hour to clean each window. If he works for 8 hours each day, 5 days per week, how many weeks will it take to clean all the small windows on the building?..... A3

B. GENERAL POST OFFICE (GPO) — OUTSIDE

Walk about 50 metres up Elizabeth St. (North) and find the stamp machines outside the GPO at the top of the steps. The one on the left takes coins up to 50c (1, 2, 5, 10, 20, 50).
To be able to purchase the lowest value stamp provided by this machine, calculate:

(i) the greatest number of coins you could use?.. B1

(ii) the least number of coins you need to use?... B2

In how many different ways could you put coins to the value of 10c into the machine?......... B3

On your left, beside that machine you will see details of posting times. If you post a letter to Perth on a Tuesday at 5.00 pm, would you expect it to arrive on the next working day?............. B4

If you post a letter to Sydney at the same time, will that one arrive on the next day? B5

C. GENERAL POST OFFICE (GPO) — INSIDE

Enter the GPO Building from Elizabeth Street. On your right is the Daily Weather Report.
From this:

(i) What is the rainfall for Melbourne so far this year, in millimetres?............................. C1

(ii) Is this above or below the average for the past 129 years?... C2

[17] Blane, 1989, p. 131.

18

(iii) Estimate the average monthly rainfall so far this year C3

(iv) If rain continues to fall at the same rate for the rest of the year, estimate the total rainfall for this year C4

(v) What might cause your answer to be different from the actual answer?
....
....
.... C5

The River Report is also on this wall. From the information given, calculate:
(i) The height of the Murray River at Bringenbrong.. C6
(NB. Use another town on the Murray River if the Bringenbrong figures are not available.)

(ii) How far, in metres, does the Murray have to rise at the town that you used before it floods? C7

***CHALLENGE QUESTION*?**

If the change in the height of the river for the past 24 hours continued at the same rate for some time, calculate how long it would be before the river:
(i) floods? (if the change is positive) ....
(ii) dries up? (if the change is negative).. C8

Moving further into the GPO from the entrance, you will see hundreds of small private mail boxes on your left. Look at the top five rows on Board A. If we use a co-ordinate system with 1 to 36 along the bottom and 1 to 4 up the left hand edge, then Box 5A is at position (1, 1).
What box is at (17, 3)?.. C9

Give the co-ordinates of box number 27A.. C10

Leave the GPO by the same door and walk down Elizabeth St. and enter the Bourke St. Mall. Re-enter the GPO through the Mall entrance. In the passage there is a plaque on the wall on your right. Assuming that Victoria is 150 years old in 1985, calculate how old the State of Victoria was when the Postal Hall was established?.. C11

Behind you is a painting of the only English Captain who came to Australia and failed to take a wicket or score a run.
Write down his name C12

D. BOURKE STREET MALL

Go back down the steps into Bourke St. Mall and walk past the David Jones Store.
Move to the front of "K-K-K-Katies", watching carefully for trams. Find a circular pattern in the paving.
How wide across the middle is the smallest circle in brick widths? (This is called the diameter (D)) ....D = D1

[18] Blane, 1989, p. 132.

Appendix B

Coursework Extracts from the Mathematics SEC Syllabus 2025 as available in April 2022

APPENDIX B

[19]

Scheme of Assessment

School candidates

The assessment consists of:

Coursework: 30% of the total marks; comprising 3 assignments of equal weighting i.e. 10% each; set during the three-year course programme.

Coursework assignments can be pegged at either of two categories:

- A coursework assignment at MQF level categories 1-2 must identify assessment criteria from these two MQF levels. The ACs are to be weighted within the assignment's scheme of work and marking scheme at a ratio of 40% at Level 1 and 60% at Level 2.
- A coursework assignment at MQF level categories 1-2-3 must identify assessment criteria from each of Levels 1, 2, and 3. These ACs are to be weighted within the assignment's scheme of work and marking scheme at a ratio of 30% at each of Levels 1 and 2 and 40% at Level 3.

The mark for assignments at level categories 1-2 presented for a qualification at level categories 2-3 is to be recalculated to 60% of the original mark. The mark stands in all other cases.

Controlled assessment: 70% of the total marks; comprising of a two-hour written exam; set at the end of the programme and differentiated between three tiers:

a. MQF levels 1 and 2;
b. MQF levels 2 and 3;
c. MQF level 3 and level 3 Extension Assessment Criteria.

Candidates can obtain a level higher than Level 1 if they satisfy the examiners in both coursework and controlled assessments, irrespective of the total marks obtained.

Part 1: Coursework

The **coursework** will be based on LO 1 – LO 10. An overview of the suggested coursework assignments is shown in the table below:

Part 1: Coursework – Category Levels 1-2-3 (30 %)				
Assignment 1 (10 %)	**Assignment 2** (10 %)	**Assignment 3** (10 %)	**Assignment 4** (10 %)	**Assignment 5** (10 %)
Investigating (8%) + Non-Calculator Test (2%)	Modelling (8%) + Non-Calculator Test (2%)	Solving (8%) + Non-Calculator Test (2%)	Maths Trail (8%) + Non-Calculator Test (2%)	Research Project (8%) + Non-Calculator Test (2%)

Figure 1: Suggested Coursework Assignments for School Candidates

- Candidates will be assessed through 3 assignments carried out during this three-year programme – 1 assignment in Year 9, 1 assignment in Year 10 and 1 assignment in Year 11.
- Coursework Assignment 1 or 2 is mandatory and one can choose or repeat any of the 5 suggested Assignments in completing their coursework part.

[19] Mathematics SEC Syllabus 2025, p. 65.

[20]

- All coursework assignments shall be marked out of 100 (20% from the Non-Calculator Test and 80% from the other mode) according to guidelines and marking criteria available with this syllabus.
- School candidates' assignments, forming part of coursework, are to be available at the candidates' school for moderation purposes as indicated by the MATSEC Board.

Part 2: Controlled Assessment II

Part 2: Controlled Assessment II (2 hours) (70 %)
Paper consisting of questions at Level 1-2 **OR** Paper consisting of questions at Level 2-3. **OR** Paper consisting of questions at Level 3-3*(**Extension Assessment Criteria**).

Figure 2: Part 2 Controlled assessment for School Candidates

- Controlled Assessment II:
  - will cover all learning outcomes;
  - be marked out of 100 and all questions are compulsory - answers are to be written on the examination paper provided.
    - Controlled Paper 1-2 will consist of 10 – 15 questions:
      - 40% of the marks allotted will be based on Assessment Criteria from MQF 1
      - 60% of the marks allotted will be based on Assessment Criteria from MQF 2
    - Controlled Paper 2-3 will consist of 10 – 15 questions:
      - 40% of the marks allotted will be based on Assessment Criteria from MQF 2
      - 60% of the marks allotted will be based on Assessment Criteria from MQF 3
    - Controlled Paper 3-3* will consist of 9 – 13 questions:
      - 40% of the marks allotted will be based on Assessment Criteria from MQF 3
      - 60% of the marks allotted will be based on **Extension Assessment Criteria** at MQF 3

[20] Mathematics SEC Syllabus 2025, p. 66.

[21]

Coursework Mode 4: Maths Trail

Maths Trail	
80 marks internally-assessed externally-moderated	**Defining the Maths Trail** A maths trail is an organised route involving a number of different tasks where pupils look for clues, spot mathematical designs and patterns, take measurements using appropriate instruments, make estimations, draw sketches, work out calculations, find solutions, solve problems, etc. Maths trails can be conducted in various environments such as: the boundaries of the school or outside the school which could be another building like a museum or a supermarket or any other outside space like parks and playgrounds. **The aim of Maths trails** The aim of Maths trails is to help pupils appreciate the use of mathematics in environments outside the classroom thus bringing into greater focus the utilitarian aspect of mathematics. It helps pupils make the necessary connections between the various topics learnt in class and links classroom-based exercises with real-life contexts. **Guidelines for Setting, Conducting, and Assessing a Maths Trail** The Maths Trail assignment can be set at either MQF Level 1-2, (40% at Level 1 – 60% at Level 2), or at MQF Level 1-2-3 (30% at Level 1 – 30% at Level 2 – 40% at Level 3). It will consist of 6 to 10 mathematical tasks designed by the teacher and may take the form of a set of handouts/booklet. These are set in various locations in the designated site of the trail, targeting a number of learning outcomes covered in class. Each task is made of a number of parts, some of which are marked with an asterisk (*) meaning that these will be filled on site. The assignment will be conducted in two phases: The first phase takes place on site where pupils may work either individually, or collaboratively, as indicated by the teacher. They will be supplied with all the required measuring instruments like compasses, thermometers, clinometers, pedometers, stop watches, trundle wheels and measuring tapes as required. Pupils will collect and write down all the required data at each stage of the trail. In the second phase, each pupil, working individually, uses the data gathered on site to complete the tasks. The pupil's work is then collected by the teacher for marking using a marking scheme. Each pupil is required to hand in this assignment on an individual basis. **Marking Scheme Guidelines** **Types of Marks** Method Marks Method marks (denoted by **M**) are awarded for knowing a correct method of solution and attempting to apply it. Method marks cannot be lost for non-method related mistakes. They can only be awarded if the method used would have led to the correct answer had a non-method mistake not been made. Unless otherwise stated, any valid method, even if not specified in the marking scheme, is to be accepted and marked accordingly. Accuracy Marks There are two types of Accuracy marks (denoted by A or B): **A** marks are accuracy marks given for correct answer only. Incorrect answers, even though nearly correct, score no marks. Accuracy marks are also awarded for incorrect answers which are correctly followed through (denoted by **A** f.t.) from a previous incorrect answer, provided that f.t. is indicated in the marking scheme. **B** marks are accuracy marks awarded for specific results or statements independent of the method used. Note: No Method marks (M) or Accuracy marks (A) are awarded when a wrong method leads to a correct answer. The global mark for the Maths Trail is 80. If the total mark of all the tasks is different, it should be proportionately recalculated out of 80. This mark will be added to the mark obtained in the Non-Calculator Test for a final mark of the assignment out of 100.

[21] Mathematics SEC Syllabus 2025, p. 94.

Appendix C

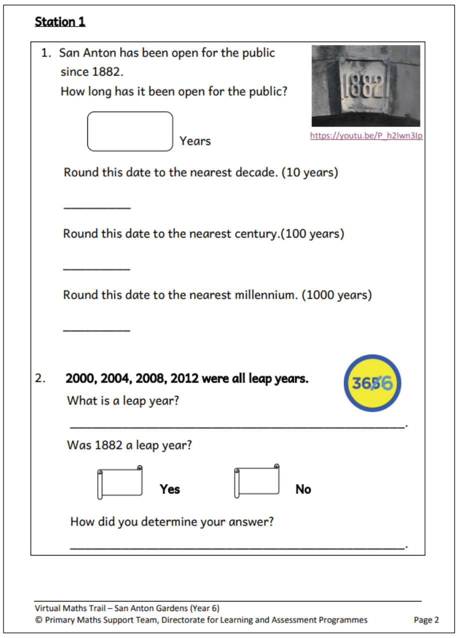
Extracts and Web Links of Local Trails[22]

[22] All web links in this appendix were last accessed on July 29, 2023.

APPENDIX C

Maths trails available to the public on websites

San Anton Gardens (Year 6):


Station 1

1. San Anton has been open for the public since 1882.
How long has it been open for the public?
[ ] Years
https://youtu.be/P_h2lwn3lp
Round this date to the nearest decade. (10 years)
____
Round this date to the nearest century.(100 years)
____
Round this date to the nearest millennium. (1000 years)
____

2. **2000, 2004, 2008, 2012 were all leap years.**
What is a leap year?
________________________________.
Was 1882 a leap year?
[ ] **Yes** [ ] **No**
How did you determine your answer?
________________________________.

Virtual Maths Trail – San Anton Gardens (Year 6)
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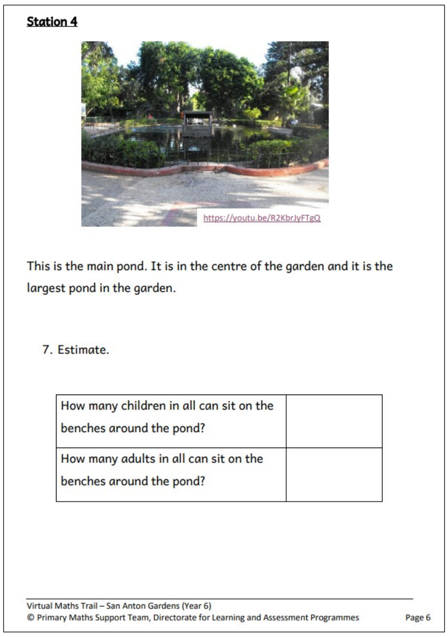
Station 4

https://youtu.be/R2KbrJyFTgQ

This is the main pond. It is in the centre of the garden and it is the largest pond in the garden.

7. Estimate.

How many children in all can sit on the benches around the pond?	
How many adults in all can sit on the benches around the pond?	

Virtual Maths Trail – San Anton Gardens (Year 6)
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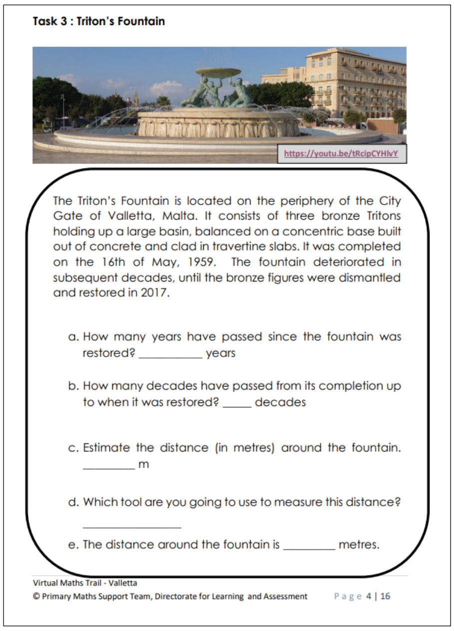
[Resource_Maria_Xuereb_0904_morn_lesson69_San_Anton_Maths_Trail-f3ec9bd6188e55d34c53d71e96cdc610.pdf (teleskola.mt)]

Valletta (Years 5 and 6):


Task 3 : Triton's Fountain

https://youtu.be/tRcipCYHlvY

The Triton's Fountain is located on the periphery of the City Gate of Valletta, Malta. It consists of three bronze Tritons holding up a large basin, balanced on a concentric base built out of concrete and clad in travertine slabs. It was completed on the 16th of May, 1959. The fountain deteriorated in subsequent decades, until the bronze figures were dismantled and restored in 2017.

a. How many years have passed since the fountain was restored? ________ years

b. How many decades have passed from its completion up to when it was restored? ____ decades

c. Estimate the distance (in metres) around the fountain. _______ m

d. Which tool are you going to use to measure this distance? ____________

e. The distance around the fountain is _______ metres.

Virtual Maths Trail - Valletta
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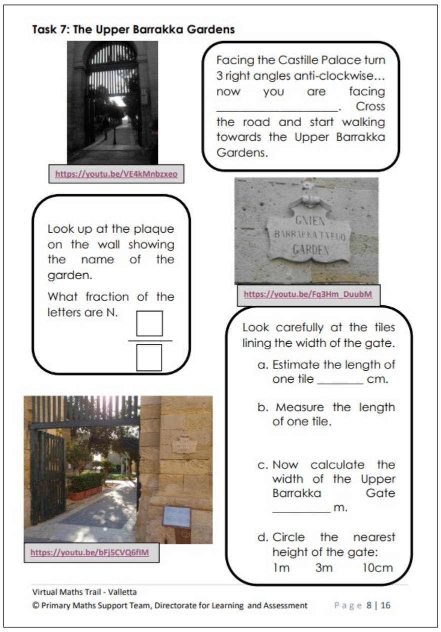
Task 7: The Upper Barrakka Gardens

https://youtu.be/VE4kMnbzxeo

Facing the Castille Palace turn 3 right angles anti-clockwise… now you are facing ______________. Cross the road and start walking towards the Upper Barrakka Gardens.

Look up at the plaque on the wall showing the name of the garden.
What fraction of the letters are N.

https://youtu.be/Fq3Hm_DuubM

Look carefully at the tiles lining the width of the gate.
a. Estimate the length of one tile ______ cm.
b. Measure the length of one tile.
c. Now calculate the width of the Upper Barrakka Gate _______ m.
d. Circle the nearest height of the gate:
1m 3m 10cm

https://youtu.be/bFjSCVQ6flM

Virtual Maths Trail - Valletta
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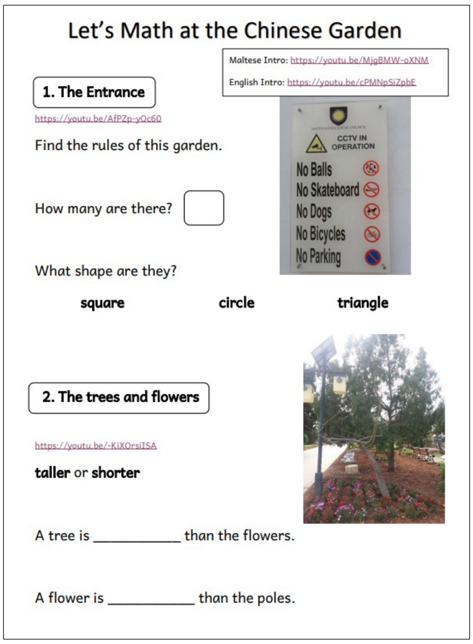
[Resource_Maria_Xuereb_1504_morn_lesson80_Valletta-Virtual-Trail-Years-5-and-6-6388d301f255ccfbc26e8f97ef557032.pdf (teleskola.mt)]

Chinese Garden (Year 1):



Let's Math at the Chinese Garden

Maltese Intro: https://youtu.be/MjgBMW-oXNM

English Intro: https://youtu.be/cPMNpSiZpbE

1. The Entrance

https://youtu.be/AfPZp-yOc60

Find the rules of this garden.

CCTV IN OPERATION
No Balls
No Skateboard
No Dogs
No Bicycles
No Parking

How many are there?

What shape are they?

square **circle** **triangle**

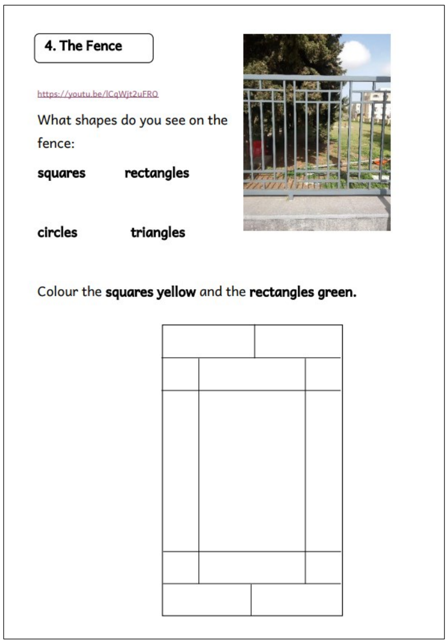
2. The trees and flowers

https://youtu.be/-KiXOrsiISA

taller or **shorter**

A tree is ___________ than the flowers.

A flower is ___________ than the poles.



4. The Fence

https://youtu.be/lCqWjt2uFRO

What shapes do you see on the fence:

squares **rectangles**

circles **triangles**

Colour the **squares yellow** and the **rectangles green.**

[Resource Analisa Magro 0705 morn Lesson 109 Maths Trail Part1-e6e5a8a89bcb1e8b042cc378bebf7267.pdf (teleskola.mt)]

Villa Rundle Gardens (Year 2):

[Maths trail 2019 - Yr2 - St. Francis School (mmargherita.org)]

Villa Rundle Gardens (Year 4):

[Yr 4 Maths Trail 2019 - St. Francis School (mmargherita.org)]

Cittadella (Year 6):

[Yr 6 Maths Trail at the Citadel - St. Francis School (mmargherita.org)]

Cittadella (Year 9):


[Maths Venture – Science Centre, Pembroke (sciencecentrepembroke.mt)](sciencecentrepembroke.mt)

Maths trails prepared by the Heads of Department within the Secretariat for Catholic Education and kindly provided by a teacher in a Church School [23]

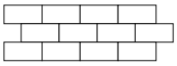
San Anton Gardens (Year 7):

Task 1: Architecture at the Eagle's Fountain

20mins.

Locate the Eagle's Fountain near the entrance of the garden.

a) Look closely at the facade of this fountain. The way in which the stones are laid are forming patterns in some areas. The following is an example of a stone pattern:

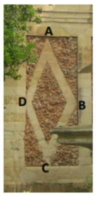


Make three drawings of other patterns that can be noted on the facade.

(3 marks)

gbonnici/12 Page 4

b) Look closely at this part of the fountain:



What is the name of the shape ABCD?

______________ (1 mark)

The four angles of this shape add up to _____°.

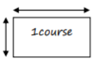
(1 mark)

List three properties of this shape:

1. ______________
2. ______________
3. ______________

(3 marks)

c) The shape ABCD is enclosed in a rectangle. By measuring the length and height of one course (filata) which is accessible to you on the side of the fountain, give an **estimate** of the Area of this rectangle. Do your calculations in metres.



Ans: ________ m²

(3 marks)

gbonnici/12 Page 5

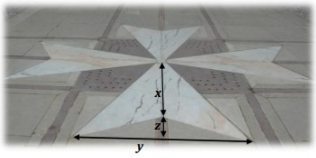
[23] S. Bartolo, personal communication, Jan 5, 2022

Mdina (Year 9):

Task 2

Maltese Cross at the National History Museum Courtyard

Time: 20 mins.

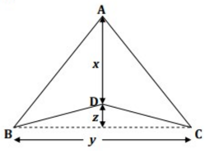


At the entrance of the National History Museum in St. Publius Square, you would find a paved Maltese cross.

a) What is the order of rotational symmetry of this cross? Ans: ________

(1 mark)

b) Using a measuring tape, measure the lengths ***x, y*** and ***z*** as indicated on the diagram:



x = __________cm

y = __________cm

z = __________cm

(3 marks)

c) Use the measurements found in part (b) to find the **Total Area** of the Maltese Cross. *Hint: Make use of the large triangle ABC and the small triangle BCD.*

(6 marks)

Valletta (Years 9 and 10):

Task 7: The Knot Monument

The 'knot' monument was erected to commemorate Malta's hosting of the Valletta Summit on migration. It symbolises Malta as a bridge between Europe and Africa. The marble was cut from an area renowned for having the finest marble in the world, from Carrara in Italy. The marble structure in this monument weighs 24 tonnes.

D a. How would you describe the structure to someone who can't see it?

____

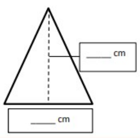
____

____

D b. Look at the triangle indicated by the arrow. What kind of triangle is this?

ANSWER

c. Measure the base and height of the triangle and record your measurements on the diagram provided.



d. Use trigonometry to find one of the base angles of the triangles.

ANSWER

e. Use Pythagoras' theorem to find one of the slant edges of the triangle.

ANSWER

f. The plaque below commemorates the 2015 Valletta Summit on Migration.

- Translate point X $\binom{-1}{0}$ and label it A
- Translate point A $\binom{0}{5}$ and label it B
- Translate point B $\binom{-1}{0}$ and label it C
- Translate point C by $\binom{2}{2}$ and label it D
- Join the points and reflect the resulting shape on the dotted

Attachment A

Teacher’s Resource Pack

BIRGU MATHS TRAIL

YEAR 10 TRACK 3

Teacher's Resource Pack

Abstract

A mathematics trail is an organized route which gives students a first-hand experience to apply mathematical knowledge in a way that is real and hands on.

Celine Vassallo Grant

August 2023

Foreword

Author's Note

My wish is for students to have various opportunities where they can experience mathematics in a way that is real and hands-on, with the ultimate aim being that of students finding the subject useful, relevant, and enjoyable. The mathematics trail is one such experience. Designing this trail has been an enriching experience from which I learnt new aspects, both about mathematics and Birgu. I hope that students carrying out this trail will be inspired in the same way too!

About the trail pack

This mathematics trail has been designed in an effort to help students appreciate the utilitarian, aesthetic, and communication aspects of mathematics, in an outdoor environment whilst working cooperatively in a group. It encompasses several activities covering the different topics related to mathematics and aspects from other curricular subjects such as History and Physics.

There are two resource packs related to this trail – the students' pack and the teachers' pack. The teachers' pack, apart from all the material presented in the students' pack, also includes a teacher's guide, the expected working and answer of each task, and additional information related to each station such as the Learning Outcomes, ideas for preparatory work, and difficulties students might encounter whilst carrying out the task. The Learning Outcomes are taken from the Mathematics SEC Syllabus 2025 as was available in April 2022.

Illustrations

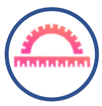
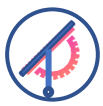
All photos shown in these resource packs were taken by the creator of this trail. The map of Birgu was purchased from the Planning Authority.

Acknowledgements

I would like to thank and express my immense gratitude towards my tutor Dr Leonard Bezzina for his constant and invaluable help in the creation of this project.

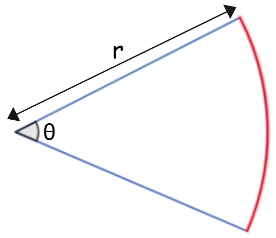
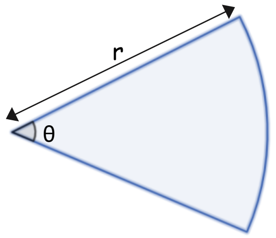
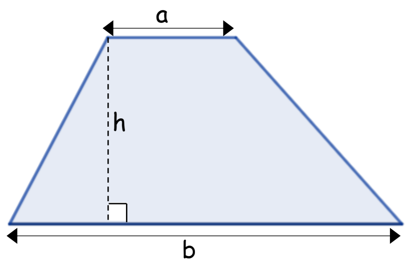
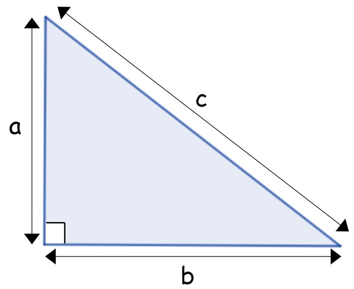
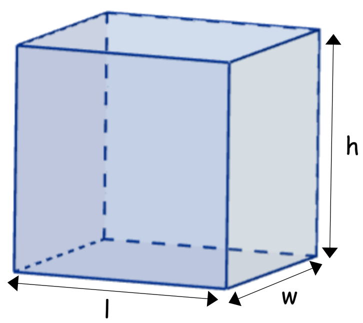
Table of Contents

List of Symbols

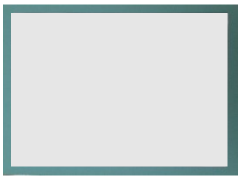
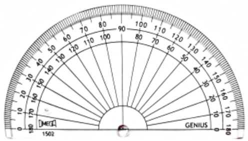
Symbol	Meaning
	Tape Measure Icon by Turkkub on freeicons.io
	Sewing Tape Measure Icon by Muhammad Naufal Subhiansyah on freeicons.io
	Calculator Icon by DotFix Technologies on freeicons.io
	Photo Camera Icon by Raj Dev on freeicons.io
	Protractor Icon by Cuby design on freeicons.io
	Clinometer Icon by Cuby design on freeicons.io
	Stopwatch Icon by MD Badsha Meah on freeicons.io
	Navigating Compass Icon by Anu Rocks on freeicons.io

Symbol	Meaning
	Think and Explain Icon by Supalerk laipawat on freeicons.io
	Time Calculations Icon by icon king1 on freeicons.io
	'Aha' Moment Icon by Hilmy Abiyyu Asad on freeicons.io
	Information Guide Icon by Melvin ilham Oktaviansyah on freeicons.io
	Formulae List Icon by Manthana Chaiwong on freeicons.io
	Work out/ Write Icon by Hilmy Abiyyu Asad on freeicons.io
	Answer Box
	Homework Icon by Anu Rocks on freeicons.io

List of Formulae

Length of arc	$\frac{\theta}{360^\circ} \times 2\pi r$	
Area of sector	$\frac{\theta}{360^\circ} \times \pi r^2$	
Area of trapezium	$\frac{1}{2}(a+b)h$	
Pythagoras' Theorem	$c^2 = a^2 + b^2$	
Volume of cube /cuboid	$V = l \times w \times h$	

List of Resources

			
Clip Board	Pencil and Biro	A4 Blank Paper	Tracing Paper
			
Ruler (30cm)	White Chalk	Protractor	Clinometer**
			
Tape Measure (5m or more)	Sewing Tape Measure (1m or more)	Disc Tape Measure (20m or more)	
			
Calculator *	Photo Camera *	Navigating Compass *	Stopwatch *

* Resource can be an app on the mobile phone

** Photo kindly provided by a teacher in a secondary school

A Teacher's Guide

Resource Pack Layout

The trail is made up of six different locations around Birgu, referred to as 'stations'. In each station students have three or four tasks to carry out. The tasks are independent of each other, thus the teacher can decide to omit certain tasks because of time constraints. The teacher can also decide to split the trail tasks over two days, for example across two terms or between Year 9 and Year 10. If the trail is to be used as an assessed coursework, the SEC Syllabus 2025 (MATSEC, 2022) suggests 6 to 10 tasks for which students need to collect on-site data and then complete the work individually off-site. It also suggests that 30% of the content is at Level 1, 30% at Level 2, and 40% at Level 3. The MQF Level of each task can be found in the teacher's notes sections. Thus, in the case of assessed work, the teacher can choose from the various tasks available in this resource pack to prepare a trail that meets the SEC Syllabus 2025 (MATSEC, 2022) requirements. The content of this trail (i.e., including all 22 tasks) consists of 31% at Level 1, 34% at Level 2, and 35% at Level 3.

The first page of each station consists of three main parts:

- A map to show the location of the station.
- The learning objectives of the tasks within that station
- A photo of the area where the station is located.

Each question has an icon assigned to it. These icons indicate the tools that students need to use for that particular task, and whether the task is to be carried out on site, or whether it is to be continued at home as individual work (following the recording of on-site data).

Preparation

"Any project takes organization and planning" (Shoaf-Grubbs et al., 2004, p. 14), and so does a mathematics trail.

Prior to the trail, the teacher needs to prepare students by:

- Giving students an overview of the trail and what it entails – students need to know what is expected of them during the trail, for the day to run smoothly and for the learning objectives to be reached. In addition, "it is most important that the children should understand the terminology used so that no time is wasted on the trail" (Lumb, 1980, p. 5).
- Letting students know the groups they will be split in - the groups should be heterogeneous as much as possible.
- Explaining the various roles students need to assume during the different tasks – for example in one task student A and B hold the measuring tape (one at each end), student C reads the measurement and Student D writes it down. Then in another task students rotate roles, for example student C looks through the clinometer, Student A checks the clinometer is not tilted during measurement, Student D reads the angle, and student B writes it down.
- Dedicating a lesson or two in which students are given exercises that enable them to use the tools, especially those that students might not be familiar with, in particular the compass, clinometer and stopwatch.
- Informing students and the adults supervising the group about the importance of time keeping - students have a total of thirty minutes to finish the tasks in each station and arrive at the next station. In case students are not ready from the task they still need to leave the station when the time is up, so that all groups can start working on the next station according to the time schedule.
- Revising the 'code of behaviour during outings'.

As Vella (2015) suggests, "preparation" and "military precision" are key for such activities to be a success (p. 30).

Trail Itinerary

The amount of time allocated to the trail is of three hours. This includes twenty minutes at each station to carry out the tasks, and ten minutes transition time in-between stations. The teacher needs to set a time schedule for the trail. The following is an example of such schedule.

The start and finish time may vary depending on the school opening hours. The break at the end of the trail is optional, it mostly depends on the time students need to be back at school.

09:00	Arrival at Birgu
09:00 to 09:30	Briefing to students at the 'Meeting Point'
09:30 to 10:00	Activity 1
10:00 to 10:30	Activity 2
10:30 to 11:00	Activity 3
11:00 to 11:30	Activity 4
11:30 to 12:00	Activity 5
12:00 to 12:30	Activity 6
12:30 to 13:00	Break (optional)
13:00	Departure from Birgu

Logistics

Meeting Point - The meeting point to start the trail is the area in front of the War Museum, as shown in photo below. It is a safe space to gather all students together for the briefing, as well as for students to have break when the trail is over. This area also offers a pay public convenience and a cafeteria.

Transport – The recommended pick-up/drop-off point for transport is the area just outside Couvre Porte area, as shown in photos below.

Birgu Events – There are several events happening at Birgu all year round. Thus, it is best to contact the Local Council when planning the trail to ensure that there aren't any activities or preparations happening on the day of the trail. Birgu, similar to other localities in Malta, has its own open market, in this case held every Tuesday, however this is located outside of Birgu and does not affect the trail.

Group work

The trail is to be carried out as group work and students need to be split into six groups. The recommended amount of students per group is four to five students. The six groups will each start from a different station and then move on to the next station according to a pre-defined schedule, as shown below.

	Activity 1	Activity 2	Activity 3	Activity 4	Activity 5	Activity 6
Group A	Station 1	Station 2	Station 3	Station 4	Station 5	Station 6
Group B	Station 2	Station 3	Station 4	Station 5	Station 6	Station 1
Group C	Station 3	Station 4	Station 5	Station 6	Station 1	Station 2
Group D	Station 4	Station 5	Station 6	Station 1	Station 2	Station 3
Group E	Station 5	Station 6	Station 1	Station 2	Station 3	Station 4
Group F	Station 6	Station 1	Station 2	Station 3	Station 4	Station 5

It must be ensured there are sufficient teachers/LSEs/parents as helpers (Baker & Selinger, 1991, p. 5) with each group, so that students are supervised at all times. However, in order to aid students to work autonomously as a group, the following information is provided in their resource pack:

- The 'Trail Sequence for Group X' map – This map is found at the end of the trail pack, there is one for each group. The aim of this map is for the directions to be "sufficiently self-explanatory" (Baker & Selinger, 1991, p. 6). Thus, the map indicates the path students need to follow to go from one activity to the next. Markers with key points, such as the activity number and the meeting point, are indicated on the map too.

- The 'Active Station' map – This map is found in the first page of each station in the trail pack. It shows the location of the station students are working on (the 'active station') with respect to the other five stations. The active station's area is highlighted in red, whereas the other stations are highlighted in yellow.
- The 'Information Guide' – This is the information given to students in order to help them carry out the task. It is normally enclosed in a box and labelled with the corresponding icon.

Suggestions

The teacher needs to assess whether students will be able to carry out all of the tasks set in each station or not. This depends on various factors such as the ability of students, the confidence of students in using the tools, whether students are used to working cooperatively, the topics the teacher wishes to cover in the trail, and whether students have been on other maths trails in previous terms or years.

Students should use white chalk, rather than pencils or biros, when marking a temporary mark during measurements, so as to leave the least possible impact on the environment.

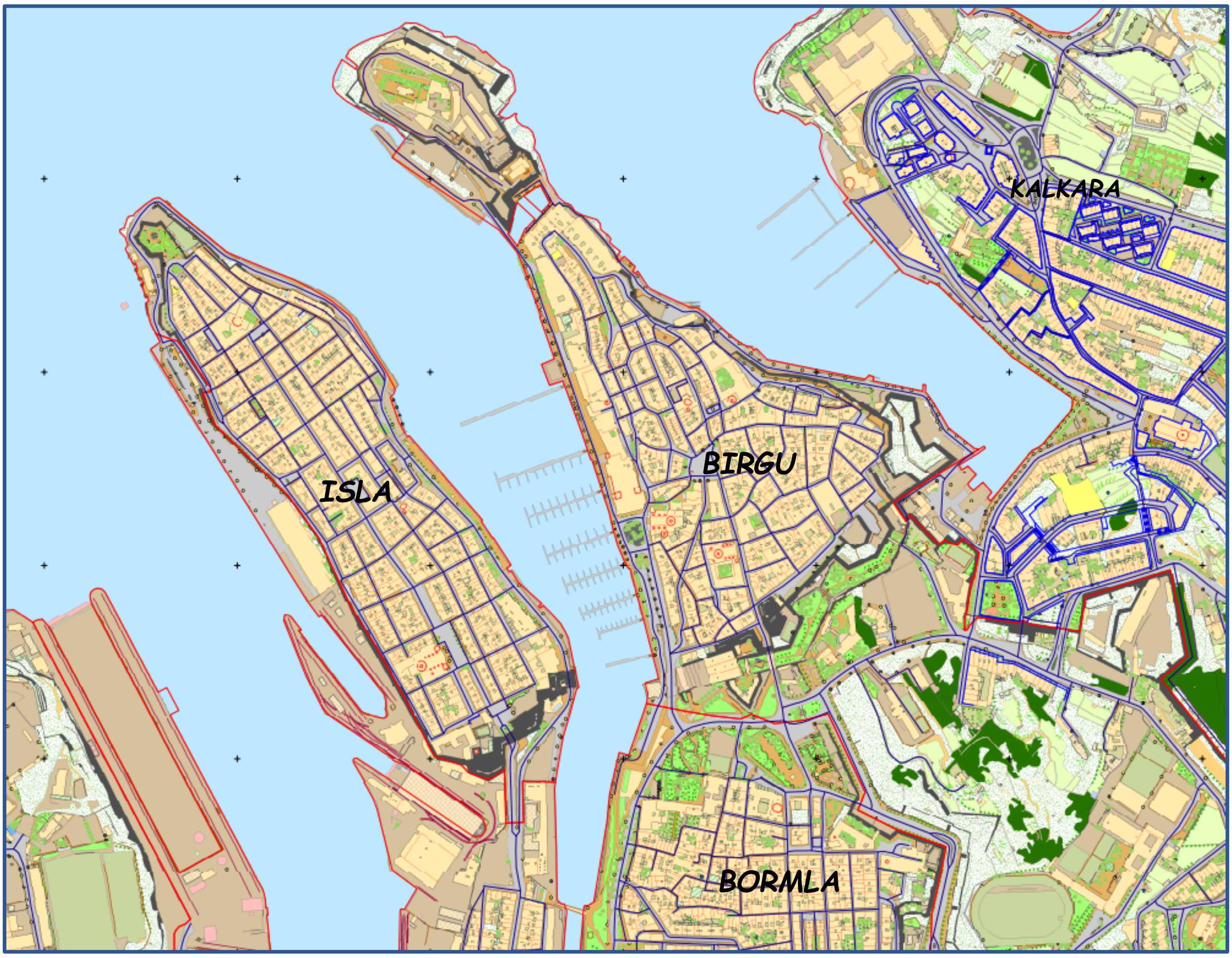
Ideally a workshop is set near the meeting point, i.e., in front of the Birgu War Museum, with supplies such as extra paper, pencils, measuring tools, and preferably some refreshments too (Baker & Selinger, 1991, p. 6).

Follow-up work

After the trail students have off-site work. This can be carried out either at home or in class. This depends on the aims the teacher is using the trail for, such as whether the trail is being used to promote cooperative work or whether it is being used as part of the assessed coursework (or school-based assessment).

A class discussion should always be held to reflect on the experience - the favourite task, the most challenging task, the 'aha' moments, the different solutions and strategies students adopted etc.

General Information on Birgu



Mapping Unit - Planning Authority (www.pa.org.mt)

Birgu, also known as Vittoriosa, is one of the Three Cities, the others being Bormla (Cospicua) and Isla (Senglea). They are all located along the shores of the Grand Harbour. Birgu is built on a peninsula, between Dockyard Creek and Kalkara Creek. It has always been a maritime settlement. The people of Birgu are called Vittoriosani and its population stands at around 2,300.

When the Knights of St. John came Malta in 1530, they first settled in Birgu and remained there for about forty years before building and eventually moving to Valletta in 1571. During the Great Siege of 1565, Birgu played a very important role and was given the other name of Citta' Vittoriosa after the victory of the Knights over the invading Turkish armada.

Although a number of buildings and bastions built by the Knights were destroyed during World War II, quite a number of buildings and fortifications have survived as we will see along this trail.

Station 1 – Teacher's Notes

Resources

- Sewing Tape Measure
- Tape Measure
- Calculator
- Pen and Resource Pack

Learning Outcomes

Task No	Strand	MQF Level	Learning Outcome
1.2	Shape, Space and Measure	1	6.1au Identify one or more of the main components of a circle (i.e. centre, radius, diameter, and circumference).
1.2	Shape, Space and Measure	2	5.2an Define the notion of π as the ratio of the circumference to the diameter.
1.3	Shape, Space and Measure	2	5.2al Use formulae to find the area of a parallelogram, a triangle and/or a trapezium.
1.3	Shape, Space and Measure	2	5.2am Calculate the area of compound shapes that include triangles, parallelograms and/or trapezia.
1.3	Shape, Space and Measure	3	5.3ax Calculate the area of compound shapes that include sectors and/or segments of circles.
1.4	Shape, Space and Measure	1	6.1ag Describe the properties of a square and/or rectangle.
1.4	Shape, Space and Measure	3	5.3av Calculate the length of an arc of a circle.
1.4	Shape, Space and Measure	3	5.3aw Calculate the area of a sector and/or segment of a circle.

Task 1.1 is related to one of the suggested ideas for a research project in the Mathematics SEC Syllabus 2025: “Conduct research on the Roman number system. Compare and contrast the Roman number system and the Hindu-Arabic number system we are using now. Include examples of the use of the Roman number system in the local context” (MATSEC, 2022, p. 113).

Additional Information on each Task

Task 1.1: Students might be familiar with roman numerals from I to XII, considering that they are used in clocks, at times. However, they might not know how to convert larger numbers, such as a year. It is important for students to know how to carry out such conversions, considering Malta's rich history and the various facades that have the year sculpted in Roman numerals on them. The Roman number system is also one of the suggested ideas for a research project in the SEC syllabus. Students can be given a few examples to be worked out in class when preparing for the trail.

Task 1.2: There are various ways how to measure the diameter, however the one proposed in this trail reinforces the concept that the diameter is the longest chord in a circle. It is recommended that, as a large planter, students measure the big one in front of the War Museum. As for the medium and small sized planters, there are plenty to choose from the ones located at the sides of the War Museum. In this task students might find difficulty in measuring the circumference and diameter accurately, for example difficulty in holding the tape measure securely from one end and sliding the other end back and forth till the maximum length is found. It is best if this is practised at school first. Through this task students will discover that for any circle the ratio of the circumference to the diameter is constant.

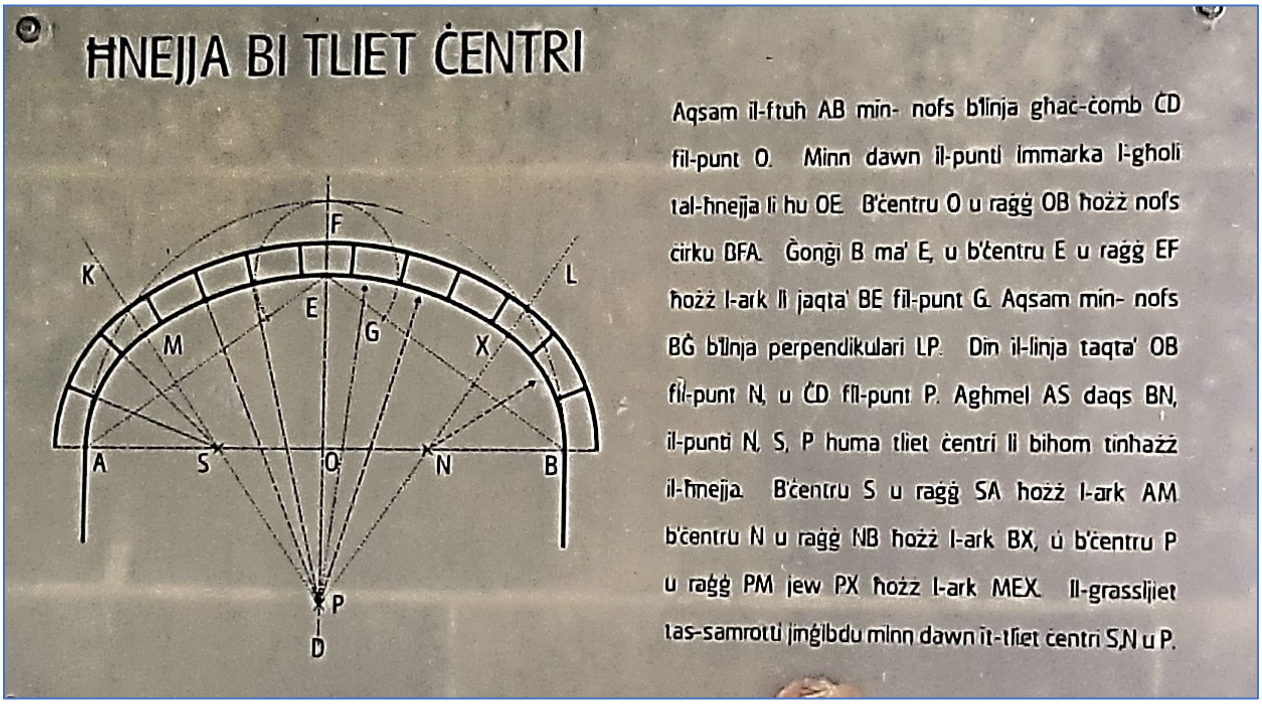
Task 1.3: The several arches present in Couvre Porte area present a good opportunity for students to visually see and compare different types of arches. The semi-circular arches and the segmental arches are two types of arches that students might be familiar with from their pre-acquired knowledge on semi-circles and segments. The basket-handle arch will probably be a new concept. However, it might be interesting to highlight to students that whereas the semi-circular and segmental arches have one centre (i.e. an arch made out of one circle), the basket-handle arch has three centres (i.e. it is made out of three circles). Refer to the following photos taken at Ġnien il-Bennej, Zurrieq.

An example of a semi-circular arch:

An example of a segmental arch:

An example of a basket-handle arch:

Details on how to construct a three-centred arch:

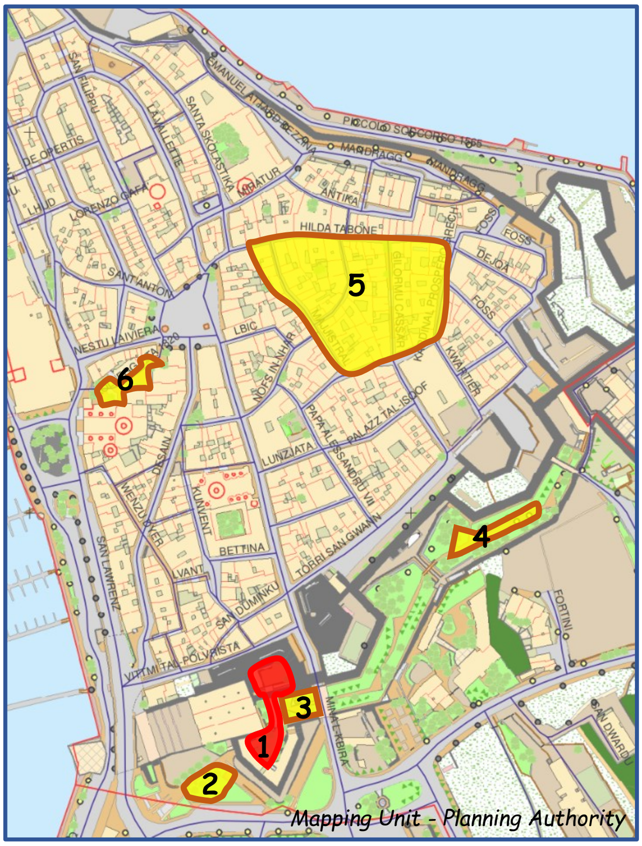


The composite area of segmental arch in Task 1.3b was split into a sector and trapezia instead of a segment and a rectangle because the area of segments is carried out in the Year 11 syllabus (Year 11 as from Sept 2024 LOF). The measurements provided to students, in particular the angle of sector, the radius of the circle and the line common to both trapezia, were found by importing the image in GeoGebra.

Task 1.4: Students might find difficulty in recognizing that the diameter of a semi-circular arch, which is not within reach, can be found by measuring the distance between the vertical supports of the arch. Hence, they were guided through this process, by referring to this structure as a semi-circle on top of a rectangle. It is worth highlighting to students that when taking such horizontal measurements, it is best that the tape measure is placed on the ground to ensure that it is correctly levelled from one end to the other. This task is also a good exercise to make use of the length of arc and area of sector formulae learnt in the classroom.

Station 1 – Couvre Porte Counterguard

Numerical Calculations; Shapes, Space and Measures (Circles)



Learning Objectives

- I can measure the diameter of a semi-circular arch.
- I can calculate the length of arc and area of sector of a circle.
- I can calculate the area of compound shapes that include sectors of circles.
- I can convert Roman numerals to the decimal number system.
- I can identify different types of circular arches.
- I can deduce that the ratio of the circumference to the diameter of a circle is constant.

The War Museum and Couvre Porte Gate

Task 1.1 - Roman Numerals

The Advanced Gate

After the Great Siege, the Knights continued to fortify Birgu with curtains (walls), bastions and gates.

To enter the city, they built a complex of three gates, the first of which is called the Advanced Gate (also known as Gate D'Aragon). The date on this gate refers to the year when works on it were completed.

This gate had to be defended by the Knights of Aragon (Spanish) in case of an attack.

Roman numerals represent a number system that uses **7 letters** to express numbers:

I, **V**, **X**, **L**, **C**, **D** and **M**

These letters correspond to integer values as follows:

Roman	I	V	X	L	C	D	M
Decimal	1	5	10	50	100	500	1000

Two of the rules **to convert Roman numerals to our system of integers** include:

Rule 1 – "When a Roman numeral is placed **after** another Roman numeral of **greater value**, the result is the **sum** of the numerals".

Example: VII = V + I + I
= 5 + 1 + 1
= 7

Rule 2 – "When a Roman numeral is placed **before** another Roman numeral of **greater value**, the result is the **difference** between the numerals".

Example: IV = V - I
= 5 - 1
= 4

(Cuemath, n.d)

a. Fill in the empty boxes to complete the following table:

Roman	I	II	III	IV	V	VI	VII	VIII	IX	X
Decimal	1	*2*	*3*	4	*5*	6	*7*	*8*	*9*	*10*

b. Write down the **Roman numeral** shown on this gate.

Answer:

MDCCXXII

c. Now **convert** this number to our system of integers.

= 1000 + 500 + 100 + 100 + 1 + 1

= 1722

Answer:

1722

d. Which **year** are we in?

Answer:

2023

e. Now **convert** the year we are in, to Roman numerals.

= 1000 + 1000 + 20 + 3

= 1000 + 1000 + 10 + 10 + 3

Answer:

MMXXIII

f. Calculate **how long** the Advanced Gate has been standing.

= 2023 – 1722

= 301

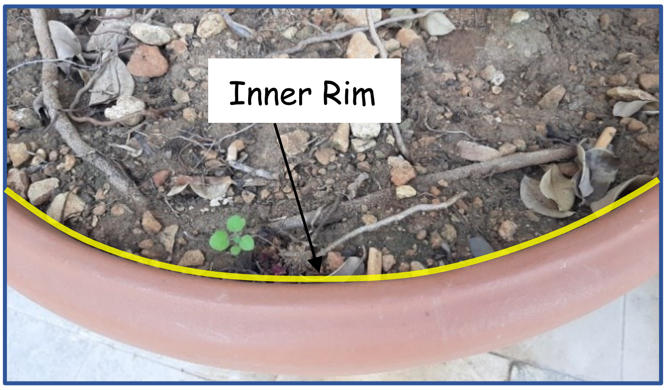
Answer:

301 years

Task 1.2 - Finding Pi (π) using Round-shaped Pots

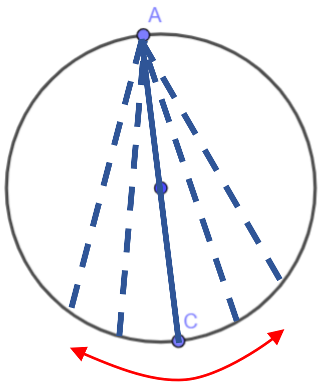
There are several planters in the area in front of the War Museum:

The War Museum



To measure the **diameter** of the opening at the top of a plant pot:

- measure across from one side to the other of the inner rim.
- hold the tape measure on one side of the pot (e.g. A) and move the other side back and forth until you measure the **longest** point (e.g. AC).



Select **three** round plant pots (large, middle-sized, and small). In order to do this, choose planters whose plant is not exactly in the centre.

For **each** planter:

a. Use the sewing tape measure to go round the **inner rim** in order to find the **circumference** of the opening at the **top**, correct to the **nearest cm**. Record your measurement in the following table.

b. Use the tape measure to find the **diameter** of the opening at the **top** (**inner rim**), correct to the **nearest cm**. Record this in the following table.

	Circumference (C) cm	Diameter (D) cm
Planter 1 (large)	*342.2*	*109*
Planter 2 (medium)	*254*	*81.6*
Planter 3 (small)	*151.2*	*47.5*

c. Use the calculator to work out the **ratio** of the **circumference (C) to its diameter (D)**, i.e. $\frac{C}{D}$. Give your answers correct to:

(i) 2 decimal places (d.p.)

(ii) 1 significant figure (s. f.)

	$\frac{\mathbf{C}\ (\text{cm})}{\mathbf{D}\ (\text{cm})}$	
	2 d.p.	1 s.f.
Planter 1 (large)	*3.14*	*3*
Planter 2 (medium)	*3.11*	*3*
Planter 3 (small)	*3.18*	*3*

d. The **ratio** of the circumference to the diameter of a circle ($\frac{C}{D}$), irrespective of the circle's size, is **constant**. What is this constant **approximately** equal to?

Tick (✓) the correct answer:

☐ 1

☑ 3

☐ 5

e. What is this **constant** called?

Answer:

Pi or π

f. The value of this constant is **3.1415926536** correct to 10 decimal places. Write down the value of this constant correct to **3 decimal places**.

Answer:

3.142

g. **Compare** the results of Task 1.2c(i) to the answer of Task 1.2f. Which answer in Task 1.2c(i) is the **most accurate**? Why do you **think** this is so?

Answer:

The planter with the largest circumference is the most accurate because as the measured length increases, the effect of measurement errors becomes less significant.

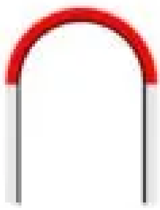
Task 1.3 – Arches and Area of a Composite Shape

Couvre Porte Gate

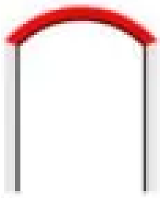
The second gate in this unique entrance to Birgu is Couvre Porte Gate (or the Covered Gate). It is located at the end of a bridge that crosses the ditch.

The aim of the Advanced Gate and Couvre Porte Gate was to shield Birgu's main entry point (known as the Main Gate or Gate of Provence) from the attacking forces.

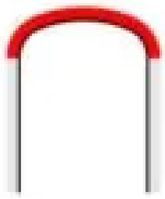
There are various types of arches, and their names typically suggest how the top part of the arch looks like. Some examples include:



Semi-circular arch



Segmental arch



Basket-handle arch

(Homestratosphere's Editorial Staff & Writers, 2022)

a. Identify the **two types of arches** that can be found on the façade of Couvre Porte Gate. Tick (✓) the correct answers.

- ☑ Semi-circular
- ☑ Segmental
- ☐ Basket-handle

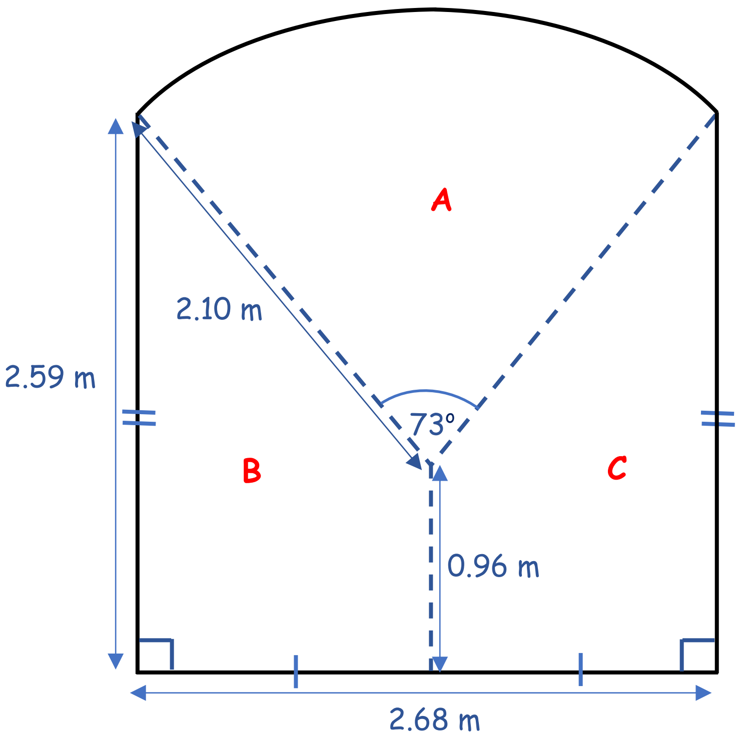
b. As you pass through the gate, you will find the monumental bust of Nestu Laiviera placed under an arch. What **type of arch** is this? Tick (✓) the correct answer.

- ☐ Semi-circular
- ☐ Segmental
- ☑ Basket-handle

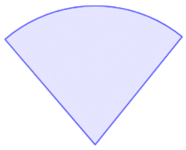
Consider the **segmental arch** at Couvre Porte Gate (marked in yellow) as a composite shape for which you need to calculate the area.

To calculate the area of a **composite shape** you need to divide the composite shape into shapes you can calculate the area of.

One way to divide this composite shape is as follows:



c. What **shape** is A? Tick (✓) the correct answer.



- ☑ sector of a circle
- ☐ semi-circle
- ☐ segment of a circle

d. Use your calculator to work out the **area of shape A**, correct to **1 decimal place**.

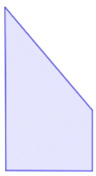
Area of sector $= \frac{\theta}{360°} \times \pi\, r^{2}$

$= \frac{73}{360°} \times \pi \times 2.1^{2}$

$= 2.8\ m^2$

Answer: $2.8\ m^2$

e. What **shape** is B? Tick (✓) the correct answer.



- ☐ parallelogram
- ☑ trapezium
- ☐ rhombus

f. Use your calculator to work out the **area of shape B**, correct to **1 decimal place**.

Area of trapezium $= \frac{1}{2}\ (a + b)\ h$

$= \frac{1}{2} \times (2.59 + 0.96) \times 1.34$

$= 2.4\ m^2$

Answer: $2.4\ m^2$

g. Given that shape B and shape C are equal, calculate the **total area** of the segmental arch, correct to **1 decimal place**.

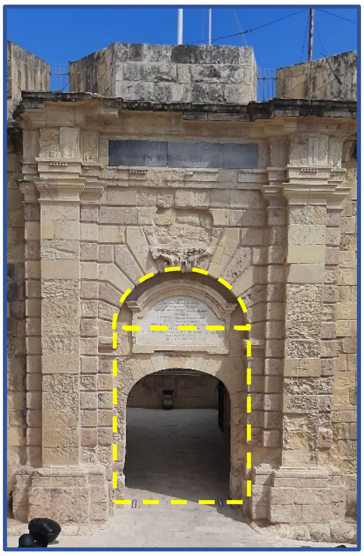
Total Area = Area of sector + (Area of trapezium x 2)

= 2.8 + (2.4 x 2)

$= 7.6\ m^2$

Answer: $7.6\ m^2$

Task 1.4 - Sectors



The Gate of Provence

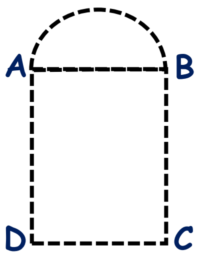
The Main Gate, also called the Gate of Provence, is the third gate that actually leads one into the city.

The Knights of Provence (French) were responsible for its defence.

Works on the three gates were recommended by the French military engineering mission led by L.F. de Tigne' and C.F. de Mondion in an effort to improve the defence system. These works were completed in the times of Grand Master Manoel de Vilhena.

The **semi-circular arch** (marked in yellow in the diagram above) forms a **semi-circle** on top of a **rectangle**.

a. If you are told that ABCD is a **rectangle** (see diagram below), is AB equal to CD? Why? Justify your answer using the properties of a rectangle.



Answer:

Yes, because the opposite sides of a rectangle are parallel and equal to each other.

b. Now, use your tape measure to find the **diameter (d)** of this semi-circle. Give your answer correct to the **nearest cm**.

Answer:

305 cm

c. So, what is the **radius (r)** of this semicircle? Give your answer correct to **1 decimal place**.

Radius = 305 ÷ 2

= 152.5 cm

Answer:

152.5 cm

The semi-circle can be divided into **sectors** as shown in blue below. Assuming that all sectors are **equal**:

d. What is the **total number** of sectors?

Answer:

16 sectors

e. Given that the angle of a semi-circle is 180°, calculate the **angle (θ)** of each sector. Give your answer correct to **2 decimal places**.

Angle of sector = 180 ÷ 16

= 11.25

Answer:

11.25 °

f. Use your calculator to find the **length of arc** of each sector. Give your answer correct to the **nearest cm**.

Length of arc $= \frac{\theta}{360°} \times 2\,\pi\,r$

$= \frac{11.25}{360°} \times 2 \times \pi \times 152.5$

= 29.94

Answer:

30 cm

g. Use your calculator to find the **area of each sector**. Give your answer correct to the **nearest cm²**.

Area of sector $= \frac{\theta}{360°} \times \pi\,r^{2}$

$= \frac{11.25}{360°} \times \pi \times 152.5^{2}$

= *2283.18*

Answer:

2283 cm^2

Station 2 – Teacher's Notes

Resources

- Tape Measure or Disk Tape Measure
- Calculator
- Pen and Resource Pack

Learning Outcomes

Task No	Strand	MQF Level	Learning Outcome
2.1	Number	3	2.3bg Work through situations that involve direct and / or inverse proportion.
2.2	Algebra	3	3.3f Describe the nth term of a linear sequence using an algebraic expression. E.g. –3n – 1.
2.3	Shape, Space and Measure	1	5.1bl Write time to the hour/half hour/quarter hour using terms 'o'clock', 'half past', 'quarter past', and 'quarter to'.
2.3	Shape, Space and Measure	1	5.1bo Read the 24-hour clock (analogue and digital).
2.3	Shape, Space and Measure	1	5.1bq Interpret a time table, a timeline and/or a calendar.
2.3	Shape, Space and Measure	1	5.1bu Work through situations involving addition and subtraction of time given in hours and minutes.
2.3	Number	2	2.2ay Work through simple situations involving directed numbers, personal and household finance. E.g. pocket money accrued, how much it will cost to prepare a meal, which item is the best buy when items come in various sizes e.g. oil in one litre bottles vs oil in two litre bottles.

Additional Information on each Task

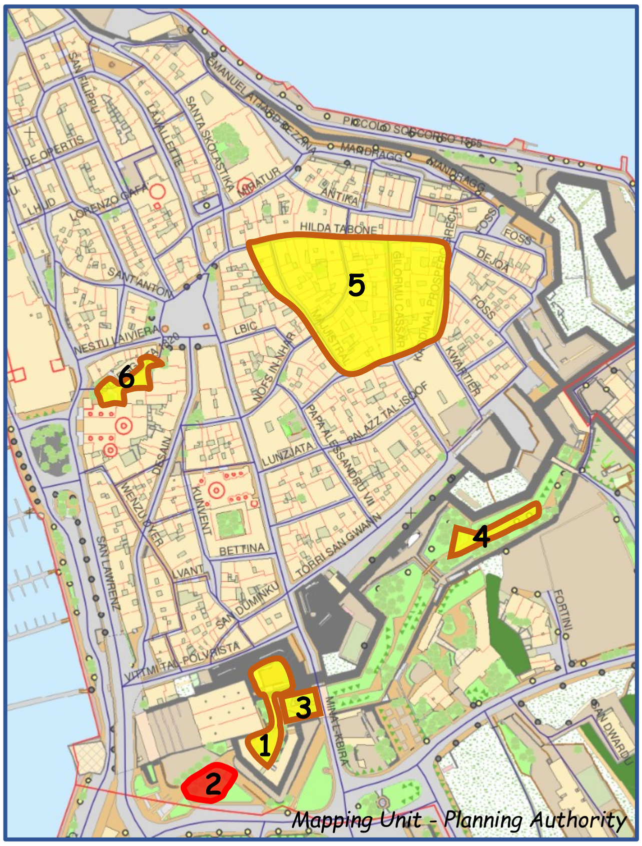
Task 2.1: Measuring heights using shadows is a task students will probably enjoy doing because they will measure their own height and shadow. The task gives students the opportunity to make use of direct proportion. There might be some inaccuracies in their answers as the measurements of shadows can be tricky (the end part is normally faded rather than clear-cut). This task cannot be carried out on cloudy days.

Task 2.2: The sequence of these tiles is rather interesting especially when observed from aerial view images, such as those by Google Earth. The task gives students the possibility to identify it as a linear sequence and to find its nth term. It would have been preferred if the length of each strip could be found by counting the tiles instead of using the tape measure or disc tape measure, but there are several tiles which are not whole tiles, mostly due to another tile pattern crossing through them. Thus, as a workaround to this 'problem', students are asked to first measure the length of the vertical strips and then finding the number of 'theoretical' whole tiles by dividing these measurements by 60cm (length of 1 tile). This way of 'normalizing' the measurements also eliminates errors due to imprecision in the tiling works. This activity shows the importance of students experiencing authentic tasks, because real-life mathematics is not 'perfect' and 'simplified' as presented in textbooks.

Task 2.3: This task makes use of the Valletta-Three Cities ferry schedule for activities related to time and money. As for the time activity, the season in which the journey is taking place is not indicated as the ferry schedule for winter and summer is the same for the time window referred to in the task. With regards to money, knowing how to buy the best alternative that meets one's needs, irrespective of any offers that there might be, is an important skill that students should learn. This task was designed with this concept in mind. There should be no particular difficulties related to this task.

Station 2 – Couvre Porte Belvedere

Numerical Calculations (Proportions; Money) and Measures (Time)



Learning Objectives

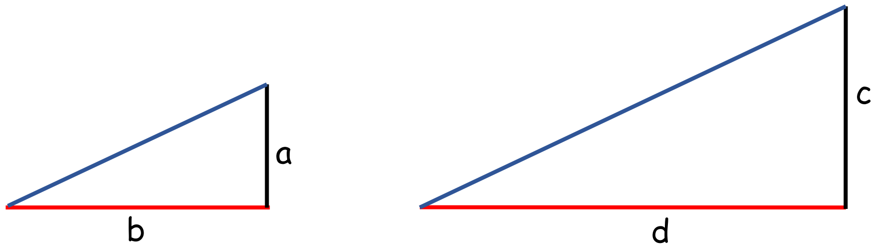
- I can work through situations that involve direct proportion.
- I can measure tall items using shadows.
- I can recognize and extend number sequences.
- I can use algebraic expressions to describe the nth term of a linear sequence.
- I can read and use a timetable and a timeline.
- I can work through simple situations involving personal finance.

Couvre Porte Belvedere

Task 2.1 – Measuring Heights using Shadows

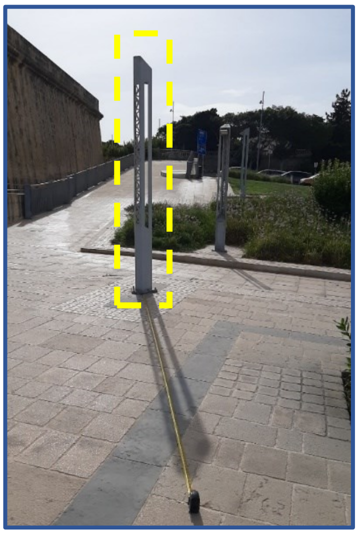
At a given time of day, objects create **shadows** that are **proportional** to one another.

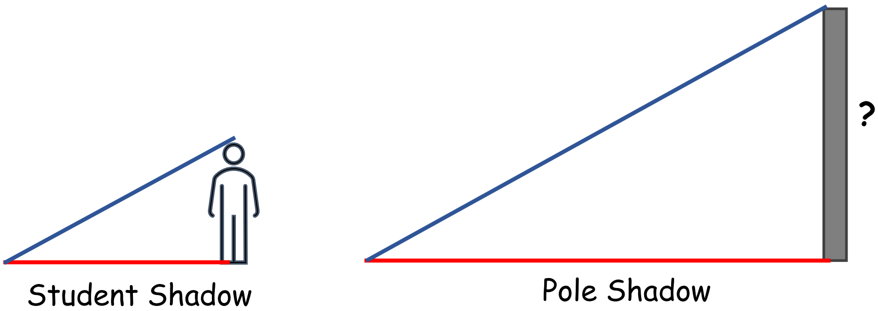
Thus, when measurements are taken **at the same time of the day**, the **ratio** of the **height of an object to the length of its shadow** is the **same** for all objects.



$$\frac{a}{b} = \frac{c}{d}$$

You can use this fact to calculate the height of the light poles (marked in yellow below) at Couvre Porte Belvedere.





$$\frac{\text{Student Height}}{\text{Student Shadow}} = \frac{\text{Pole Height}}{\text{Pole Shadow}}$$

a. Select **one student** from your group. Find a **position** where the student's **shadow** can be **clearly** seen. The student needs to remain **still** in this position whilst measurements are being taken.

The rest of the group needs to use the tape measure to find to the **nearest cm** the:

- **student's body height**

Answer:

Example: *168* cm

- **length of student's shadow**

Answer:

Example: *253* cm

b. Choose **one of the light poles** for which the shadow can be **clearly** seen. Use the tape measure to find the **length of the pole's shadow**, correct to the **nearest cm**.

Answer:

Example: *418* cm

c. Now calculate the **pole's height** using the formula below and give your answer correct to the **nearest cm**.

$$\frac{\text{Student Height}}{\text{Student Shadow}} = \frac{\text{Pole Height}}{\text{Pole Shadow}}$$

$$\frac{168}{253} = \frac{\text{Pole Height}}{418}$$

$$\text{Pole Height} = (168 \times 418) \div 253$$

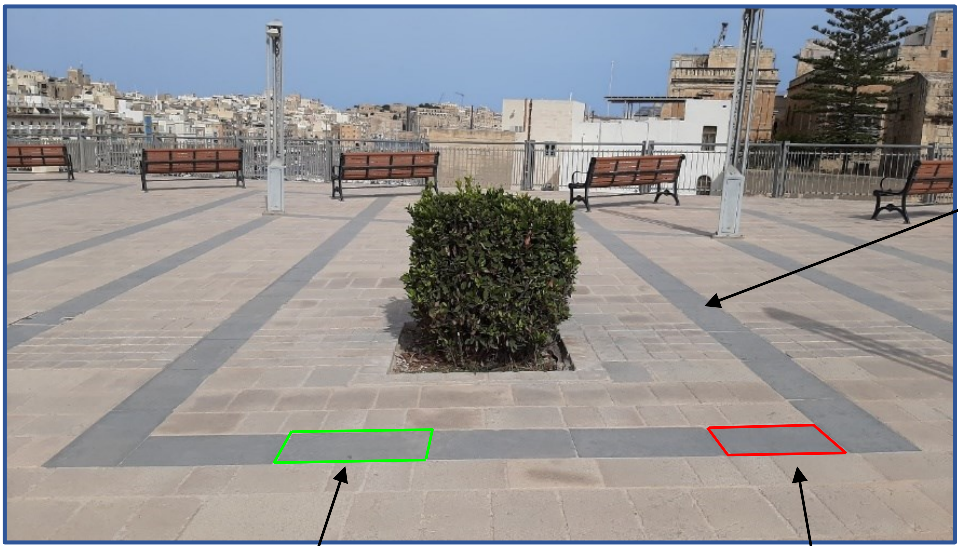
$$= 277.56$$

(Actual Pole Height = 281 cm)

Answer:

Example: *278* cm

Task 2.2 – Sequences



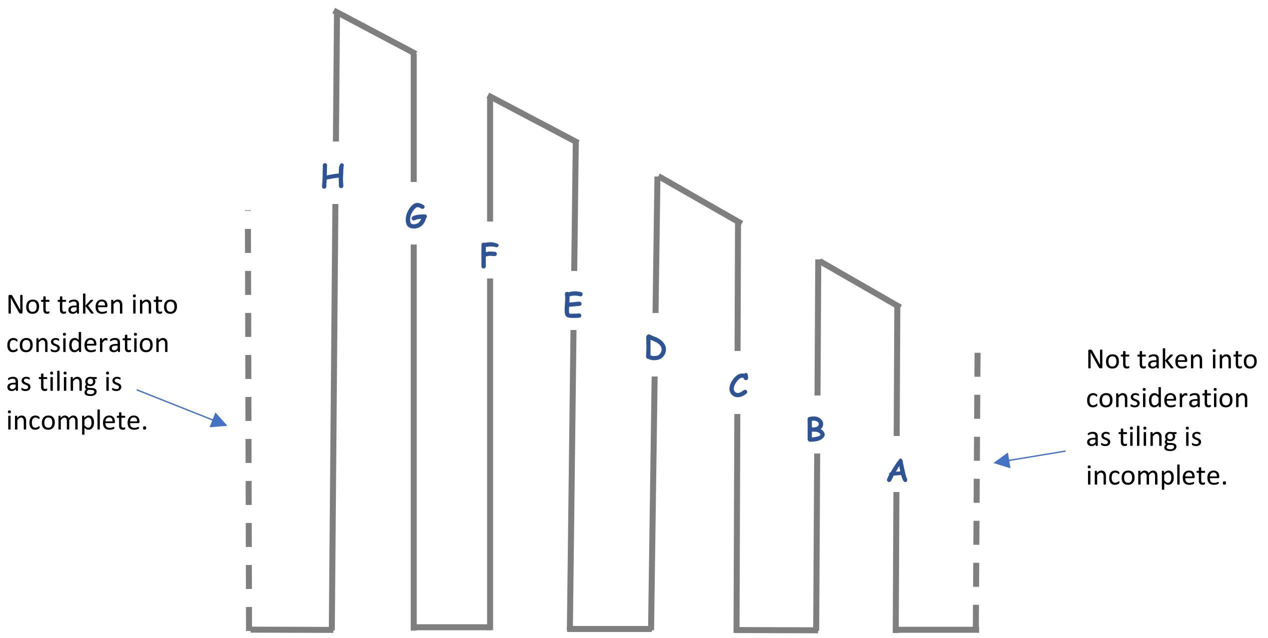
a. Observe the **grey tiles**. Focus on a tile that is a **whole one** and not on a tile that has a part cut off. Use your tape measure to find the **length and width of the tile**. Tick (✓) the correct answer.

- ☐ 60 x 60cm
- ☑ 30 x 60cm
- ☐ 60 x 90cm

Part of the grey tiling follows a particular **sequence**, highlighted in yellow below:

Aerial View of Couvre Porte Belvedere

This sequence is represented in the following diagram.



b. (i) Use your tape measure to find the **lengths of the vertical strips** A, B, C, D, E, F, G and H, correct to the **nearest cm**. Write your answers in the empty boxes below. Some of them have been already measured.

(ii) Divide these lengths by 60cm to find the number of **whole tiles.**

For example, 1789 ÷ 60 = **29**.817.

This means that there are **29 whole tiles** (ignore the decimal part).

Write your answers in the empty boxes below. Some of them have been already worked out.

Vertical Strip	A	B	C	D	E	F	G	H
Length (cm)	900	*1030*	*1154*	*1284*	*1413*	1538	*1663*	1789
No of whole tiles	15	*17*	19	*21*	*23*	25	*27*	29

c. Considering the number of whole tiles for each vertical strip, **is this a linear sequence**? Why do you think so?

Answer:

Yes, it is a linear sequence because the difference between terms is constant.

To go from one term to the next add the same value, 2.

d. The following table shows the number of whole tiles per vertical strip according to their position 'n' in the sequence. For instance, vertical strip A has 15 whole tiles. Complete the following table.

Vertical Strip	A	B	C	D	E	F	G	H
n	1	2	3	*4*	*5*	*6*	*7*	8
nth term	15	*17*	19	21	*23*	25	*27*	29

e. Find the **nth term** of this sequence **using an algebraic expression**. You can use:

either:

$\pm an \pm b$, where

a is the difference

b is the adjustment

a = 2

when n = 1:

15 = 2(1) + b

b = 15-2

b = 13

nth term = 2n + 13

or:

$a + (n-1)d$, where

a is the 1st term

d is the common difference

a = 15

d = 2

nth term = 15 + (n-1)2

= 15 + 2n - 2

= 13 + 2n

= 2n + 13

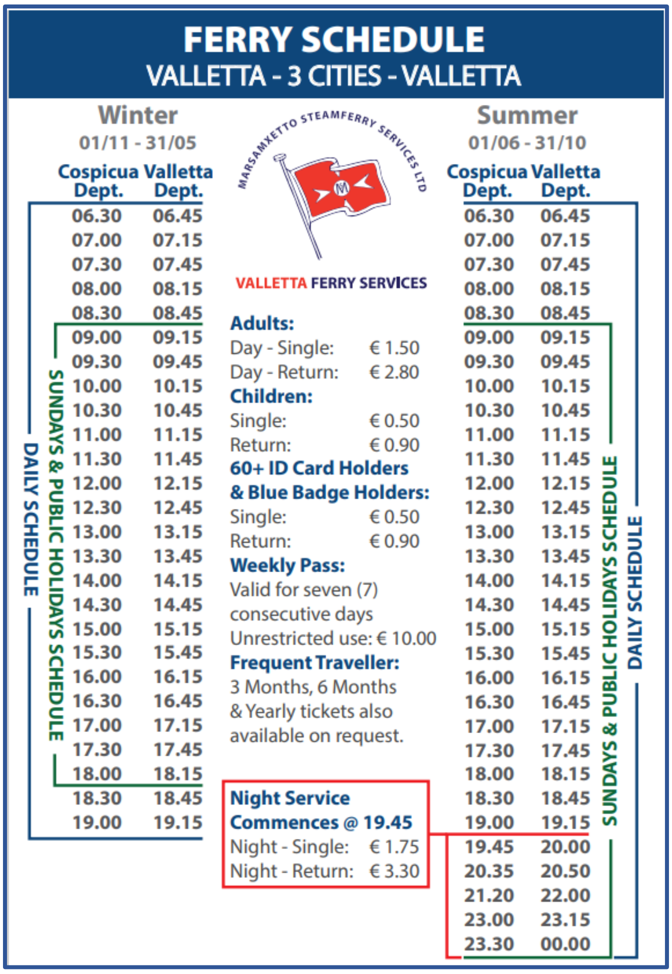
Answer:

2n + 13

Task 2.3 – Time and Money

Dockyard Creek

Dockyard Creek (originally called Galley Creek) is a very sheltered area in the Grand Harbour. In the Middle Ages, the Spanish and Sicilians used it for their commercial ships. The Knights used it to berth and repair their galleys. The British used it for Royal Navy ships. It used to be the location for Dock No.1. Nowadays it is the Vittoriosa Yacht Marina and also has a small ferry quay.



FERRY SCHEDULE
VALLETTA - 3 CITIES - VALLETTA

MARSAMXETTO STEAMFERRY SERVICES LTD

VALLETTA FERRY SERVICES

Winter
01/11 - 31/05

Cospicua Dept.	Valletta Dept.
06.30	06.45
07.00	07.15
07.30	07.45
08.00	08.15
08.30	08.45
09.00	09.15
09.30	09.45
10.00	10.15
10.30	10.45
11.00	11.15
11.30	11.45
12.00	12.15
12.30	12.45
13.00	13.15
13.30	13.45
14.00	14.15
14.30	14.45
15.00	15.15
15.30	15.45
16.00	16.15
16.30	16.45
17.00	17.15
17.30	17.45
18.00	18.15
18.30	18.45
19.00	19.15

DAILY SCHEDULE (06.30 – 19.00)
SUNDAYS & PUBLIC HOLIDAYS SCHEDULE (09.00 – 18.00)

Summer
01/06 - 31/10

Cospicua Dept.	Valletta Dept.
06.30	06.45
07.00	07.15
07.30	07.45
08.00	08.15
08.30	08.45
09.00	09.15
09.30	09.45
10.00	10.15
10.30	10.45
11.00	11.15
11.30	11.45
12.00	12.15
12.30	12.45
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14.00	14.15
14.30	14.45
15.00	15.15
15.30	15.45
16.00	16.15
16.30	16.45
17.00	17.15
17.30	17.45
18.00	18.15
18.30	18.45
19.00	19.15
19.45	20.00
20.35	20.50
21.20	22.00
23.00	23.15
23.30	00.00

DAILY SCHEDULE (06.30 – 23.30)
SUNDAYS & PUBLIC HOLIDAYS SCHEDULE (09.00 – 23.30)

Adults:
Day - Single: € 1.50
Day - Return: € 2.80

Children:
Single: € 0.50
Return: € 0.90

60+ ID Card Holders & Blue Badge Holders:
Single: € 0.50
Return: € 0.90

Weekly Pass:
Valid for seven (7) consecutive days
Unrestricted use: € 10.00

Frequent Traveller:
3 Months, 6 Months & Yearly tickets also available on request.

Night Service Commences @ 19.45
Night - Single: € 1.75
Night - Return: € 3.30

This is the **Valletta - Three Cities ferry schedule**.

Refer to this schedule for tasks in this section.

(Ferry Schedule v8, n.d.)

Lawrence lives in Birgu and travels to and from Valletta every day during **weekdays** for work. He starts work at **09:00**.

a. If Lawrence leaves his house at **08:10** and arrives near the Three Cities ferry terminal at **08:23**:

i. Calculate the **amount of time** he spends walking from his house to the ferry terminal.

From 08:10 to 08:23 = 13 minutes

Answer:

13 minutes

ii. How long does he have to **wait** to catch the **next ferry**?

Next ferry is at 08:30
He has to wait from 08:23 to 08:30 = 7 minutes

Answer:

7 minutes

b. Lawrence manages to catch the **08:30** ferry. The duration of the ferry trip is **12 minutes**. If Lawrence takes **7 minutes** to get from the Valletta ferry terminal to the Upper Barrakka Gardens using the Barrakka Lift, and then has a **10-minute** walk to his workplace:

(i) Fill in the following **timeline** to represent Lawrence's journey from the Three Cities ferry terminal to his office in Valletta.

08:30	08:42	08:49	08:59
Three Cities Ferry Departure	Valletta Ferry Arrival	Valletta Upper Barrakka Gardens	Office in Valletta

(ii) Does he manage to arrive to work **on time**?

He arrives at 08:59. Work starts at 09:00.

Yes, he manages to arrive on time.

Answer: *Yes*

c. Lawrence finishes work at **17:00**. He takes the same amount of time to reach the Valletta ferry terminal from his office as it takes him in the morning.

(i) Which is the **next ferry** from Valletta to the Three Cities that he can catch?

Amount of time to reach the ferry: 10 + 7 + 12 = 29 mins

He is at the ferry at 17:29

Next ferry from Valletta is at 17:45

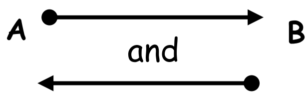
Answer: *17:45*

(ii) Give the equivalent answer in **words**.

Answer: quarter to *six*

Single ticket means **one-way** (either from A to B or from B to A)

Return ticket means **two-way** (from A to B and from B back to A)



d. What is the **cost** of the Adults Day-Return ticket for **one day**?

Answer: *€2.80*

e. What is the **weekly travelling cost** for Lawrence to go to work if he buys the Adults Day-Return tickets?

1 ticket = €2.80
5 tickets = ?

= (2.80 x 5) ÷ 1
= 14

Answer: *€14*

f. How much will he **save** if he buys the Weekly Pass instead of the Adults Day-Return tickets?

Weekly Pass = €10

He will save: €14 – €10 = €4

Answer: *€4*

g. If for a particular week he has 2 days off work, which is the **cheapest option**?

☑ Adults Day-Return ticket

☐ Weekly Pass

Tick (✓) the correct answer. Show your working to justify your answer.

Answer:

Cost of tickets for 3 days of work:

Adults Day-Return ticket option = €2.80 x 3

= €8.40

Weekly Pass option = €10

The cheapest option is the Adults Day-Return ticket.

Station 3 – Teacher's Notes

Resources

- Tracing Paper (per student)
- A4 Paper (per student)
- Pencil (per student)
- Photo Camera (per student)
- Clinometer
- Tape Measure or Disk Tape Measure
- Calculator
- Pen and Resource Pack

Learning Outcomes

Task No	Strand	MQF Level	Learning Outcome
3.1	Shape, Space and Measure	1	5.1h Estimate angles up to 180^0.
3.1	Shape, Space and Measure	2	5.2l Distinguish between right, acute, obtuse, and reflex angles.
3.2	Shape, Space and Measure	2	6.2v Use the properties of one or more types of triangles (i.e. equilateral, isosceles, scalene, and/or right-angled triangles).
3.2	Shape, Space and Measure	3	5.3v Interpret angles of elevation and depression.
3.3	Shape, Space and Measure	3	5.3t Define the trigonometric ratios (sine, cosine and tangent) as the ratios of sides in a right-angled triangle.
3.3	Shape, Space and Measure	3	6.3ac Identify similar shapes.

3.3	Shape, Space and Measure	3	6.3ab Explain the concept of similarity.
3.4	Shape, Space and Measure	3	5.3r Apply the Converse of Pythagoras' Theorem to check whether a triangle is right-angled.

Additional Information on each Task

Task 3.1: In this task students need to use an A4 paper as a 'non-standard' tool to check if two sides of a triangle are equal, or if an angle is greater or smaller than 90°. This is a good opportunity for students to learn such skills as in real-life there are several occasions where one estimates measurements using the resources available at hand. As a pre-trail activity students can use an A4 paper to check if the desks in the classroom have an equal surface or if an item will fit in the locker without actually trying it out.

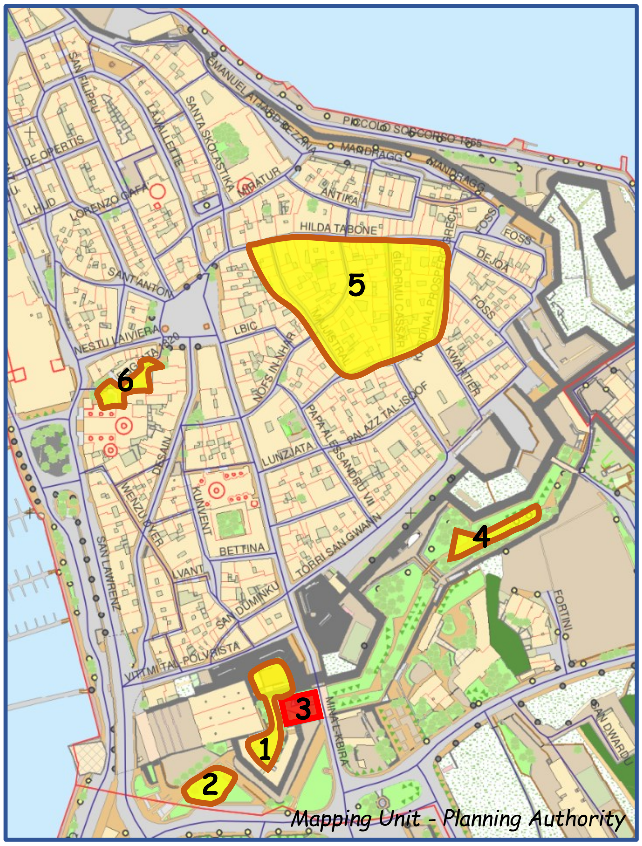
Task 3.2: Students learn about the properties of a right-angled isosceles triangle in the classroom. The aim of this task is to render this information relevant and use it to measure the height of the bridge. Students might forget to include the height of the person using the clinometer in the total height, and thus the task was guided accordingly. Students should be reminded that when measuring the horizontal distance, it is best to place the tape measure on the ground, so as to ensure that the tape measure is levelled properly. Students might have difficulty in using the clinometer correctly, considering that it is not a frequently used tool. Thus, it is recommended that students are given tasks, at school, in which they work as a group to measure particular angles of elevation or depression before the trail.

Task 3.3: The nested triangles structure has the potential for various activities, such as to explore similar triangles or the area of a trapezium, but in this trail, it is being used to discover the tangent ratio. Through measurements students can find that for the same angle the tangent ratio is always the same, irrespective of the size of the triangle. The base of the triangles in this structure is 'elevated' from the ground level by 8cm, thus when finding the heights of the triangles, students need to take away this extra measurement. Students might have difficulties to understand this idea, thus it is best that it is explained to them before the trail. A pre-trail activity for students to be familiar with such a situation is to measure the height of the school stairs should the first flight of the stairs is L-shaped.

Task 3.4: The way this fountain is positioned might give the false impression that it is not a right-angled triangle, however students can use Pythagoras' Theorem to find this out. The measurements of sides AB and AC are provided to students as they are not easily accessible. It is important to inform students not to place any of their belongings on the edges of the fountain as this is filled with water. Students can prepare for this activity by measuring the height (x^2) and horizontal length of a normal flight of stairs (y^2), as well as its slanting length (z^2), and then use Pythagoras' Theorem to confirm that $z^2 = x^2 + y^2$.

Station 3 – Birgu Ditch (Couvre Porte Area)

Euclidean Geometry (Triangles); Shapes, Space and Measures (Trigonometry)



Learning Objectives

- I know the properties of different types of triangles.
- I can use resources at hand to identify and distinguish between acute, obtuse right angles, and to distinguish between equal and non-equal lengths.
- I can measure angles of elevation.
- I can measure tall objects using the properties of an isosceles triangle.
- I can deduce that the Tangent ratio is the same for all triangles for a given angle.
- I can interpret and use Pythagoras' Theorem.

Birgu Ditch (Couvre Porte Area)

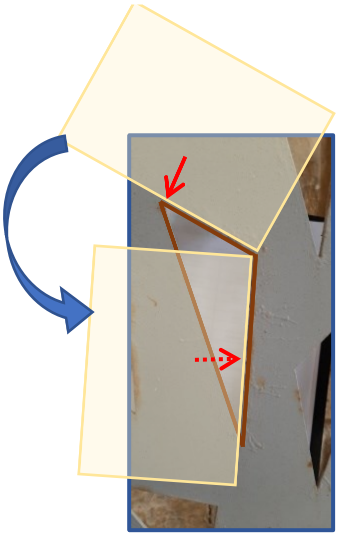
Task 3.1 – Types of Triangles

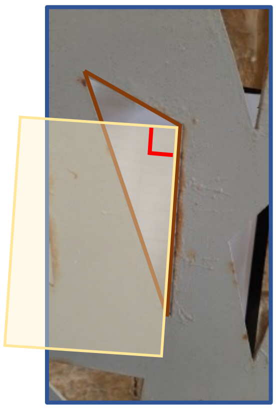
Before the Knights came to Malta, Birgu had a demarcation line on its land front, referred to as 'tagliata'. This 'tagliata' probably is the same location where, following the Great Siege, the Knights excavated a dry ditch to cut off the city from mainland and hence protect the city from an attack. Works on it by the Knights continued until the 1720s.

The ditch, nowadays referred to as 'il-Foss tal-Birgu', underwent restoration works that were completed in 2016.

To **trace** a triangle:

- clip a plain paper to the clipboard file (or any other hard backing).
- position the paper at the back of the iron panel.
- use a pencil and draw the chosen triangle by stencilling accurately inside it.





To find out **if two lengths are equal**, use one side of an A4 paper to mark the first length. Then compare this marking to the second length.

To find out **if an angle is equal to, greater than, or less than 90°**, use the corner of an A4 paper to estimate if the angle is =, > or < 90 °.

Choose **one of the iron panels** showing different types of triangles.

a. Use an A4 paper to identify the following triangles:

- Right-angled
- Isosceles
- Obtuse
- Acute
- Scalene

b. **Each member** in the group selects **one of these triangles**. Make sure to choose a triangle that fits on an A4 paper. Each member then needs to:

Take a **photo** of the selected triangle.

Trace the selected triangle on a blank A4 paper.

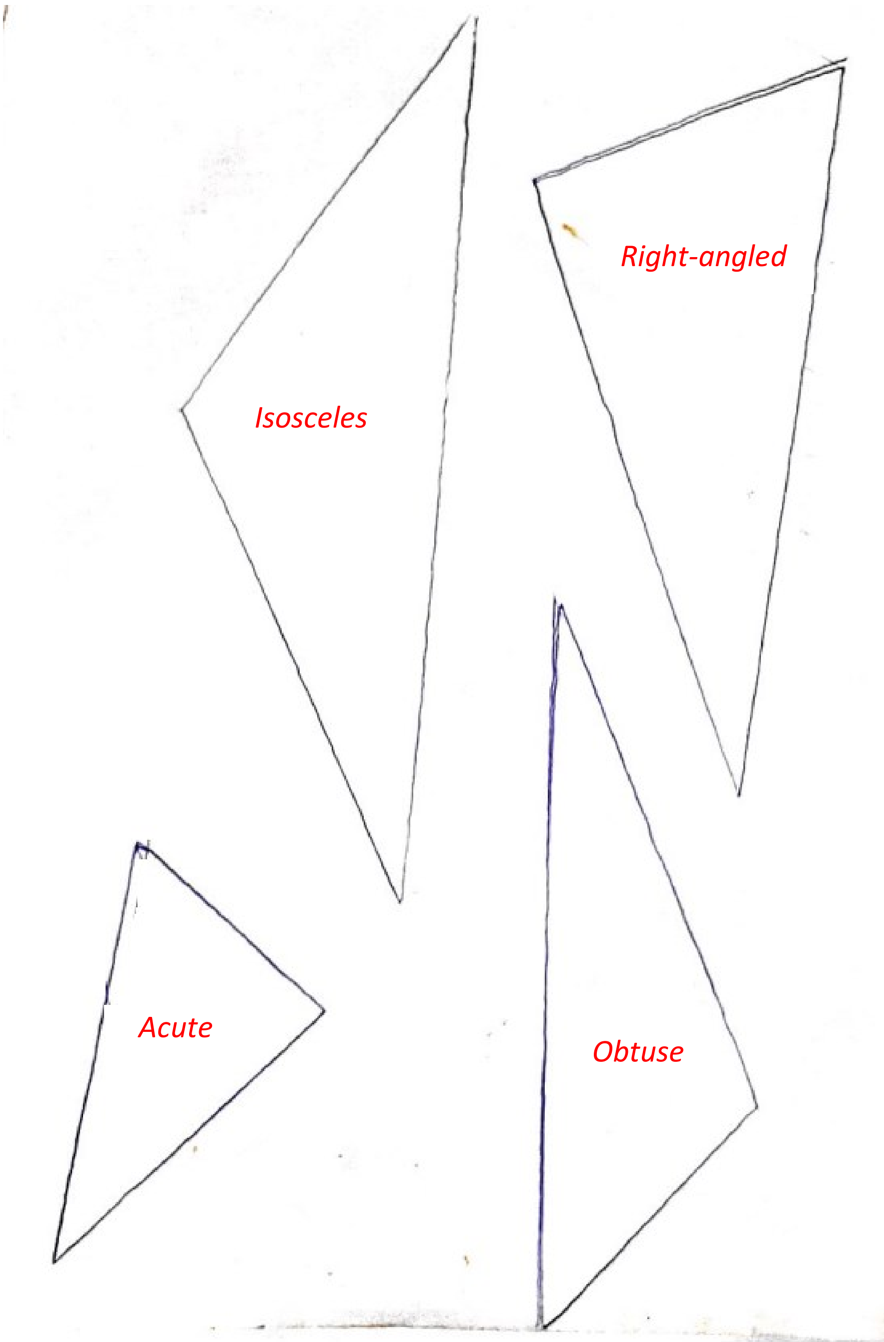
Attach the triangle traced in the step above onto the following page.

Use a **ruler** to measure the sides and a **protractor** to measure the angles of the selected triangle.

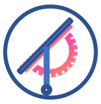
Mark these measurements on the A4 paper and **name** the triangle accordingly.

Selected triangle:

(Examples)



Task 3.2 – Measuring Heights using Isosceles Triangles



a. Select **one student** from your group. The student needs to measure the **angle of elevation** using the clinometer as follows:

- Student needs to stand and face the vertical strip highlighted in yellow.
- With the help of the group, the student needs to move slowly backwards till the angle of elevation from the ditch to the top of the bridge is **45°**.

b. The student **needs to remain** in the position where the angle of elevation is 45°. The rest of the group needs to use the tape measure to find:

(i) the **horizontal distance** between the student and the bridge (BC). Give your answer correct to the **nearest cm**.

Note: The ground level gives you the most accurate horizontal measurement.

Answer:

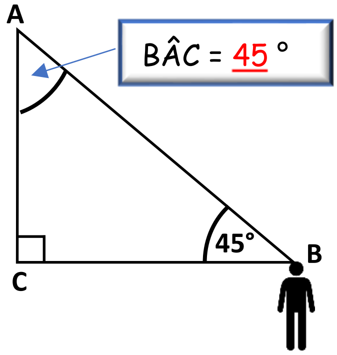
Example: *412* cm

(ii) the **height** of the student that is using the clinometer. Give your answer correct to the **nearest cm**.

Answer:

Example: *168* cm

c. Calculate the **missing angles** in the following diagram:



d. What is this **triangle** called? Tick (✓) the correct answer.

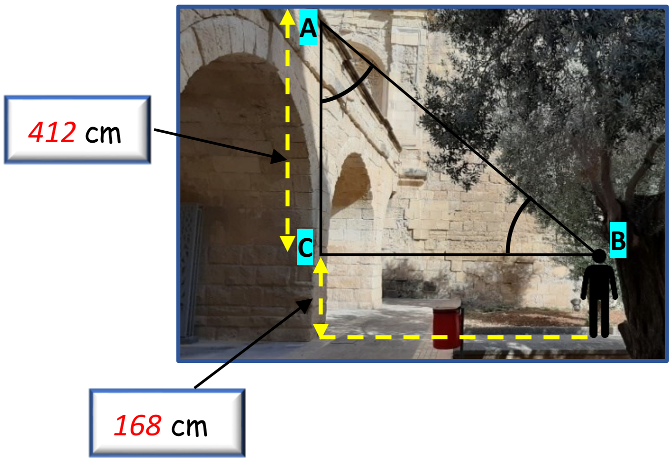
- ☐ Equilateral
- ☑ Isosceles
- ☐ Scalene

e. Using the **properties of this triangle**, what is height AC?

Answer:

Example:
412 cm

f. Fill in your measurements in the following diagram and calculate the **total height of the bridge**.

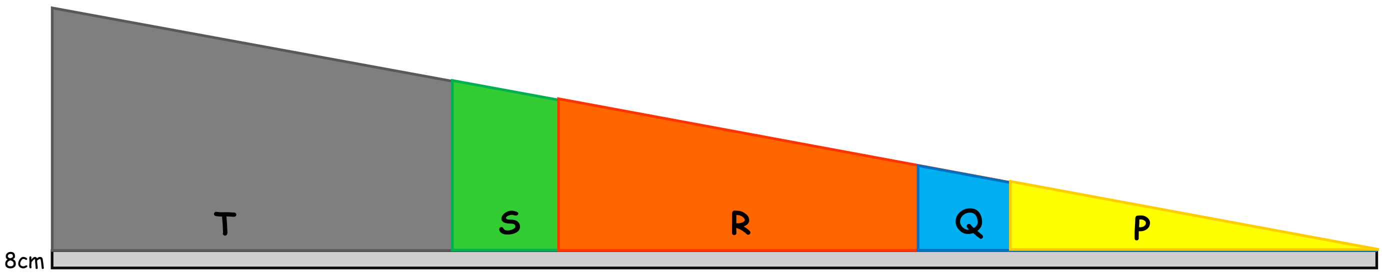


Answer:

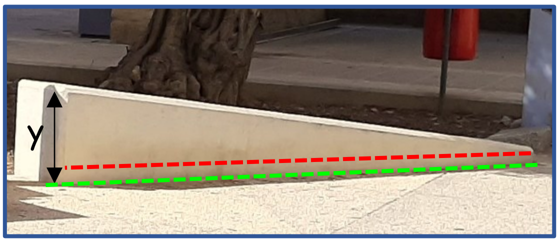
580 cm

Task 3.3 – The Tangent Ratio

The structure shown above is made up of **nested triangles** of various sizes. These triangles can be depicted as follows:

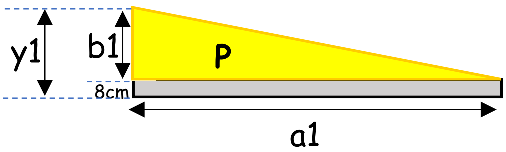


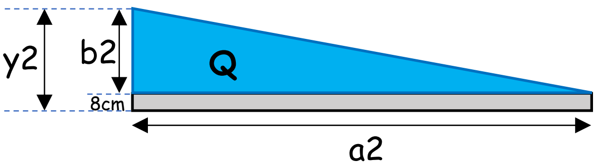
Measuring Guide

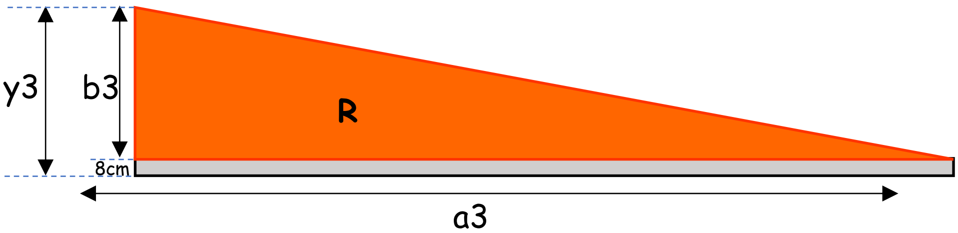


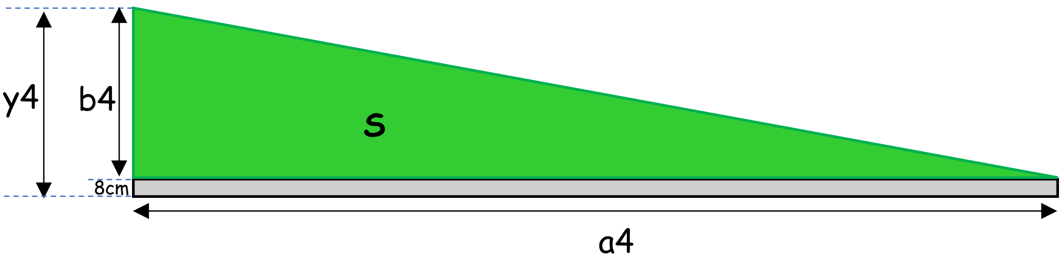
The triangle structure is 8cm above ground level, marked in red above. To measure heights (y), **measure till ground level**, i.e. up to the green line. You will then be asked to deduct the extra 8cm in the task itself.

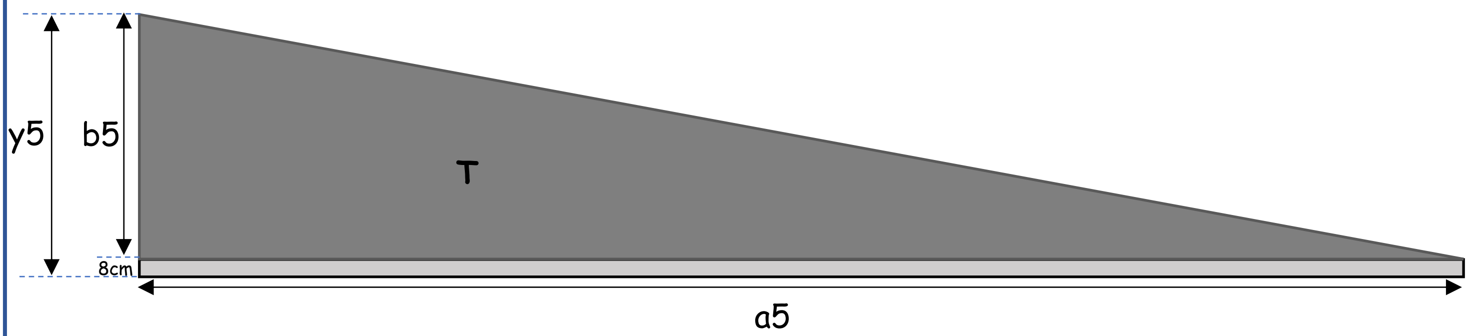
When measuring, use the **inner edges** of the structure.











a. Use the tape measure to find to the **nearest cm**: a1, a2, a3, a4, a5, y1, y2, y3 and y4, y5 and write them in the table below. Find the **'b' values by subtracting 8** from the 'y' values, for example b1 = y1 – 8. Some of the measurements have been filled in already.

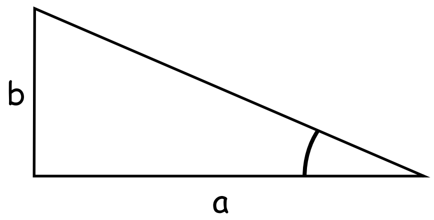
Triangle	y (cm)	b (cm) = y - 8	a (cm)
P (1)	49	41	*339*
Q (2)	*58.5*	*51*	*426*
R (3)	93	85	714
S (4)	*103*	*95*	*800*
T (5)	137	129	1087

b. Use your calculator to find the **ratio of 'b' to 'a' for each triangle** and write it in the table below. Give your answers correct to **2 decimal places**.

Triangle	$\frac{b}{a}$
P (1)	*0.12*
Q (2)	*0.12*
R (3)	*0.12*
S (4)	*0.12*
T (5)	*0.12*

c. For **a given angle**, the **ratio** of the **opposite side** to the **adjacent side** is always the **same**, irrespective of the length of the sides.

What is this ratio called? Tick (✓) the correct answer.



☑ Tangent ratio

☐ Sine ratio

☐ Cosine ratio

d. Are triangles P, Q, R, S and T **similar**? Why do you think this is so?

Answer:

Yes, triangles P, Q, R, S and T are similar because:

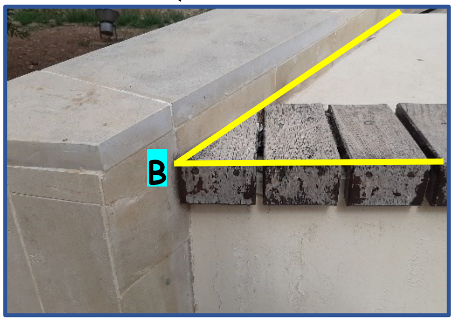
Either:

AAA – three angles of one triangle are equal to the angles of the other triangle.

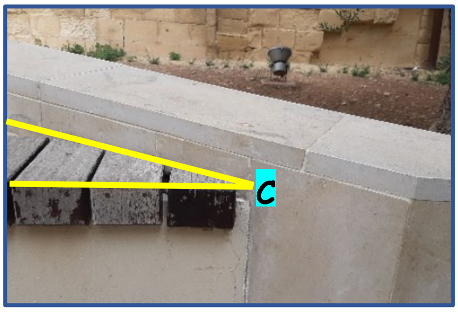
Or:

R.A.R. - two pairs of corresponding sides are in the same ratio and the included angle is equal (e.g. a1:a2 = b1:b2 or a2:a3 = b2:b3 etc.).

Task 3.4 – Pythagoras' Theorem



A close-up photo of part B above.



A close-up photo of part C above.

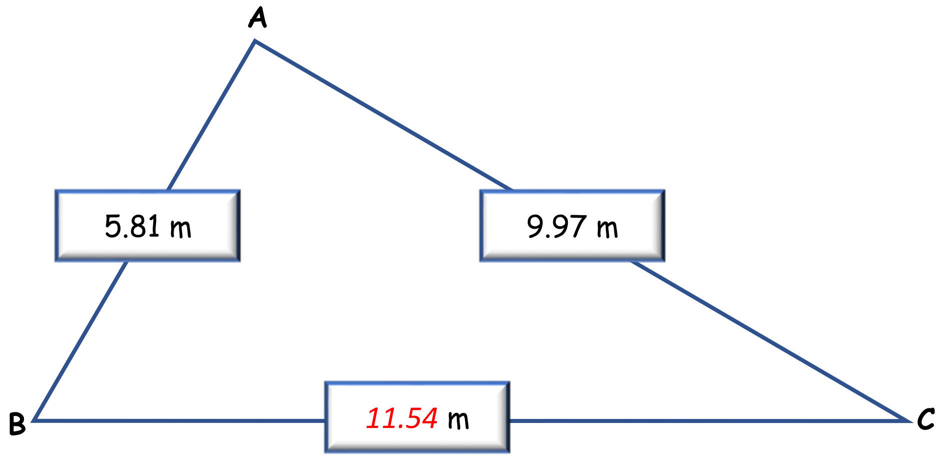
This is a triangular shaped fountain. Sides AB and AC were measured using the **inner edge** of the triangle as shown above.

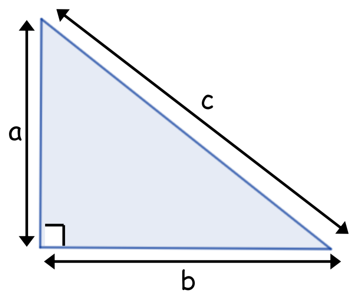
a. Use the tape measure to find BC, correct to the **nearest cm**. Make sure to measure the length from point B to point C as shown in yellow in the close-up diagrams above.

Answer:

1154 cm

b. Fill in the missing length in this diagram:





The converse of **Pythagoras' Theorem** states that if $c^2 = a^2 + b^2$, then:

- the triangle is a right-angled triangle
- the right-angle is directly opposite the hypotenuse

c. The converse of Pythagoras' Theorem allows us to check if the fountain, shown as triangle ABC above, is a **right-angled triangle**. We can do so by calculating CB^2, BA^2, and AC^2. Is CB^2 approximately equal to BA^2 and AC^2? So, what can you say about triangle ABC above?

Answer:

$CB^2 = 11.54^2 = 133.1716$

$BA^2 + AC^2 = 5.81^2 + 9.97^2$

$= 133.157$

Yes, CB^2 is approximately equal to $BA^2 + AC^2$, hence triangle ABC is a right-angled triangle.

Station 4 – Teacher's Notes

Resources

- Tracing Paper
- Protractor
- Photo Camera (per student)
- Stopwatch
- Tape Measure or Disk Tape Measure
- Calculator
- Pen and Resource Pack

Learning Outcomes

Task No	Strand	MQF Level	Learning Outcome
4.1	Shape, Space and Measure	1	6.1br Count the faces, edges and/or vertices of simple 3D shape (i.e. cube, cuboid, triangular prism, and pyramid) from a 2D drawing and/or a 3D model.
4.1	Shape, Space and Measure	2	6.2bt Identify a net as being either that of a cuboid, triangular prism, square-based right pyramid, or of neither.
4.1	Shape, Space and Measure	2	5.2at Calculate the volume of cubes and cuboids using the appropriate formulae.
4.1	Shape, Space and Measure	3	5.3bf Find missing dimensions of 3D shapes. E.g. A cube has a volume of $1000cm^3$. Calculate the length of the side of the cube.
4.2	Shape, Space and Measure	1	6.1am Name a polygon using the number of sides (from 3 to 10 sides).
4.2	Shape, Space and Measure	3	6.3aq Derive the sum of the interior angles of any polygon.

4.2	Shape, Space and Measure	1	5.1j Measure angles up to 180^0 with a protractor.
4.2	Shape, Space and Measure	1	6.1p State that angles around a point add up to 360°.
4.3	Shape, Space and Measure	1	5.1bt Measure time using seconds, minutes and/or hours.
4.3	Shape, Space and Measure	2	5.2bj Convert between larger and smaller units of time (year, days, hours, minutes, seconds).
4.3	Shape, Space and Measure	2	5.2ah Convert between larger and smaller metric units of mass (kg, g) length (km, m, cm, mm) and capacity (l, ml).
4.3	Data Handling and Chance	2	9.2ac Work through situations involving the mean, mode, median and/or range.
4.3	Data Handling and Chance	2	9.2ad Decide when best to use each type of average using words like "outlier".

Additional Information on each Task

Task 4.1: In this task, students make use of a cuboid structure to explore the properties of cuboids. They are also asked to find its volume by measuring the sides. Students need to make sure to exclude the base when taking measurements. The volume is then used to find the mass of the structure and also to find the dimensions of cube, that is to be made from the same volume of material. This gives students a practical situation where the concept of volume is used.

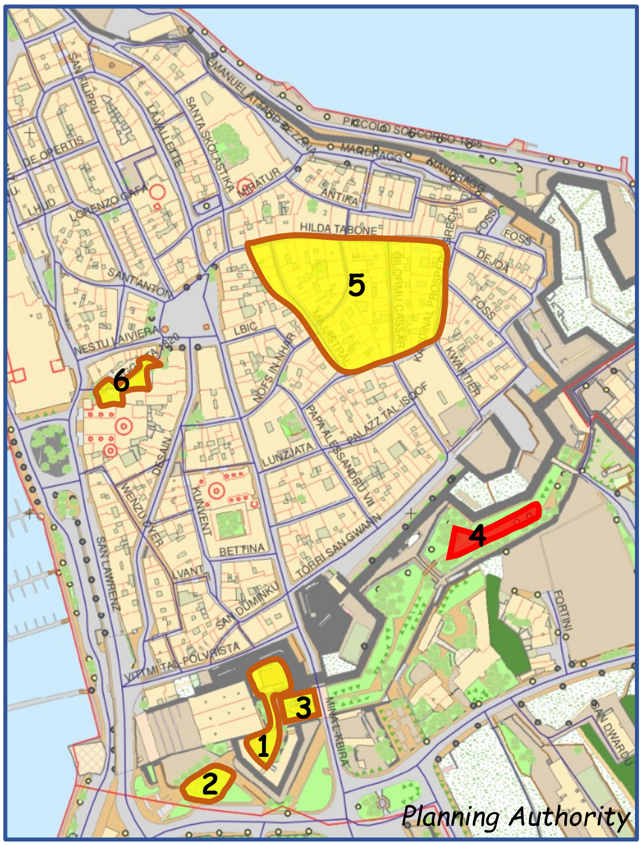
Task 4.2: The tiling in some parts of the Birgu ditch is an ideal opportunity for students to explore different irregular polygons and to confirm that the sum of interior angles truly adds up to $(n – 2) \times 180^o$, where n is the number of sides of the polygon. Students also get to measure the angles at a point and confirm that they really add up to 360^o. These are two concepts that students normally see in textbook diagrams but rarely in real-life. The angle measurements will be carried out in two different ways – for the angle at point students will use the protractor whilst on-site, whereas for the sum of interior angles students will import a photo of their chosen polygon and use GeoGebra, when back at home, to measure the angles.

This gives students the chance to use GeoGebra in a practical way. It is thus important that students are already familiar with GeoGebra, in particular how to create line segments and measure an angle.

Task 4.3: In this task, each student in the group will take turns running a short distance. Both the running time and distance need to be measured, and students will use these measurements to find the average speed. While preparing for the trail or during follow-up work, the teacher can discuss with students how reaction time when handling the stopwatch may lead to measurement inaccuracies and how to address this issue. For instance, it can be suggested to take an average of multiple readings or to have the same student using the stopwatch in order to maintain consistent reaction times for all measurements. The path that students need to run is a safe area in the ditch. It is important that students are positioned as instructed in the task and to explain to students that the spirit of this task is to collect data rather than a competition. Students are then provided with a data set that uses the same context, but it also includes an outlier. For this data students are asked to find the mean, median and mode. It is important for students to realize that when analysing data, the mean is the most impacted by outliers, whereas the median is a statistical measure that is more robust and gives a better picture of the central tendency when outliers are present.

Station 4 – Birgu Ditch (St. James Bastion Area)

Shapes, Space and Measures (Cube/Cuboids); Euclidean Geometry; Algebra; Data Handling (Statistics)

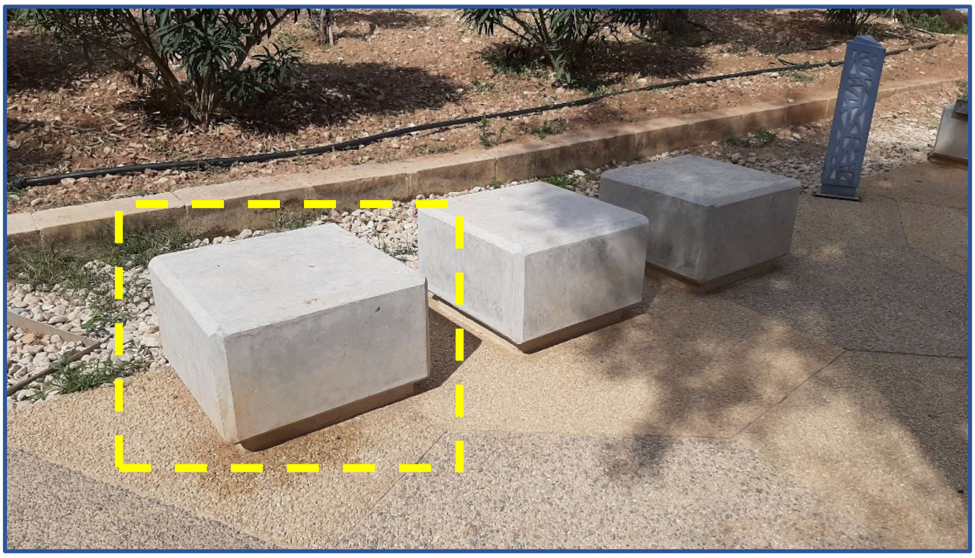


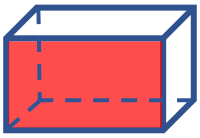
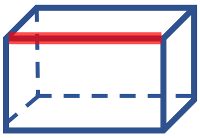
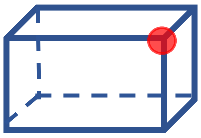
Learning Objectives

- I can use formulae to calculate the volume of cubes and cuboids.
- I can identify nets that are possible or not possible for a cuboid.
- I can use the density of a material to find the mass of an object.
- I can calculate the sum of the interior angles of irregular polygons.
- I can measure the angles at a point.
- I can measure time and distance to calculate the average speed.
- I can convert m/s to km/h or vice-versa.
- I can calculate the mean, mode and median of a data set and identify which is the most robust in case of outliers in the data set.

Birgu Ditch (St. James Bastion Area)

Task 4.1 – Volume and 3D-shapes



Face	Edge	Vertex
		
A **face** of a 3D shape is a flat surface on the outside of a shape.	An **edge** of a 3D shape is a straight line segment where two faces meet.	A **vertex** of a 3D shape is a point where two or more edges meet.

(BBC, 2020)

The three structures shown in photo above are **cuboids**. Consider the **one** marked in yellow:

a. How many **faces** does this cuboid have?

Make sure to count all the faces, and not only those shown in the photo.

Answer:

6 faces

b. How many **edges** does this cuboid have?

Make sure to count all the edges, and not only those shown in the photo.

Answer:

8 edges

c. How many **vertices** does this cuboid have?

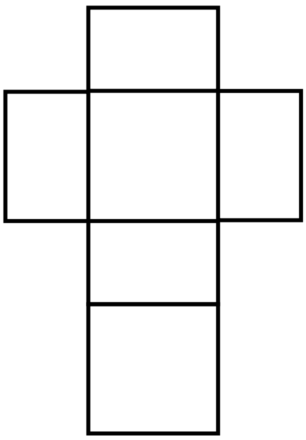
Make sure to count all the vertices, and not only those shown in the photo.

Answer:

8 vertices

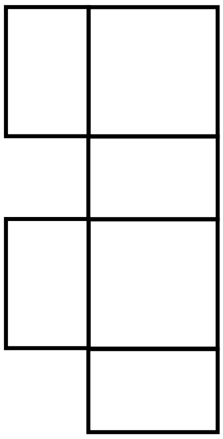
d. Which of the following diagrams represent the **net of a cuboid**? Tick (✓) the correct answer for each net.

(i)



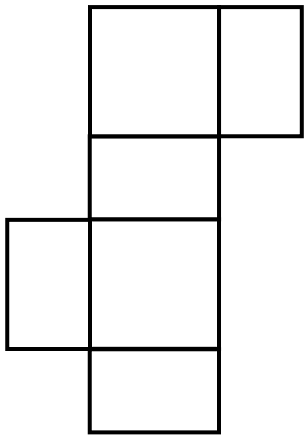
☑ Yes ☐ No

(ii)



☐ Yes ☑ No

(iii)



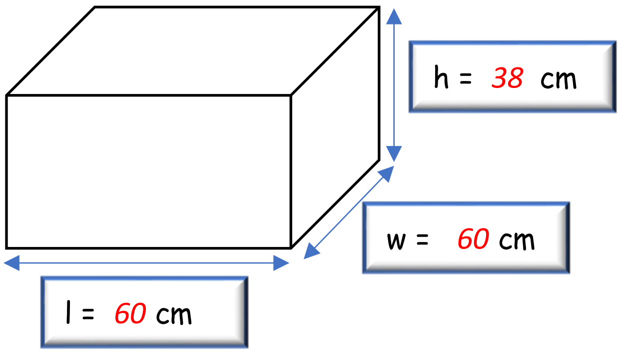
☑ Yes ☐ No

Exclude this base

When measuring:

- use the **inner edges** of the structure, as shown red in this photo.
- focus only on the cuboid, i.e. **exclude the base**.

e. Use your tape measure to find the **sides of the cuboid** (l, w, h) and fill in the measurements in the diagram below. Give your answer correct to the **nearest cm.**



f. Now, calculate the **volume of the cuboid**. Give your answer correct to 3 significant figures.

Volume of cuboid = l x b x h

= 60 x 60 x 38

= 136, 800

Answer:

137, 000 cm^3

Density is how much mass there is in a particular volume. The denser an object is, the heavier it is.

The density of an object is found by dividing the mass of the object by its volume.

The formula for density is:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

g. The concrete used for these structures has a density of **2.4g/cm^3**. Use the result of Task 4.1f to find the **mass of one cuboid**. Give your answer **in kg**, correct to the **nearest ten**.

$$\text{Density} = \frac{Mass}{Volume}$$

Mass = Density x Volume

Mass = 2.4 x 137000

= 328, 800

= 326.4kg

Answer:

330 kg

h. Which of the following statements about **cubes are correct**? Write True or False for each statement.

Statement	**True or False**
A cube is a special type of cuboid.	*True*
A cube has 15 edges.	*False*
A cube is a square.	*False*
A cube has 6 equal faces.	*True*
A cube is a prism.	*True*

i. If the **same amount of concrete** as that found in Task 4.1f is used to create a cube instead of a cuboid, what will the dimensions of the **cube** be? Give your answer in metres, correct to the **1 decimal place**.

Volume of cube = l x b x h

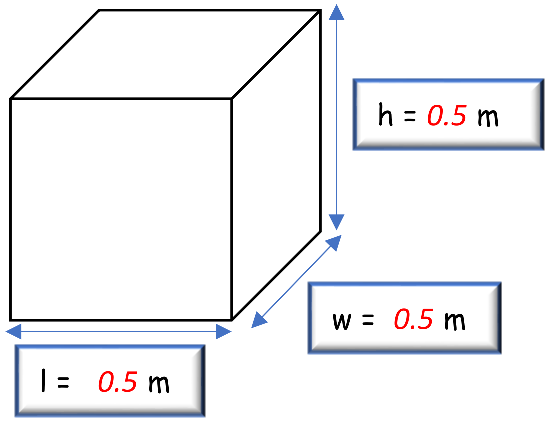
Volume of cube = side x side x side

Volume of cube = $side^3$

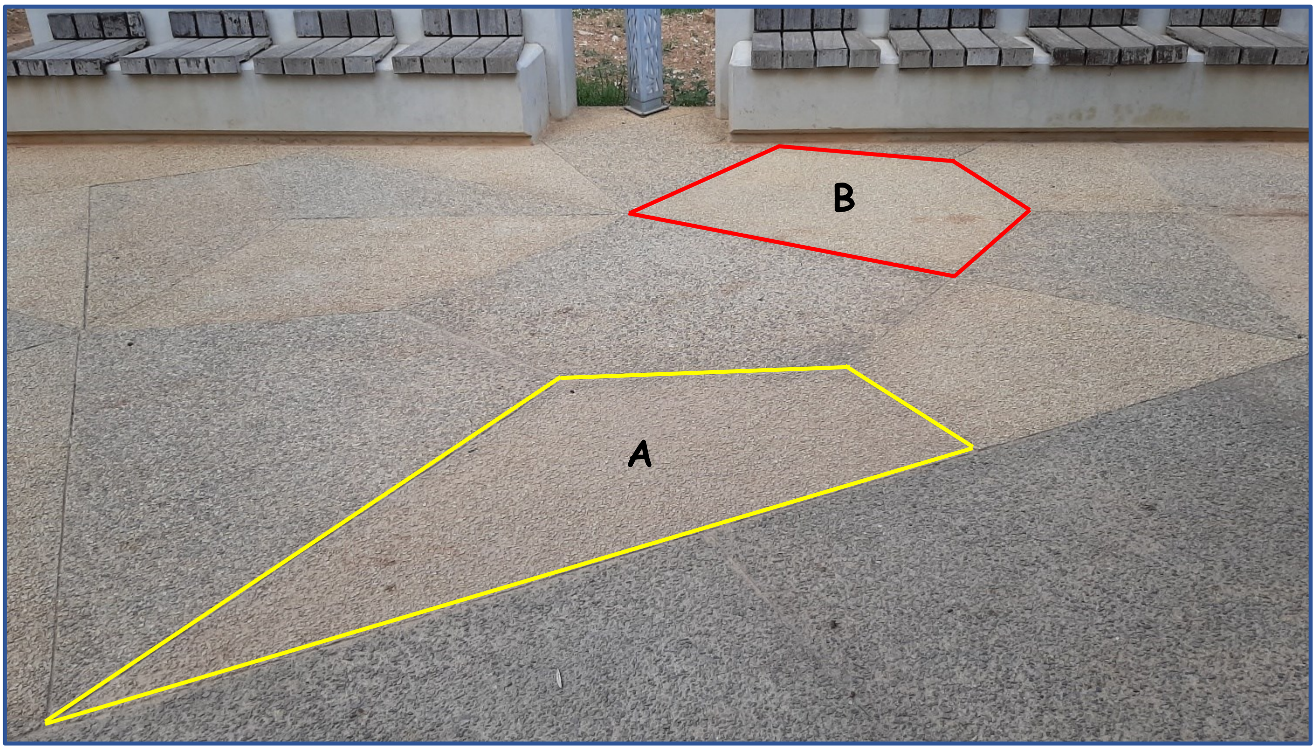
136, 800 = $side^3$

side = $\sqrt[3]{136800}$

= 51.5cm



Task 4.2 – Polygons and Angles at a Point



This tiling is made up of different polygons of various sizes.

a. Which of the following describes **shape A** (marked in yellow in the diagram above)?

Tick (✓) the correct answers.

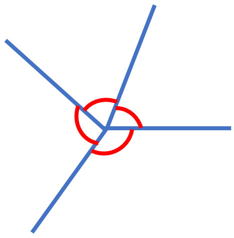
- Quarilateral ☑ Yes ☐ No
- Pentagon ☐ Yes ☑ No
- Hexagon ☐ Yes ☑ No
- Irregular polygon ☑ Yes ☐ No

b. Which of the following describes **shape B** (marked in red in the diagram above)?

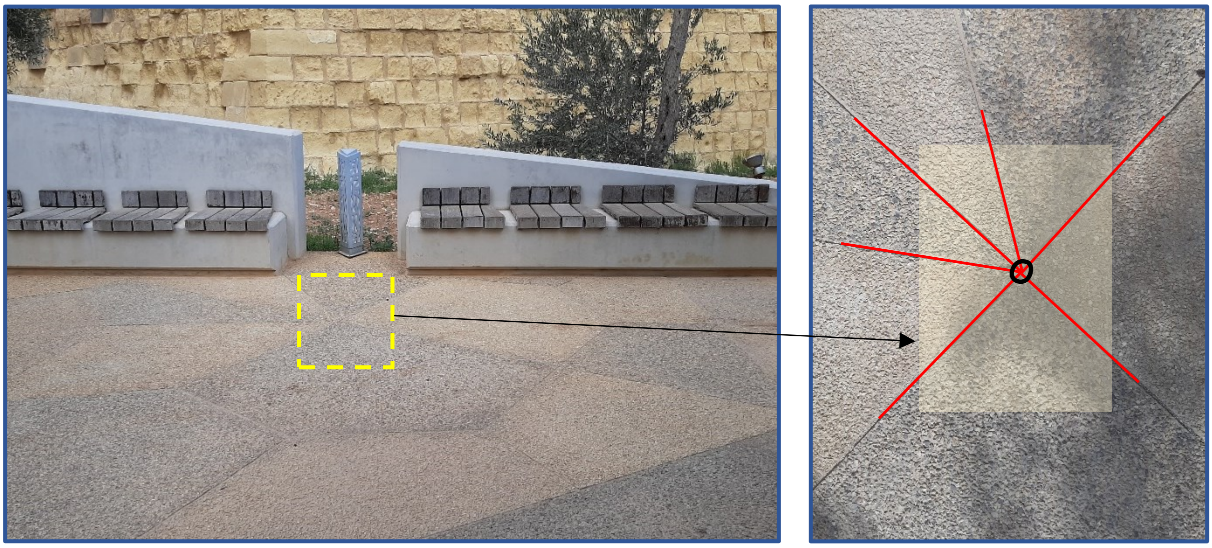
Tick (✓) the correct answers.

- Quadrilateral ☐ Yes ☑ No
- Pentagon ☑ Yes ☐ No
- Hexagon ☐ Yes ☑ No
- Irregular polygon ☑ Yes ☐ No

The angles formed by several rays having a common initial point are called **angles at a point**.

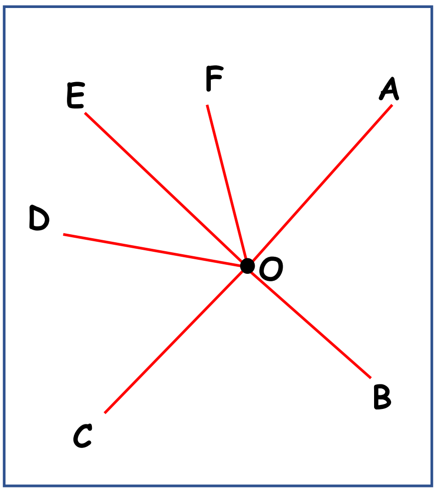


Focus on the part of tiling marked in yellow below:



c. Use a tracing paper to **trace the rays at point O**. Label the traced diagram as follows:

- O is common initial point
- A, B, C, D, E and F are the rays



d. Use your protractor to measure the following **angles**. Write your measurements in the table below, correct to the **nearest degree**. Then, work out the sum of these angles.

	Angle
Angle AÔB	89 °
Angle BÔC	91 °
Angle CÔD	55 °
Angle DÔE	35 °
Angle EÔF	33 °
Angle FÔA	57 °
Total	360 °

e. So, what is the **sum of angles** at point O?

Answer:

360 °

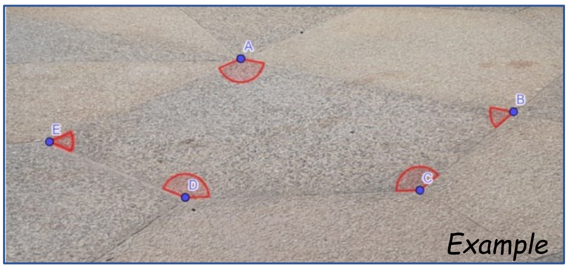
f. **Angles at a point** always add up to 360 °.

g. **Each member in the group** needs to select **one quadrilateral or one pentagon** from the tiling. Then, each member needs to:

Take a **photo** of the selected polygon. Position the camera such that the photo is taken directly from **above** the polygon (like bird's eye view). Make sure that all of the polygon **fits** in the photo.

Import the photo on **GeoGebra** and:

- Use [Angle] to measure all of the interior angles. Measurement is to be clearly marked on your GeoGebra file.



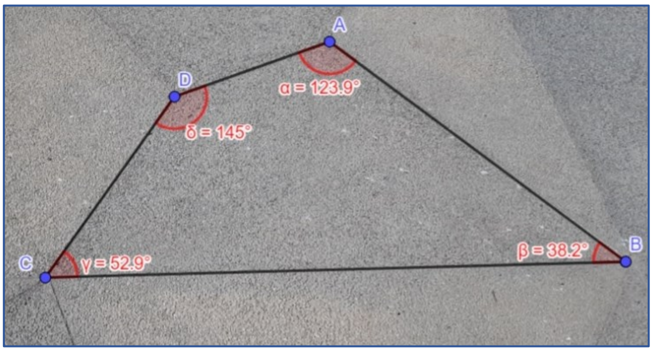
- Record these interior angles in the table below.
- Work out the sum of these interior angles and write your answer in the 'Total' box below.

If you selected a quadrilateral, use this:

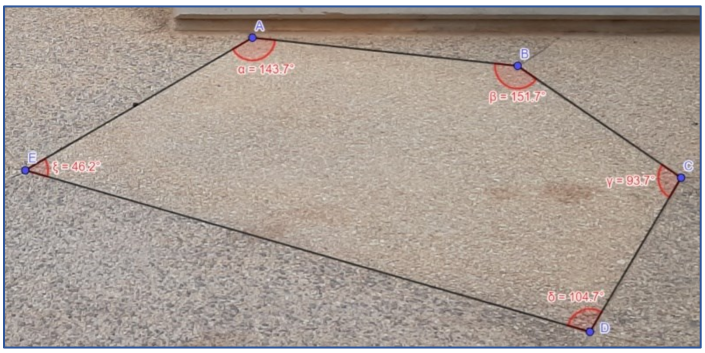
	Quadrilateral *(Example)*
Angle A	*123.9* °
Angle B	*38.2* °
Angle C	*52.9* °
Angle D	*145* °
Total	***360*** °

If you selected a pentagon, use this:

	Pentagon *(Example)*
Angle A	*143.7* °
Angle B	*151.7* °
Angle C	*93.7* °
Angle D	*104.7* °
Angle E	*46.2* °
Total	***540*** °



Example of a quadrilateral

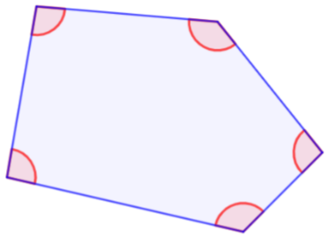


Example of a pentagon

The **sum of interior angles** of a polygon can be found using the formula:

$$(n-2) \times 180^{\circ}$$

where *n* is the number of sides



h. Find the **sum of interior angles** of your polygon using this formula.

Quadrilateral:

Sum of interior angles $= (4 - 2) \times 180$

$= 360^{\circ}$

Pentagon:

Sum of interior angles $= (5 - 2) \times 180$

$= 540^{\circ}$

Answer:

Quadrilateral $= 360^{\circ}$

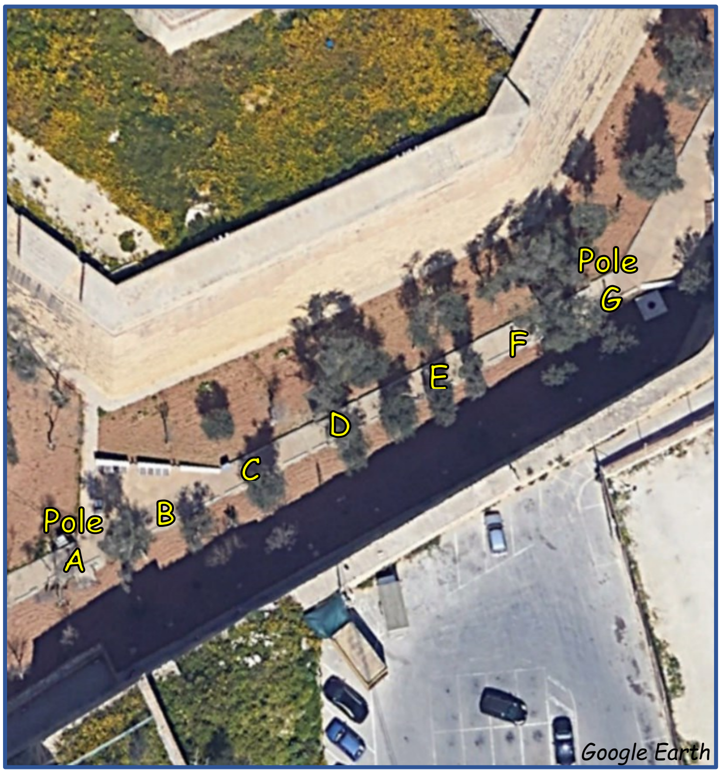
Pentagon $= 540^{\circ}$

i. Compare the result of Task 4.2g to the answer of Task 4.2h. What can you notice?

Answer:

The sum of the interior angles of a polygon is equal to $(n - 2) \times 180^{\circ}$ *where n is the number of sides of the polygon.*

Task 4.3 – Statistics



Aerial View of Birgu Ditch (St. James Bastion Area)

In the following activity you are going to take it in turns so that **each student** runs from **Pole A to Pole G, and back to Pole A**. In each round there needs to be a student who is a:

- '**Pole A keeper**' situated next to Pole A responsible for the start and finish line;
- '**Time keeper**' situated next to Pole A responsible to measure the time taken for the 'runner' to run from the start line to the finish line;
- '**Pole G keeper**' situated next to Pole G responsible to ensure that the 'runner' runs up to Pole G before heading back to Pole A;
- '**Runner**' to run from Pole A to Pole G and back to Pole A.

a. Use your tape measure to find the **distance** between Pole A and Pole G as marked in the diagrams above. Use this measurement to find the **total distance travelled** from the start line to the finish line. Give you answer in **metres**.

Distance between Pole A and Pole G = 54m

Distance from the start line to the finish line = 54 x 2

= 108 m

Answer:

108 m

b. Record the time on your stopwatch whenever a student runs **from the start line to the finish line**.

Write these measurements in the table below.

	Running Time	Running Time in seconds
Student 1 *Example*	*00 : 28 . 34*	*28s*
Student 2	__ : __ . __	
Student 3	__ : __ . __	
Student 4	__ : __ . __	

Speed is a measure of how fast an object is moving.

When objects do not move at a constant speed, we measure the **average speed**. The formula for average speed is:

$$\text{Average Speed} = \frac{\textbf{Total Distance Travelled}}{\textbf{Total Time Taken}}$$

c. Calculate the average speed of **the fastest student** in your group.

(The average speed of the fastest student is the speed of the student with the shortest running time)

Answer:

m/s

d. Which of the following statements is correct. Tick (✓) the correct answer. Show your working to justify your answer.

- ☐ 4m/s is less than 14.4km/h
- ☑ 4m/s is equal to 14.4km/h
- ☐ 4m/s is greater than 14.4km/h

Answer:

To convert km/hr to m/s: x1000, ÷3600

14.4 km/hr = 14.4 x 1000 ÷ 3600

= 4 m/s

Mean, median and mode

Mean is the **average** value.

- To find the mean, add all numbers together and then divide by the number of values.

Median is the **middle** value.

- To find the median, order the numbers lowest to highest and see which value is in the middle of the list. If there are two middle values, the median is the value halfway between them.

Mode is the number that appears the **most**.

- To find the mode, order the numbers lowest to highest and see which value appears the most often.

(BBC, 2023a)

e. A group of 10 students carried out a similar run. The table below shows their running times.

Student	A	B	C	D	E	F	G	H	I	J
Running Time (seconds)	28	26	25	30	100	32	27	29	35	28

Calculate the **mean**, the **median** and the **mode** of this data set. Give your answers correct to **one decimal place**.

Data set in ascending order = 25, 26, 27, 28, 28, 29, 30, 32, 35, 100

Mean = (25 + 26 + 27 + 28 + 28 + 29 + 30 + 32 + 35 + 100) ÷ 10

= 36 s

Mode = 28 s

Median = (28 + 29) ÷ 2

= 28.5 s

Mean:	**Median:**	**Mode:**
36 s	*28.5* s	*28* s

f. As you can notice, the running time of Student E is much larger than the rest of the group. This was recorded incorrectly because of an issue with the timer. We say that this measurement is an '**outlier**' to the rest of the data set.

Eliminate the record of student E from the data set as follows:

Student	A	B	C	D	F	G	H	I	J
Running Time (seconds)	28	26	25	30	32	27	29	35	28

Re-calculate the **mean**, the **median** and the **mode**. Give your answers correct to **one decimal place**.

Data set in ascending order = 25, 26, 27, 28, 28, 29, 30, 32, 35

Mean = (25 + 26 + 27 + 28 + 28 + 29 + 30 + 32 + 35) ÷ 9

= 28.9 s

Mode = 28 s

Median = 28 s

Mean: *28.9* s

Median: *28* s

Mode: *28* s

g. Compare the results of the **mean** in Task 4.3e and Task 4.3f. What can you observe?

Answer:

The mean changes from 36s to 28.9s after eliminating the outlier from the data set.

Compare the results of the **median** in Task 4.3e and Task 4.3f. What can you observe?

Answer:

The median changes slightly from 28.5s to 28s after eliminating the outlier from the data set.

Compare the results of the **mode** in Task 4.3e and Task 4.3f. What can you observe?

Answer:

The mode does not change after eliminating the outlier from the data set.

h. Which of these calculations do you think is **the most distorted** by the outlier?

Answer:

The mean is the most distorted by the outlier.

Station 5 – Teacher's Notes

Resources

- Compass
- Protractor
- Ruler
- Pen and Resource Pack

Learning Objectives

Task No	Strand	MQF Level	Learning Outcome
5.1	Data Handling and Chance	1	9.1a Collect data.
5.2	Data Handling and Chance	1	9.1g Construct a frequency table using a tally column.
5.2	Data Handling and Chance	2	9.2k Construct a bar chart using grouped and/or ungrouped discrete data from a frequency table.
5.2	Data Handling and Chance	2	9.2p Construct pie charts.
5.3	Number	1	1.1j Identify odd and even numbers.
5.3	Number	1	1.1x Identify prime numbers up to 100.
5.3	Algebra	2	3.2h Represent unknowns, variables and constants in algebraic expressions using letters.
5.3	Algebra	2	3.2z Work through situations leading to the solution of linear equations in one unknown. E.g. mystery numbers, geometric shapes, etc.

5.4	Shape, Space and Measure	3	5.3b Interpret three figure bearings.
5.4	Shape, Space and Measure	2	5.2k Draw angles up to 360^0 with a protractor.
5.4	Number	3	2.3bd Draw scale drawings involving bearings; angles of elevation and depression.
5.4	Number	3	2.3be Interpret scale drawings. E.g. bearings; angles of elevation and depression.

Additional Information on each Task

Task 5.1: The old buildings in the Collacchio area have several features that can be used for data collection purposes. In the case of this trail, students will walk along Triq Hilda Tabone, a prominent street in Birgu, to gather information about the door number, knocker, and colour. Although traffic in this street is minimal and the occasional car passes by at a low speed, students should be made aware that this is not a pedestrian road and hence need to be careful. There are no particular difficulties associated with this and the subsequent two tasks. The aim of the activity is for students to be involved in the data collection process in order to enhance the meaningfulness and relevance of the data they will then try to represent. It also gives students the opportunity to appreciate the uniqueness of the Maltese doors and the origins of the door knockers ('il-ħabbata')

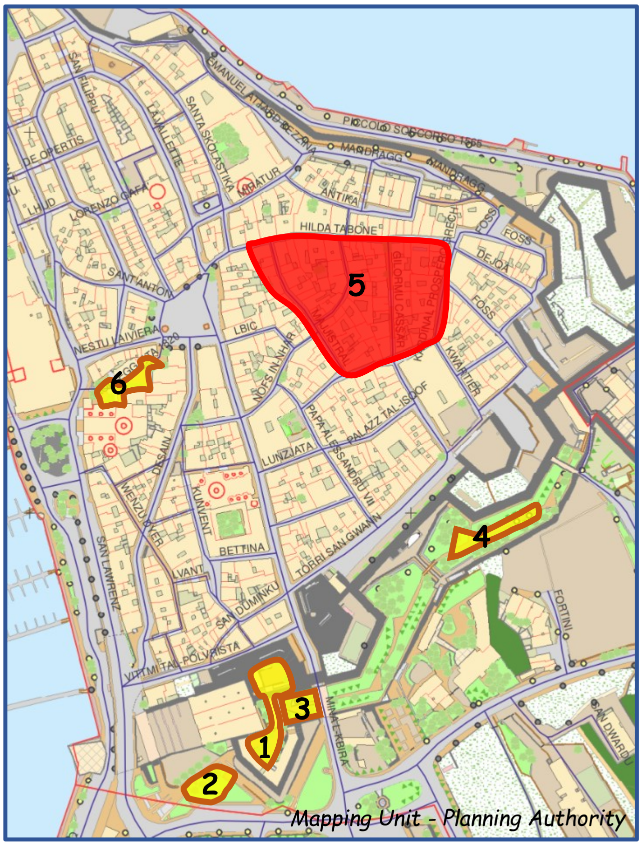
Task 5.2: Students will create frequency tables from the data collected in Task 5.1 in order to construct a pie chart and a bar chart. This will be done at home. Whilst carrying out this task, students can reflect and make a connection with the data they are illustrating, as for example, they would already have an idea of the most common door colour and knocker whilst gathering the information on-site.

Task 5.3: In this task students will use the set of numbers collected in Task 5.1 to carry out number work. Students might need to be reminded what prime numbers, square numbers and triangular numbers are when preparing for the trail. The set of numbers will also be used to create a fun activity using algebra in which students learn how to do a 'magic trick' with numbers.

Task 5.4: The Collacchio area is the perfect place for students to navigate through its narrow and winding roads using bearings. The compass is not a tool students use often; at times they would not know that they need to position it flat on a metal-free surface. Thus, before the trail students need to be given exercises where they practice how to measure bearings. In this task students also get to pass by the Norman House and can thus admire its notable façade.

Station 5 – Il Collacchio

Statistics (Data Handling); Number (Numerical Calculations); Algebra



Learning Objectives

- I can collect data and use this data to construct a frequency table.
- I can construct pie charts and bar charts with the collected data.
- I can interpret data from frequency tables, bar charts and pie charts.
- I can identify even, odd, square, triangular, and prime numbers.
- I can work through situations leading to the solution of a linear equation to find an unknown value.
- I can use a navigation compass to measure bearings.
- I can draw scale drawings, including bearings.

Triq Hilda Tabone – a main street in the Collacchio area

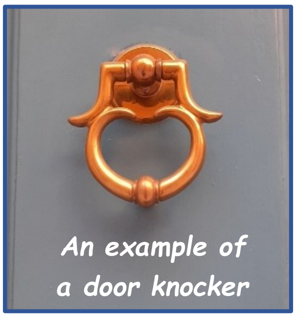
Task 5.1 – Collecting Data

Aerial View of Il Collacchio Area

In 1562, Grand Master La Valette, established the boundary within which the Knights' palaces and houses had to be built and where the Knights were ordered to live. The boundary wall was marked by a number of stone bollards, like low columns, one of which can be seen in Victory Square. The Collacchio has some of the oldest buildings in Birgu and one can still find the Auberge de France and the Auberge d'Angleterre still in good condition. An auberge was the residence of a group of knights of the same nationality.

Door Knockers ('Ħabbata')

Door knockers are a feature on most of the traditional Maltese doors. It was a way for a visitor to alert the house owner of his presence before electronic doorbells and intercoms were invented. Door knockers can be found in numerous shapes and sizes. Typically made out of metal or brass, they often feature the Maltese Cross or maritime motifs such as dolphins, seahorses and fish, amongst others. In the past, elaborate door knockers indicated families that had more status or wealth.



An example of a door knocker

Coat of arms of Malta from 1964 to 1975

The dolphin door knocker became popular post-independence as the dolphin was depicted in Malta's first coat of arms.

(Wikipedia Foundation, 2023, n.d.)

Note that a doorknob ('pum') is not a door knocker. A doorknob is used to push or pull the door whereas the door knocker is used instead of a doorbell.

An example of a door knob

In this task you are going to walk from one end of **Triq Hilda Tabone** (start) to the other end (finish), as shown in yellow below. As you walk along this street, you are going to:

- Record the **door numbers** as shown in table in Task 5.1a
- Record the **door colours** as shown in table in Task 5.1b
- Record the **door knockers** as shown in table in Task 5.1c

a. When recording the **door numbers** in the grid below, consider **only** the door numbers written on the Local Council's ceramic number plate, as shown.

Door Number					
1	*17*	*19*	*21*	*22*	*23*
24	*25*	*27*	*34*	*35*	*43*
46	*47*	*48*	*48a*	*49*	*50*
53	*59*	*62*	*64*	*68*	*73*
75	*85*	*86*	*87*		

b. When recording the **door knockers**:

- Fill in the table below with appropriate tally marks.
- **Count** the door knockers **in singles** rather than in pairs as some doors might not have two identical door knockers.

Example:

Dolphin = ||

Seahorse = | and Fish = |

- **Do not** take into consideration any doors **without** door knockers.
- Remember to group the tally marks in groups of 5 as 𝍸.

Door Knocker	Examples	Tally
Seahorse	 	𝍸 II
Fish	 	𝍸 𝍸 III
Dolphin with trident tail		𝍸 III
Ring Handle	  	𝍸 𝍸 𝍸 I

c. Record the door colours of **all** doors as you walk from the Start to the Finish points. When recording the **door colours**, fill in the table below with appropriate tally marks.

Door Colour	Tally
White	I
Cream	II
Yellow	IIII
Mustard	I
Beige	卌
Brown	卌 卌
Orange	I
Red (Wine)	II
Blue (Royal/Light/Dark)	卌 IIII
Green (Light/Dark)	卌 III
Grey	II
Black	I

Task 5.2 – Illustrating Data

Pie Charts and Bar Charts

A **pie chart** is a circular graph divided into different **sectors**, where each sector represents a category. The angle of each sector is a fraction of 360°.

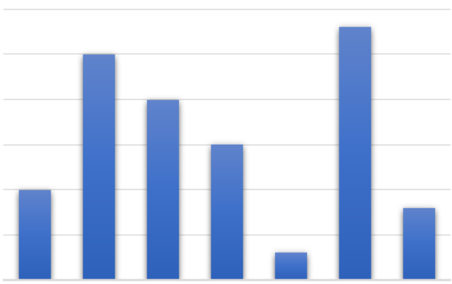
Pros

- A pie chart gives a visual representation of the part-to-whole comparison, i.e. what portion (percentage) of the whole group each category is.

Cons

- A pie chart cannot be used when the number of categories is more than six as it will be difficult to read.

A **bar chart** plots data using rectangular **bars**. It consists of two axes - one represents the category and the other represents the frequency of events occurring i.e. the height of each bar is the frequency.



Pros

- A bar chart gives a visual representation of how each category compares with the others.

Cons

- It is not easy to do a whole-to-part comparison with a bar chart.

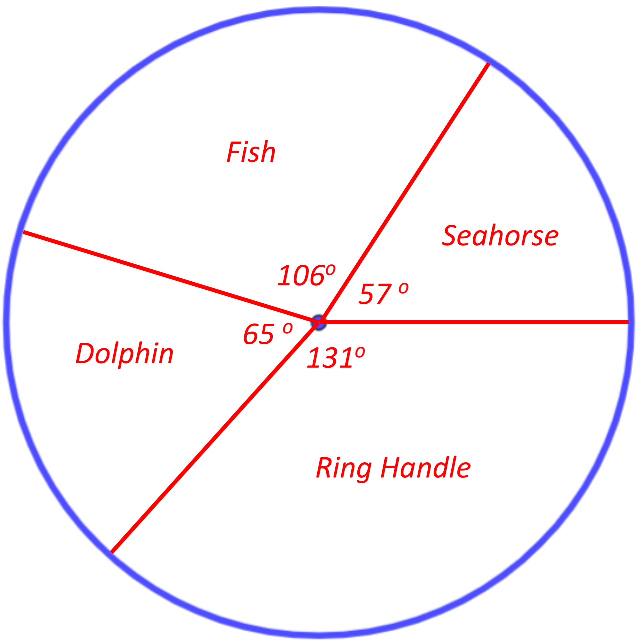
BBC (2022) and Arif (2022)

a. In this task you need to construct a **pie chart** to represent the **door knockers**.

(i) Refer to the data collected in Task 5.1b to fill in the table below:

Shape	Seahorse	Fish	Dolphin	Ring Handle	Total
Frequency	7	13	8	16	44
Fraction of total	$\frac{7}{44}$	$\frac{13}{44}$	$\frac{8}{44}$	$\frac{16}{44}$	$\frac{44}{44}$
Angle= Fraction of total × 360° (correct to the nearest degree)	57°	106°	65°	131°	359°

(ii) Use your protractor to draw the pie chart below, labelling each category and its corresponding angle on the diagram.



(iii) The total of the angles might not add up to 360°. Why do you think this is so?

Answer:

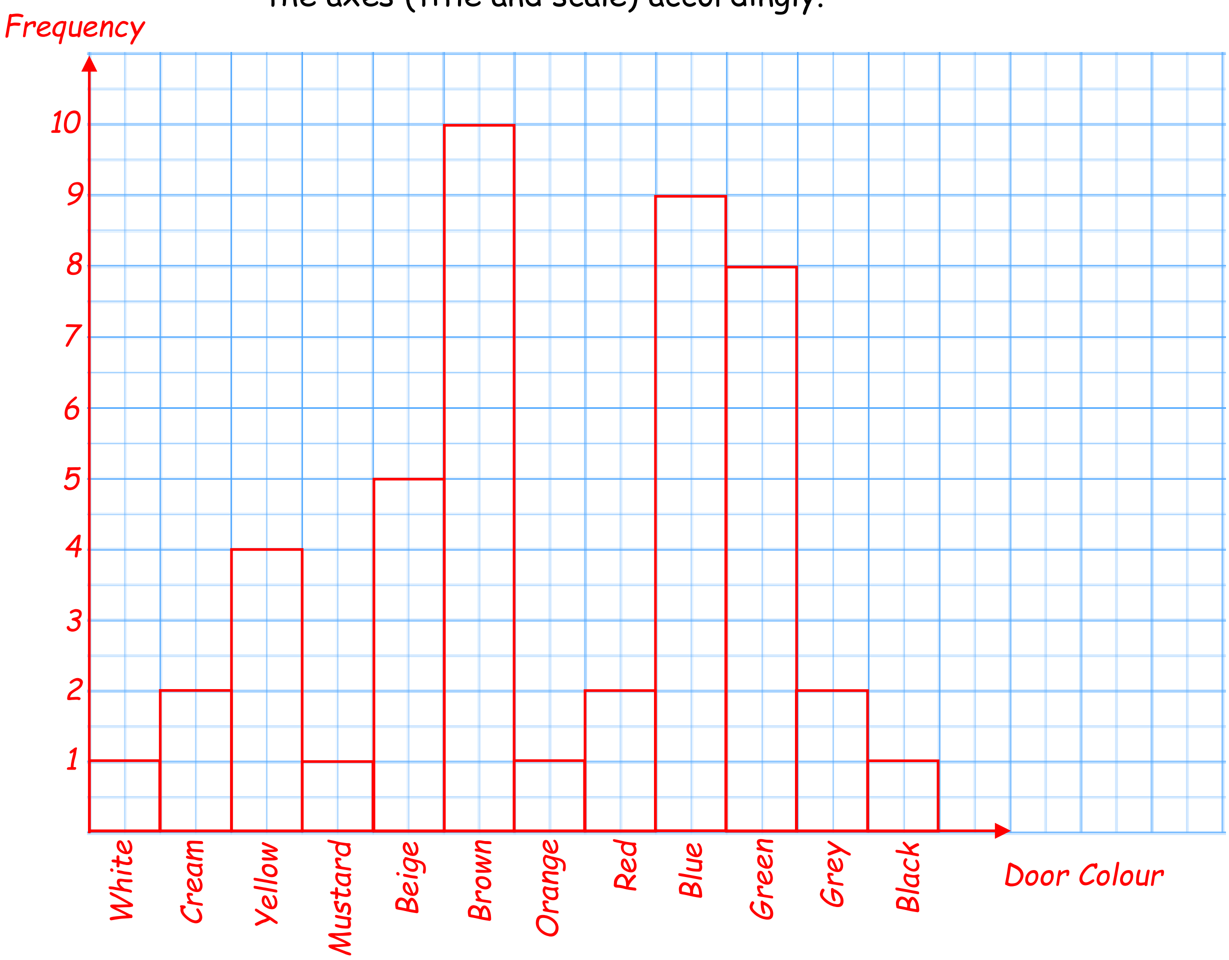
Because the angles are calculated to the nearest degree.

b. In this task you need to construct a **bar chart** to represent **the door colours**.

(i) Refer to the data collected in Task 5.1c to fill in the table below:

Colour	**White**	**Cream**	**Yellow**	**Mustard**	**Beige**	**Brown**	**Orange**	**Red**	**Blue**	**Green**	**Grey**	**Black**
Frequency	*1*	*2*	*4*	*1*	*5*	*10*	*1*	*2*	*9*	*8*	*2*	*1*

(ii) Use your ruler to draw a **bar chart** on the squared paper below. Label the axes (title and scale) accordingly.



c. Which is the most frequent door knocker?

Answer:

Ring Handle

Which is the most frequent door colour?

Answer:

Brown

Task 5.3 – The Number System

a. Refer to the number set of door numbers in Task 5.1a and identify all of the:

Answer:

- even numbers ………… 22, 24, 34, 46, 48, 50, 62, 64, 68, 86
- odd numbers ………… 1, 17, 19, 21, 23, 25, 27, 35, 43, 47, 49, 53, 59, 73, 75, 85, 87
- prime numbers ……… 17, 19, 23, 43, 47, 53, 59, 73
- square numbers …… 1, 25, 49, 64
- triangular numbers 1, 21

b. Refer to the table in Task 5.1a and find **3 consecutive numbers**, for example 102,103,104.

(i) Write down these 3 numbers and find their sum.

Example:

21 + 22 + 23 = 66

Answer: *Example: 66*

(ii) Is the sum a multiple of the middle number?

66 ÷ 22 = 3

Answer: *Yes*

(iii) Find another set of 3 consecutive numbers from your table in Task 5.1a and check if the sum is also a multiple of the middle number.

Answer:

22 + 23 + 24 = 69	*-> 69 ÷ 23 = 3*
23 + 24 + 25 = 72	*-> 72 ÷ 24 = 3*
46 + 47 + 48 = 141	*-> 141 ÷ 47 = 3*
47 + 48 + 49 = 144	*-> 144 ÷ 48 = 3*
48 + 49 + 50 = 147	*-> 147 ÷ 49 = 3*
85 + 86 + 87 = 258	*-> 258 ÷ 86 = 3*

(iv) Let **n** be the middle number of 3 consecutive numbers. Show that the sum of these 3 numbers is always 3 times the middle number.

Answer:

$$\text{Total} = (n - 1) + (n) + (n+1)$$
$$= n - 1 + n + n + 1$$
$$= 3n$$

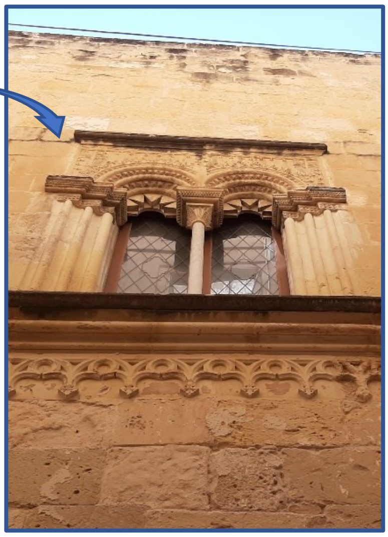
Magic trick with numbers:

Ask a friend to think of 3 consecutive positive whole numbers, add them up, and give their total.

When you have the total, divide it by 3, take note of the middle number, deduct 1 from it and add 1 to it. This will give you the sequence that your friend thought about.

Say the sequence and notice the surprise on your friend's face!

Task 5.4 – Bearings and Scale Drawing



The Norman House

The Norman House is one of the oldest buildings in Birgu. It may have been built in the 13th century and has one of the few medieval windows in Malta. The style in which the window is built is called Siculo-Norman because the Normans built many like it in Sicily.

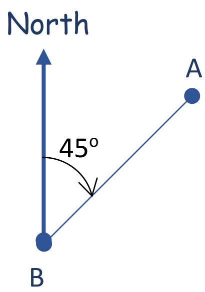
In Malta this style was imported by the wealthy people under the influence of their counterparts in Sicily.

The **bearing to a point** is the **angle** measured in a **clockwise** direction from the **north** line.

A **three-figure bearing** is a bearing given in **three figures**.

When the number of digits in a bearing is less than 3 digits, **add extra zero/s** to make the number of digits equal to 3.

For example, bearing of A from B is 045°.



To use the **navigating compass**:

- Start by **facing** your target location (e.g. a road that you need to follow).
- Hold your navigating compass **flat** with the direction of travel arrow pointing in the direction you are walking.
- If using the mobile app, read the angle given by the app.
- If using a physical navigating compass, rotate the dial so that the 'N' aligns with the red part of the compass needle. Read the angle on the rim.
- This angle is the angle between the North line and your target location.

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Lora is in Triq Hilda Tabone and walks around the Collacchio area to find the Norman House. She starts from Point A (Triq Hilda Tabone corner with Triq Ġilormu Cassar), then goes to Point B, C, D, E and then back to Triq Hilda Tabone (Point F).

a. Use your navigating compass to find the following bearings. Give your answers using a **three-figure bearing**. Make sure that you are holding your navigating compass in the '**from**' location.

For example:

- the bearing of Triq Hilda Tabone is 274 °
  (bearing of F **from** A)

i) The bearing of Triq Ġilormu Cassar (bearing of B from A)

Answer:

175°

ii) The bearing of Triq Paċifiku Scicluna (bearing of C from B)

Answer:

250°

iii) The bearing of Triq il-Majjistral (bearing of D from C)

Answer:

328°

iv) The bearing of the upper part of Triq it-Tramuntana (bearing of E from D)

Answer:

040°

v) The bearing of the lower part of Triq it-Tramuntana (bearing of F from E)

Answer:

348°

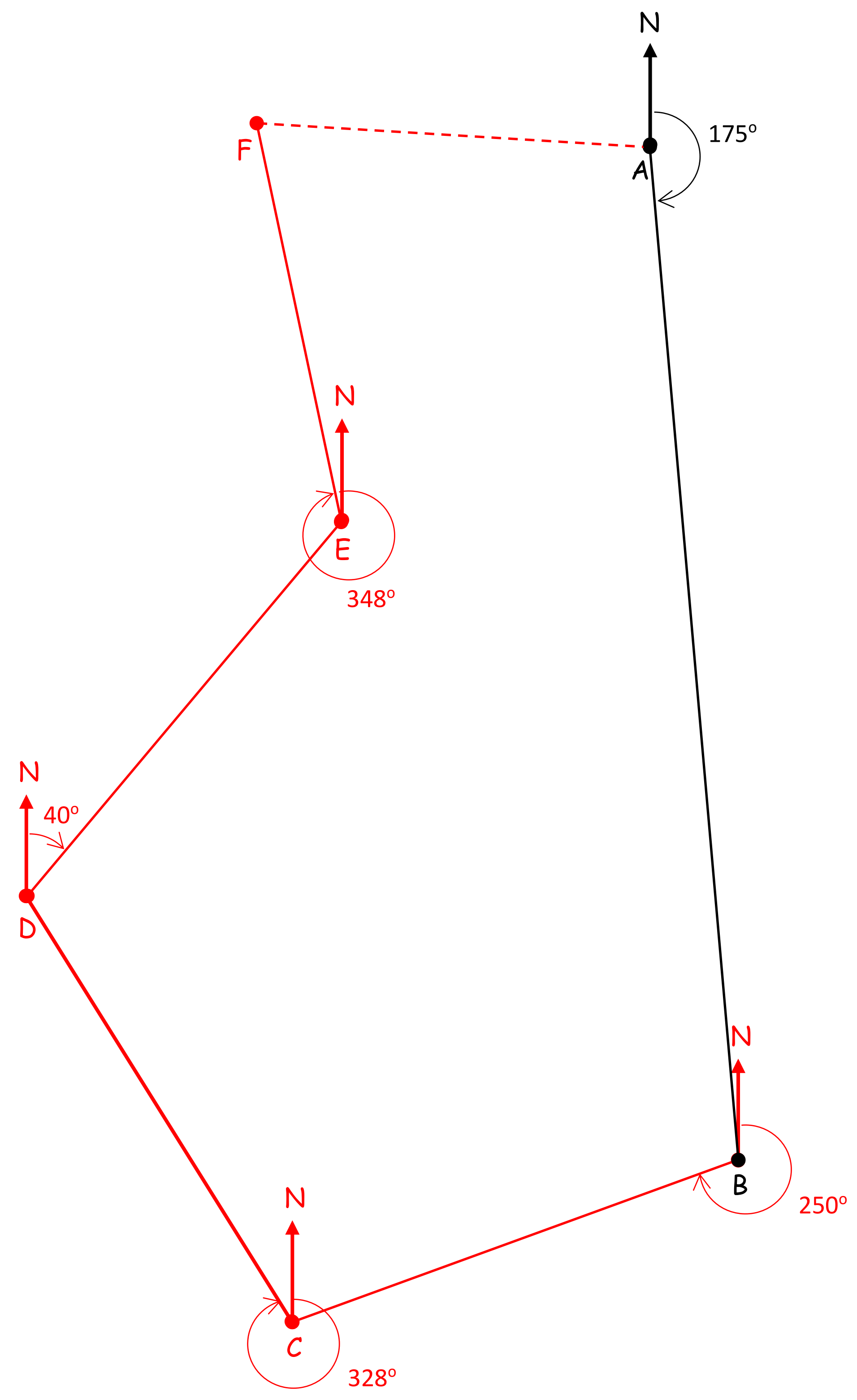
b. The distances between Points A, B, C, D, E and F, as shown on the map above are as follows:

- Point A to Point B = 75m *-> 15cm*
- Point B to Point C = 35m *-> 7cm*
- Point C to Point D = 37m *-> 7.4cm*
- Point D to Point E = 36m *-> 7.2cm*
- Point E to Point F = 30m *-> 6cm*

These distances were measured using Google Earth.

Draw an **accurate scale drawing** of Lora's route by using your ruler and protractor.

Use a **scale of 1cm to 5m**.



c. Use your diagram to find the **distance** (in metres) Lora needs to walk to go **from Point F to Point A.**

Distance from F to A = 5.8cm

Scale Factor: 1cm to 5m
1cm = 5m
5.8cm = ?

(5 x 5.8) ÷ 1 = 29m

(Actual distance using Google Earth = 31.1m)

Answer:

29 m

Station 6 – Teacher's Notes

Resources

- Clinometer
- Tape Measure or Disk Tape Measure
- Photo Camera (per student)
- Calculator
- Pen and Resource Pack

Learning Outcomes

Task No	Strand	MQF Level	Learning Outcome
6.1	Shape, Space and Measure	3	5.3v Interpret angles of elevation and depression.
6.1	Shape, Space and Measure	3	5.3u Find unknown lengths and angles in right-angled triangles using the trigonometric ratios.
6.1	Number	2	2.2ao Express a quantity as a percentage of another where the percentage < 100%.
6.2	Data Handling and Chance	3	10.3j Work out the probability of mutually exclusive events.
6.2	Data Handling and Chance	3	10.3n Construct a probability tree (tree diagram) for up to three independent and/or dependent events.
6.2	Data Handling and Chance	3	10.3o Use a probability tree (tree diagram) to work out the probability of up to three independent and/or dependent events.
6.3	Number	1	1.1h Recognise the place value of any digit in a whole number up to one billion.

6.3	Number	2	1.2w Express any integer as a product of prime factors.
6.4	Shape, Space and Measures	1	8.1c Draw lines of symmetry in 2D shapes and pictures. E.g. flags and dominoes.
6.4	Shape, Space and Measures	2	8.2j State the order of rotational symmetry of 2D shapes and/or partly shaded 2D shapes.

Additional Information on each Task

Task 6.1: In this task students will first measure the height of one stone course ('filata') and then count the number of stone courses to estimate the height of a pilaster on the side wall of the Oratory of the Holy Crucifix. Students should consider counting the courses of the wall adjacent to the pilaster to be more accurate because it is difficult to estimate the height of the cornice on top of the pilaster in terms of courses. Such an estimation is an important exercise as there are several real-life situations where one needs to estimate the height of a building or of a room by referring to the number of stone courses. The height of one stone course in modern buildings is typically of 27cm, however considering this an old building it is best if first students measure the height of one stone course. Students will then use trigonometry to actually measure the height of this pilaster. The angle of elevation will be measured using the clinometer. Students can choose any of the trigonometric ratios to find the height BO. Students can then compare the estimated height and the calculated height to find the percentage error.

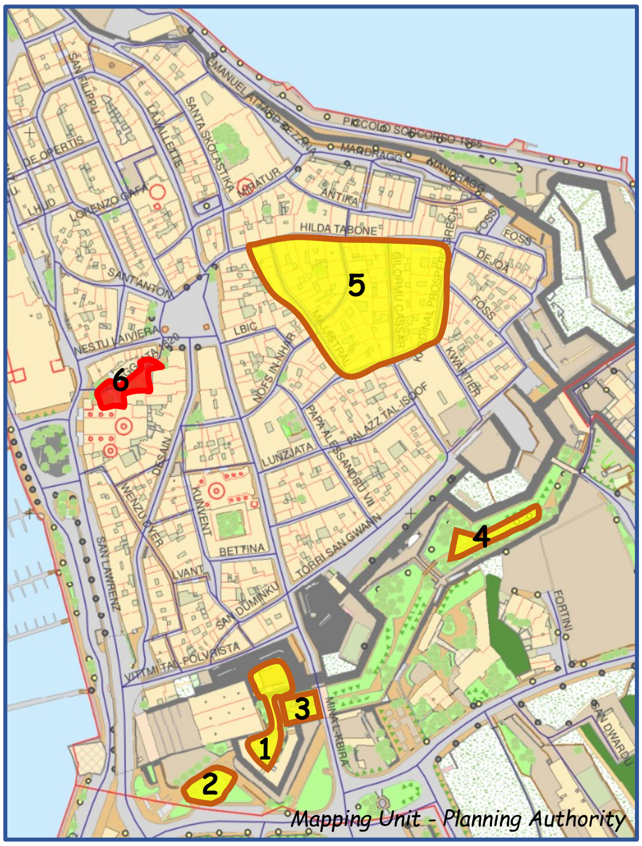
Task 6.2: The list of names commemorated on the plaque at the side of St. Lawrence's church can be used for various activities. However, considering there are several names dedicated to St. Lawrence, a task about the probability of Vittoriosani's names being dedicated to this patron developed automatically. Of course, this is only a small sample of names and not representative of the names of the people currently living in Birgu. Nevertheless, even though the probability of meeting someone from Birgu whose name is dedicated to St. Lawrence might not be 20%, it is still quite a common occurrence.

Task 6.3: The years displayed on the monumental bust of Mons. Dun Ġużepp Caruana are used in this task to ask students questions about facts of the number system. Students might find this task difficult if they do not have a sound understanding of the place value system or do not know the properties of an even or odd number, for example. It is recommended that students are reminded how to write a number as a product of its prime factors when preparing for the trail.

Task 6.4: Birgu has ample features that can be used in a task about symmetry. The ones chosen for this task are intended to highlight the difference between reflective symmetry and rotational symmetry. It is important for students to have opportunities where they have time to observe the symmetry in the environment that surrounds us, rather than just referring to diagrams on textbooks.

Station 6 – Wesgħat il-Kolleġġjata

Shapes, Space and Measures (Trigonometry and Euclidean Geometry); Data Handling & Chance (Probability); Numerical Calculations



Learning Objectives

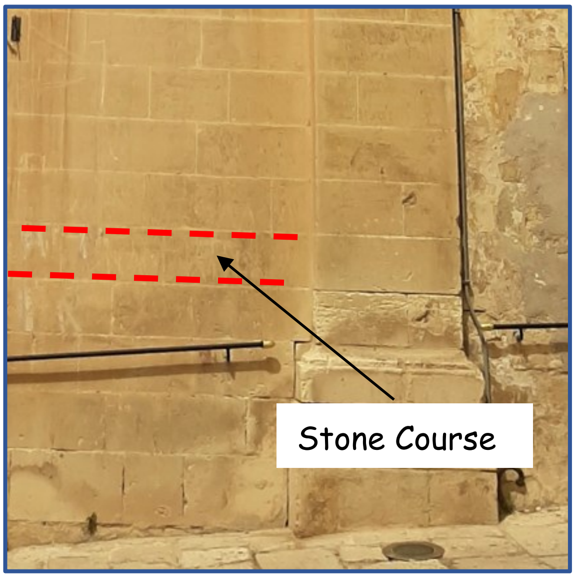
- I can use the trigonometric ratios to find unknown lengths in right-angled triangles.
- I can estimate the height of a building using the number of stone courses.
- I know the value of each digit in each number according to its position in the number.
- I can express any integer as a product of its prime factors.
- I can identify the lines of reflective symmetry and the order of rotational symmetry.
- I can construct and use a probability tree diagram to work out the probability of independent events.

Wesgħat il-Kolleġġjata

Task 6.1 – Measuring Heights using Trigonometry

A **stone course** is a layer of stone **running horizontally** in a wall. It can also be defined as a continuous row of any masonry unit such as bricks, stone, etc.

(Wikipedia Foundation, 2022, n.d.)



a. Use your tape measure to find the **height** of one stone course ('filata'). Record your answer correct to the **nearest cm**.

Answer:

28 cm

b. Count the number of stone courses to **estimate** the height AB. Give your answer correct to the **nearest cm**.

Number of stone courses = 32

AB ≈ 32 x 28

Answer:

896 cm

c. Select **one student** from your group to measure the **angle of elevation** using the clinometer as follows:

- Student needs to find a position **in front of** the vertical column AB and far enough away to see edge at point A through the clinometer.
- Record the **angle of elevation**, **θ**, correct to the **nearest degree**.

Answer:

Example:
52 °

d. The student needs **to remain** in the position where the angle of elevation is measured. The rest of the group needs to use the tape measure to find:

- the **horizontal distance** between the student and the vertical column AB. Give your answer correct to the **nearest cm**.

  Note: The ground level gives you the most accurate horizontal measurement.

Answer:

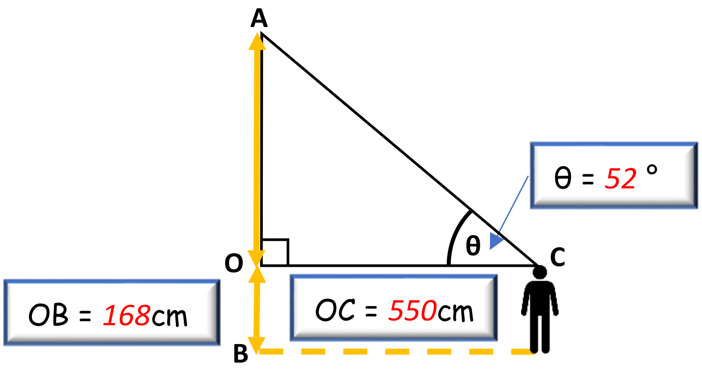
Example:
550 cm

- the **height** of the student that is using the clinometer. Give your answer correct to the **nearest cm**.

Answer:

Example: *168* cm

e. Fill in the empty boxes in the following diagram:



f. Use trigonometry to find the **height AO**, correct to the **nearest cm**.

$\tan \vartheta = \frac{Opp}{Adj}$

$\tan 52 = \frac{AO}{OC}$

AO = (tan 52) x 550

= 703.97 cm

Answer:

Example: *704* cm

g. Now, calculate the **total height AB**, correct to the **nearest cm**.

Total Height AB = AO + OB

= 704 + 168

= 872 cm

Answer:

872 cm

h. Use the result of Task 7.1b and Task 7.1g to fill in the empty boxes below. What is the **% error** between the estimated height and the measured height? Give your answer correct to **2 decimal places**.

$$\% \text{ Error} = \frac{\textbf{Estimated Height} - \textbf{Measured Height}}{\textbf{Measured Height}} = \frac{896-872}{872} \times 100 = 2.75\%$$

Answer:

2.75 %

Task 6.2 – Probability

St. Lawrence Church was founded in 1091 but the one standing now, designed by Lorenzo Gafà, was built between 1681 and 1697. The church is dedicated to St. Lawrence. Among the treasures found in it is a painting of the famous Italian artist Mattia Preti, representing The Martyrdom of St. Lawrence.

The Vittoriosani show a lot of devotion to St. Lawrence. The name Lawrence (or variations of it such as Lorenzo, Lora, Lorenza) is very common in Birgu.

At the side of St. Lawrence's church, you can find a plaque with **names** of World War II Vittoriosani victims, as shown.

a. **Each student** in the group needs to take a photo of the plaque. Make sure that the names on your photo are readable.

b. If a name is selected at random:

(i) What is the probability that the name is of a male?

P (male):

Answer:

$\frac{43}{65}$

(ii) What is the probability that the name is of a female?

P (female):

Answer:

$\frac{22}{65}$

(iii) What is the probability that the name is of a male **and** his name is dedicated to St. Lawrence?

P (male ∩ his name is dedicated to St. Lawrence):

Answer:

$\frac{7}{43}$

(iv) What is the probability that the name is of a male **and** his name is not dedicated to St. Lawrence?

P (male ∩ his name is not dedicated to St. Lawrence):

Answer:

$\frac{36}{43}$

(v) What is the probability that the name is of a female **and** her name is dedicated to St. Lawrence?

P (female ∩ her name is dedicated to St. Lawrence):

Answer:

$\frac{5}{22}$

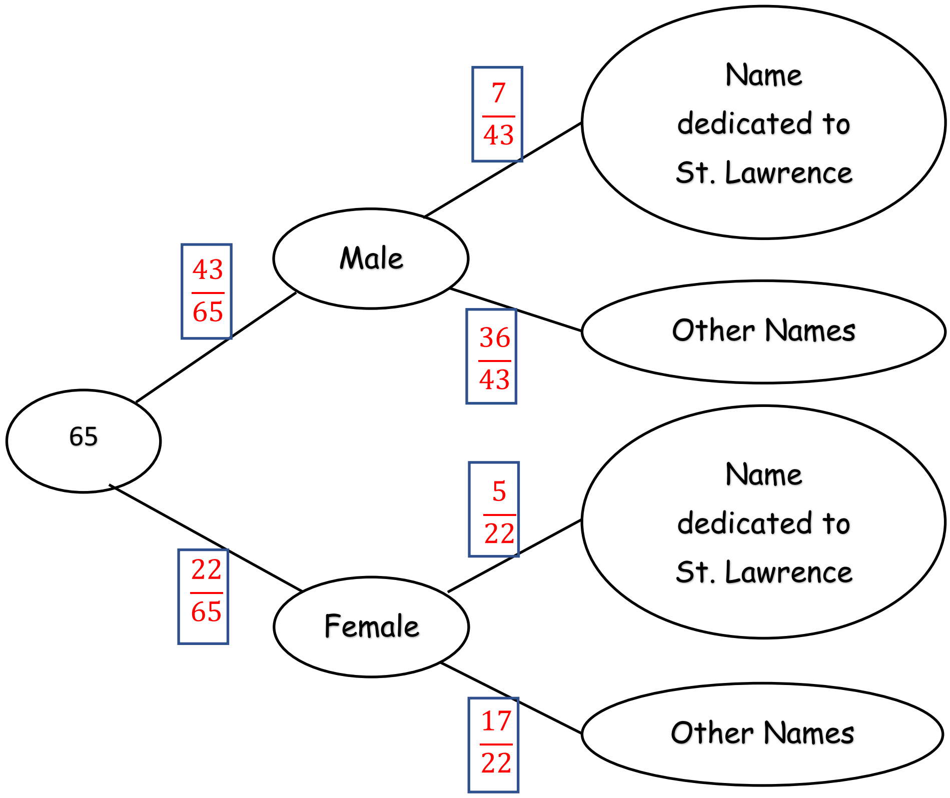
(vi) What is the probability that the name is of a female **and** her name is not dedicated to St. Lawrence,

P (female ∩ her name is not dedicated to St. Lawrence):

Answer:

$\frac{17}{22}$

c. Use the answers you obtained in Task 7.2c to complete the following probability tree:



d. Assume that this data is a good indication of the names of the people now living in Birgu. What is the **% probability** that if you meet someone from Birgu, his or her name is dedicated to St. Lawrence? Give your answer correct to the **nearest ten.**

P (name is dedicated to St. Lawrence) $= (\frac{43}{65} \times \frac{7}{43}) + (\frac{22}{65} \times \frac{5}{22})$

Or $= (\frac{7+5}{65})$

$= 0.185$

Answer: 20 %

e. Use your answer in Task 7.2d to complete the following statements:

- out of 100, 20 people are named Lawrence/Lorenz/Lorenza/Lora

- out of 10, 2 people are named Lawrence/Lorenz/Lorenza/Lora

Task 6.3 – Place Value

Place value is the basis of our number system. Each digit of a number has a place value. In the decimal system, the place values are all powers of 10, including thousands, hundreds, tens, and units.

In every number, each place to the left is 10 times greater and each place to the right is 10 times smaller.

(BBC, 2023b)

At the side of St. Lawrence Church, you can find the monumental bust of Mons. Dun Ġużepp Caruana, as shown.

a. Fill in the following spaces:

Mons. Dun Ġużepp Caruana was born in the year *1933* and passed away in the year *2016*.

b. How old was Mons. Dun Ġużepp Caruana when he died?

2016 – 1933 = 83

Answer:

83 years

c. Using the digits in the year that Mons. Dun Ġużepp Caruana was born:

(i) What is the **smallest** possible **4-digit odd number**?

Answer:

1339

(ii) What is **largest** possible **4-digit odd number**?

Answer:

9331

(iii) Is it possible to have a **4-digit even number**? Why do you think this is so?

Answer:

No, because all 4 digits are odd numbers.

In order to have an even number, the unit value needs to be an even number.

d. Now, consider the year that Mons. Dun Ġużepp Caruana passed away:

(iv) Write this year as a product of its **prime factors**.

2016 ÷ 2 = 1008

1008 ÷ 2 = 504

504 ÷ 2 = 252

252 ÷ 2 = 126

126 ÷ 2 = 63

63 ÷ 3 = 21

21 ÷ 3 = 7

7 ÷ 7 = 1

Answer:

$2^5 \times 3^2 \times 7$

(v) What is the smallest possible **4-digit multiple of 2**?

Answer:

0126

Task 6.4 – Symmetry

The Oratory of St. Joseph

The Oratory of St. Joseph was built in 1832. It is the place where one can find the hat and sword that La Valette used during the Great Siege. He had left them in the chapel as a thanksgiving to Our Lady of Damascus for the victory over the Turks in the Great Siege.

This chapel was one of the early churches in Birgu that was used by the Greek community who had come from Rhodes with the Knights.

Reflective Symmetry is when a shape is folded (**reflected**) along the line of symmetry (mirror line), and half the shape would fit exactly over the other half. Some shapes can have more than one line of symmetry.

Rotational Symmetry is when a shape is **rotated** about a fixed point to a new position and still looks the same as it did before the rotation. The number of positions in which a shape can be rotated and still looks the same as it did before the rotation is called the **order of rotational symmetry**.

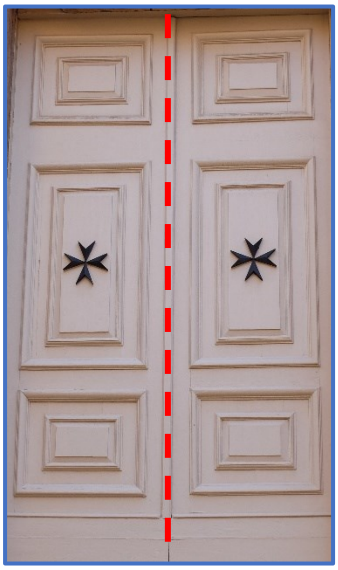
Thus, a shape can have no lines of symmetry, but the order of rotational symmetry is always **at least 1** (as the shape will always look the same as it did before being rotated a full turn).

(Bostock et al., 1992)

The following are photos of l-'Oratorju' main door. For each photo you need to:

- **Draw** and **write** the number of lines of symmetry
- **Write** the order of rotational symmetry

a.



i) Draw the line/s of symmetry.

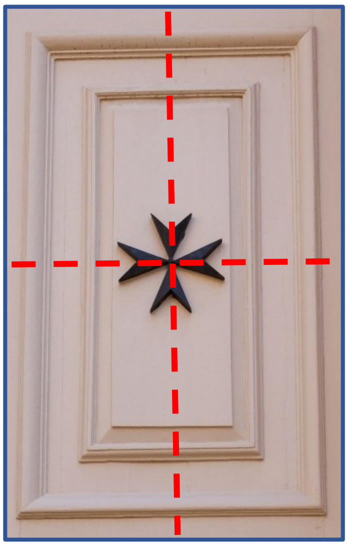
ii) The number of line/s of symmetry is equal to:

1

iii) The order of rotational symmetry is equal to:

1

b.



i) Draw the line/s of symmetry.

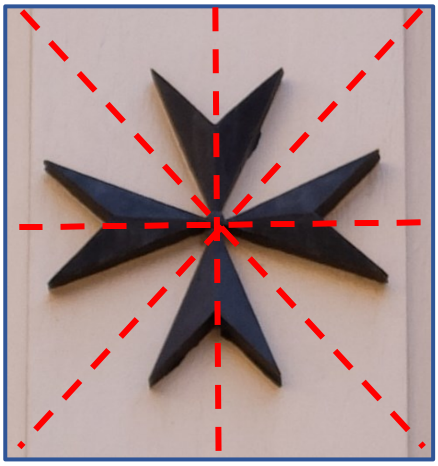
ii) The number of line/s of symmetry is equal to:

2

iii) The order of rotational symmetry is equal to:

2

c.



i) Draw the line/s of symmetry.

ii) The number of line/s of symmetry is equal to:

4

iii) The order of rotational symmetry is equal to:

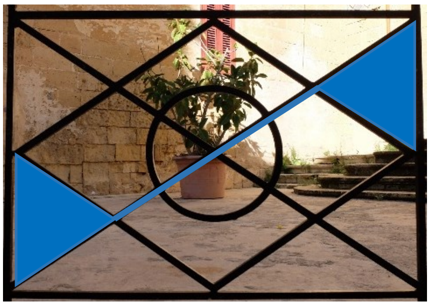
4

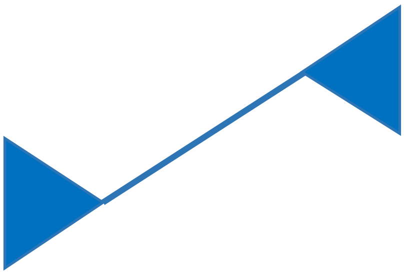
d. Is the order of rotational symmetry **always** equal to the number of lines of symmetry? Justify your answer by giving an example of a quadrilateral.
(Hint: Is this true in the case of a rhombus?)

Answer:

No, because a parallelogram or a rhombus has 0 lines of symmetry but rotational symmetry of order 1.

e. Consider the railings next to the stairs. Part of these railings form the shape coloured in blue below:





i) How many **lines of symmetry** does the blue-coloured shape have?

Answer:

0

ii) What is the **order of rotational symmetry** of this shape?

Answer:

2

The order of rotational symmetry of a shape is **always** equal to or greater than the number of lines of symmetry.

Trail Sequence for Group A

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Activity No.   Path   Stairs   Meeting Point

Trail Sequence for Group B

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Activity No. — Path — Stairs — Meeting Point

Trail Sequence for Group C

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Activity No. — Path — Stairs — Meeting Point

Trail Sequence for Group D

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Activity No. Path Stairs Meeting Point

Trail Sequence for Group E

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Activity No. Path Stairs Meeting Point

Trail Sequence for Group F

Planning Authority – www.pa.org.mt, mappingshop@pa.org.mt

Activity No. | Path | Stairs | Meeting Point

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Attachment B

Student's Resource Pack

BIRGU MATHS TRAIL

YEAR 10 TRACK 3

Student's Resource Pack

Abstract

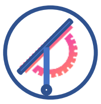
A mathematics trail is an organized route which gives students a first-hand experience to apply mathematical knowledge in a way that is real and hands on.

Celine Vassallo Grant

August 2023

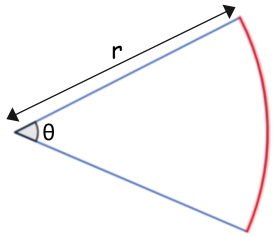
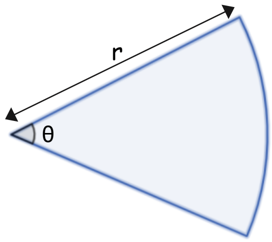
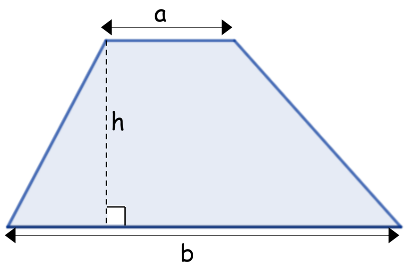
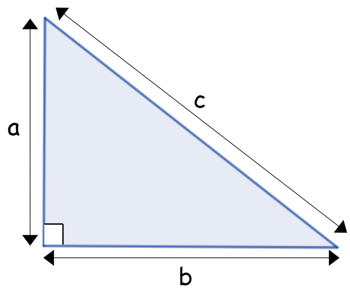
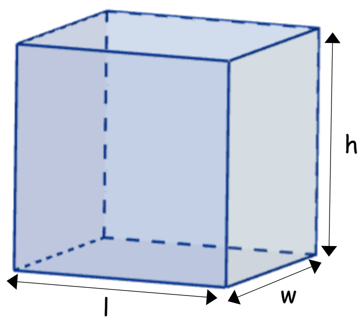
Table of Contents

List of Symbols

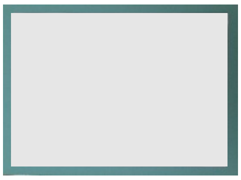
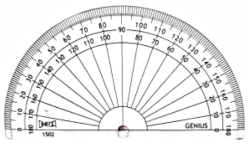
Symbol	Meaning
	Tape Measure
	Sewing Tape Measure
	Calculator
	Photo Camera
	Protractor
	Clinometer
	Stopwatch
	Navigating Compass

Symbol	Meaning
	Think and Explain
	Time Calculations
	'Aha' Moment
	Information Guide
	Formulae List
	Work out/ Write
	Answer Box
	Homework

List of Formulae

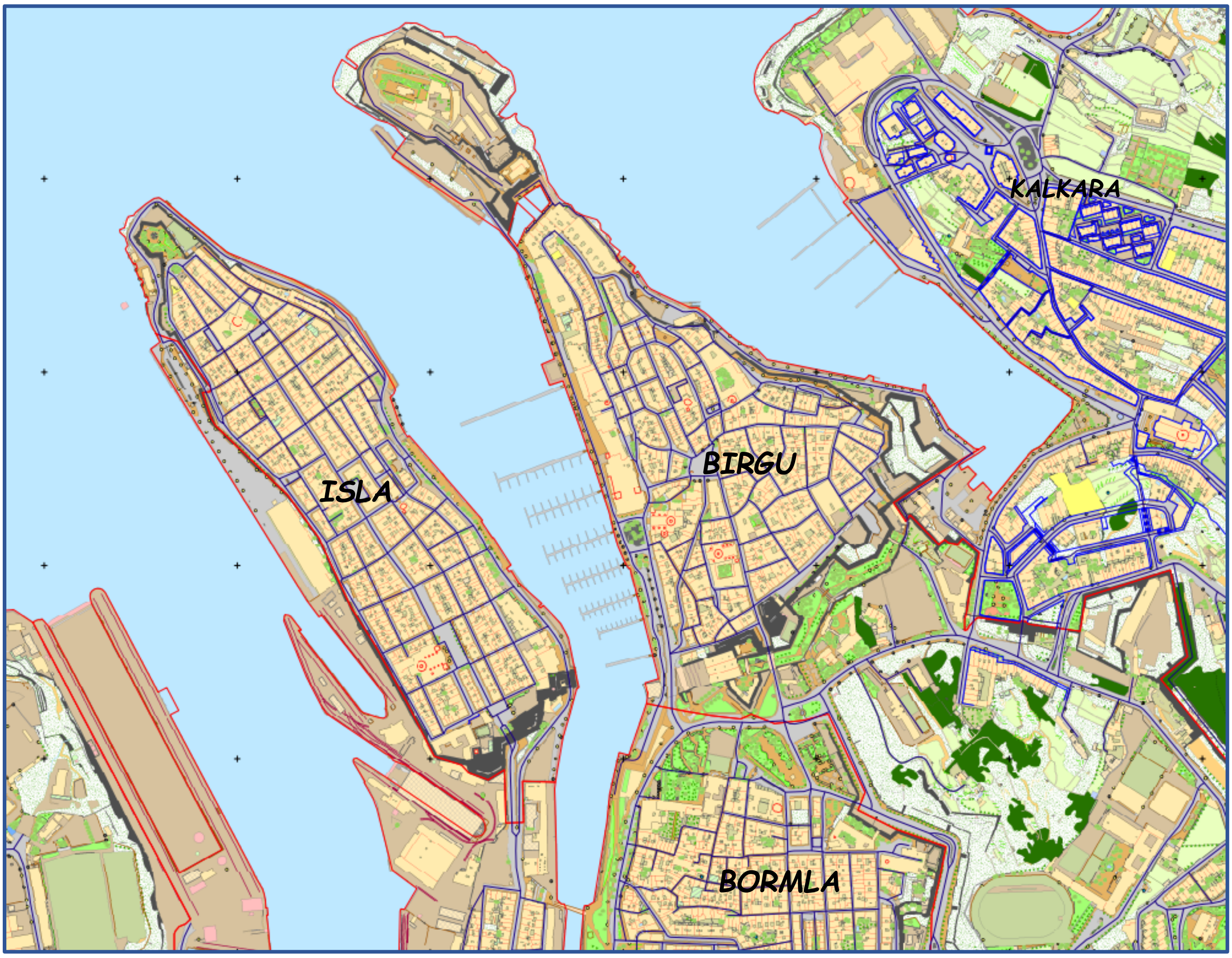
Length of arc	$\frac{\theta}{360°} \times 2\pi r$	
Area of sector	$\frac{\theta}{360°} \times \pi r^2$	
Area of trapezium	$\frac{1}{2}(a+b)h$	
Pythagoras' Theorem	$c^2 = a^2 + b^2$	
Volume of cube /cuboid	$V = l \times w \times h$	

List of Resources

			
Clip Board	Pencil and Biro	A4 Blank Paper	Tracing Paper
			
Ruler (30cm)	White Chalk	Protractor	Clinometer
			
Tape Measure (5m or more)	Sewing Tape Measure (1m or more)	Disc Tape Measure (20m or more)	
			
Calculator *	Photo Camera *	Navigating Compass *	Stopwatch *

* Resource can be an app on the mobile phone

General Information on Birgu



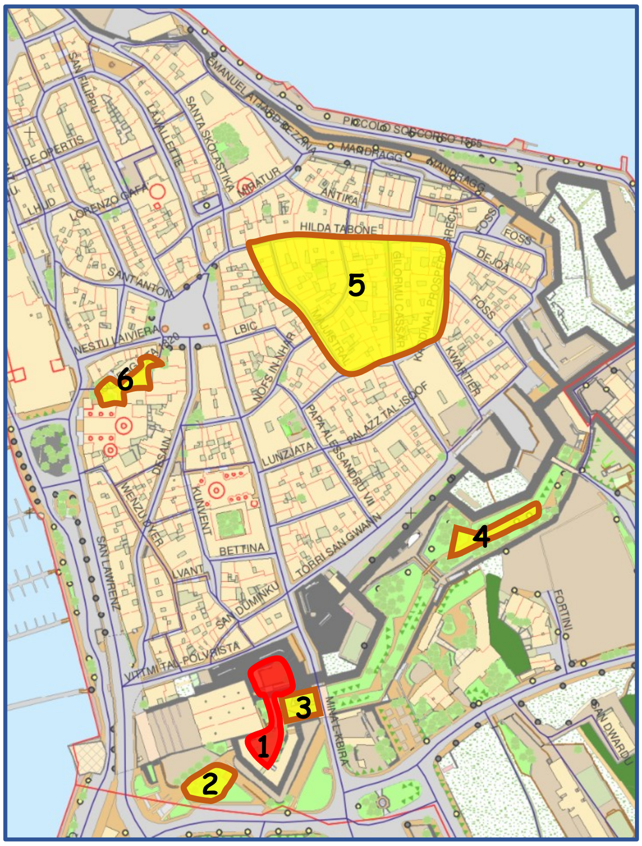
Birgu, also known as Vittoriosa, is one of the Three Cities, the others being Bormla (Cospicua) and Isla (Senglea). They are all located along the shores of the Grand Harbour. Birgu is built on a peninsula, between Dockyard Creek and Kalkara Creek. It has always been a maritime settlement. The people of Birgu are called Vittoriosani and its population stands at around 2,300.

When the Knights of St. John came Malta in 1530, they first settled in Birgu and remained there for about forty years before building and eventually moving to Valletta in 1571. During the Great Siege of 1565, Birgu played a very important role and was given the other name of Citta' Vittoriosa after the victory of the Knights over the invading Turkish armada.

Although a number of buildings and bastions built by the Knights were destroyed during World War II, quite a number of buildings and fortifications have survived as we will see along this trail.

Station 1 – Couvre Porte Counterguard

Numerical Calculations; Shapes, Space and Measures (Circles)



Learning Objectives

- I can measure the diameter of a semi-circular arch.
- I can calculate the length of arc and area of sector of a circle.
- I can calculate the area of compound shapes that include sectors of circles.
- I can convert Roman numerals to the decimal number system.
- I can identify different types of circular arches.
- I can deduce that the ratio of the circumference to the diameter of a circle is constant.

The War Museum and Couvre Porte Gate

Task 1.1 - Roman Numerals

The Advanced Gate

After the Great Siege, the Knights continued to fortify Birgu with curtains (walls), bastions and gates.

To enter the city, they built a complex of three gates, the first of which is called the Advanced Gate (also known as Gate D'Aragon). The date on this gate refers to the year when works on it were completed.

This gate had to be defended by the Knights of Aragon (Spanish) in case of an attack.

Roman numerals represent a number system that uses **7 letters** to express numbers:

I, **V**, **X**, **L**, **C**, **D** and **M**

These letters correspond to integer values as follows:

Roman	I	V	X	L	C	D	M
Decimal	1	5	10	50	100	500	1000

Two of the rules **to convert Roman numerals to our system of integers** include:

Rule 1 – "When a Roman numeral is placed **after** another Roman numeral of **greater value**, the result is the **sum** of the numerals".

Example: VII = V + I + I
= 5 + 1 + 1
= 7

Rule 2 – "When a Roman numeral is placed **before** another Roman numeral of **greater value**, the result is the **difference** between the numerals".

Example: IV = V - I
= 5 - 1
= 4

a. Fill in the empty boxes to complete the following table:

Roman	I	II	III	IV	V	VI	VII	VIII	IX	X
Decimal	1			4		6				

b. Write down the **Roman numeral** shown on this gate.

Answer:

c. Now **convert** this number to our system of integers.

Answer:

d. Which **year** are we in?

Answer:

e. Now **convert** the year we are in, to Roman numerals.

Answer:

f. Calculate **how long** the Advanced Gate has been standing.

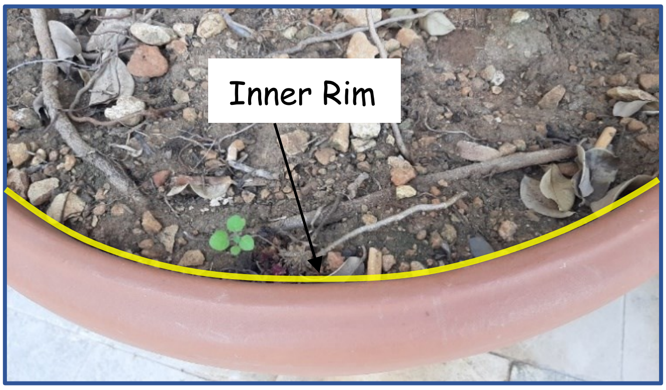
Answer:

years

Task 1.2 - Finding Pi (π) using Round-shaped Pots

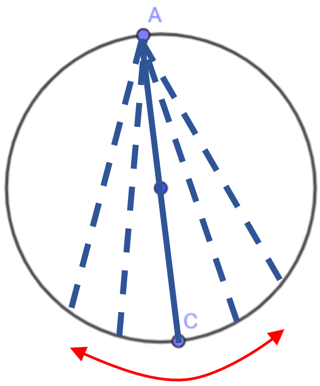
There are several planters in the area in front of the War Museum:

The War Museum



To measure the **diameter** of the opening at the top of a plant pot:

- measure across from one side to the other of the inner rim.
- hold the tape measure on one side of the pot (e.g. A) and move the other side back and forth until you measure the **longest** point (e.g. AC).



Select **three** round plant pots (large, middle-sized, and small). In order to do this, choose planters whose plant is not exactly in the centre.

For **each** planter:

a. Use the sewing tape measure to go round the **inner rim** in order to find the **circumference** of the opening at the **top**, correct to the **nearest cm**. Record your measurement in the following table.

b. Use the tape measure to find the **diameter** of the opening at the **top** (**inner rim**), correct to the **nearest cm**. Record this in the following table.

	Circumference (C) cm	Diameter (D) cm
Planter 1 (large)		
Planter 2 (medium)		
Planter 3 (small)		

c. Use the calculator to work out the **ratio** of the **circumference (C) to its diameter (D)**, i.e. $\frac{C}{D}$. Give your answers correct to:

(i) 2 decimal places (d.p.)

(ii) 1 significant figure (s. f.)

	$\frac{\mathbf{C}\text{ (cm)}}{\mathbf{D}\text{ (cm)}}$	
	2 d.p.	1 s.f.
Planter 1 (large)		
Planter 2 (medium)		
Planter 3 (small)		

d. The **ratio** of the circumference to the diameter of a circle ($\frac{C}{D}$), irrespective of the circle's size, is **constant**. What is this constant **approximately** equal to?

Tick (✓) the correct answer:

- ☐ 1
- ☐ 3
- ☐ 5

e. What is this **constant** called?

Answer:

f. The value of this constant is **3.1415926536** correct to 10 decimal places. Write down the value of this constant correct to **3 decimal places**.

Answer:

g. **Compare** the results of Task 1.2c(i) to the answer of Task 1.2f. Which answer in Task 1.2c(i) is the **most accurate**? Why do you **think** this is so?

Answer:

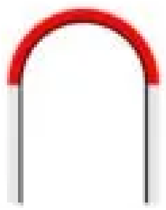
Task 1.3 – Arches and Area of a Composite Shape

Couvre Porte Gate

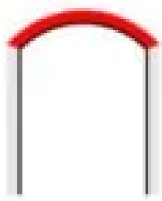
The second gate in this unique entrance to Birgu is Couvre Porte Gate (or the Covered Gate). It is located at the end of a bridge that crosses the ditch.

The aim of the Advanced Gate and Couvre Porte Gate was to shield Birgu's main entry point (known as the Main Gate or Gate of Provence) from the attacking forces.

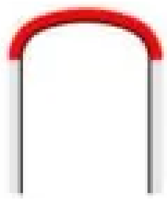
There are various types of arches, and their names typically suggest how the top part of the arch looks like. Some examples include:



Semi-circular arch



Segmental arch



Basket-handle arch

a. Identify the **two types of arches** that can be found on the façade of Couvre Porte Gate. Tick (✓) the correct answers.

- ☐ Semi-circular
- ☐ Segmental
- ☐ Basket-handle

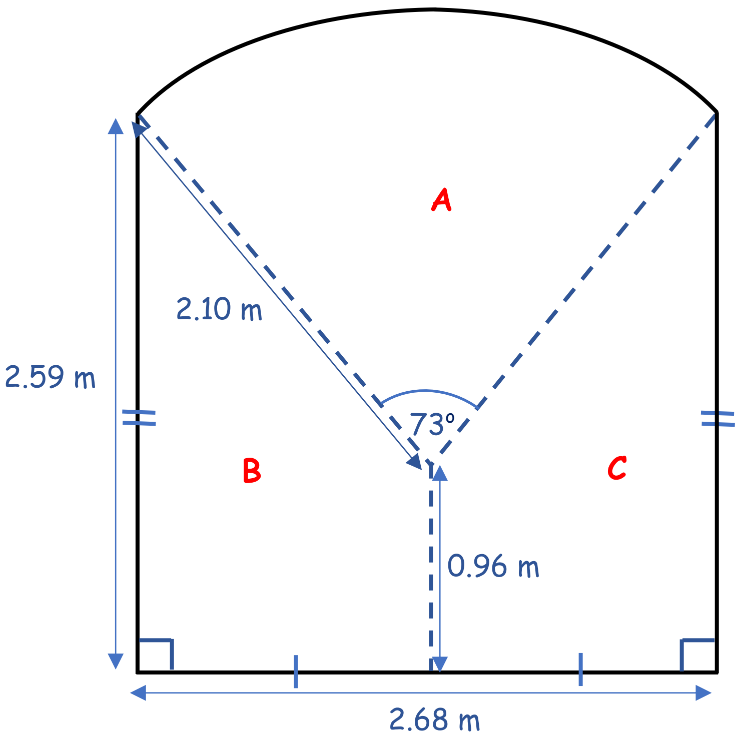
b. As you pass through the gate, you will find the monumental bust of Nestu Laiviera placed under an arch. What **type of arch** is this? Tick (✓) the correct answer.

- ☐ Semi-circular
- ☐ Segmental
- ☐ Basket-handle

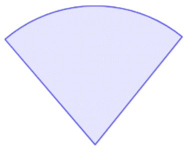
Consider the **segmental arch** at Couvre Porte Gate (marked in yellow) as a composite shape for which you need to calculate the area.

To calculate the area of a **composite shape** you need to divide the composite shape into shapes you can calculate the area of.

One way to divide this composite shape is as follows:



c. What **shape** is A? Tick (✓) the correct answer.



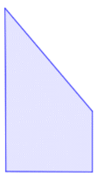
- ☐ sector of a circle
- ☐ semi-circle
- ☐ segment of a circle

d. Use your calculator to work out the **area of shape A**, correct to **1 decimal place**.

Answer:

m^2

e. What **shape** is B? Tick (✓) the correct answer.



- ☐ parallelogram
- ☐ trapezium
- ☐ rhombus

f. Use your calculator to work out the **area of shape B**, correct to **1 decimal place**.

Answer:

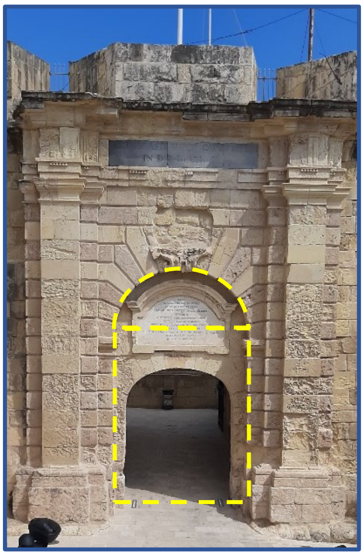
m^2

g. Given that shape B and shape C are equal, calculate the **total area** of the segmental arch, correct to **1 decimal place**.

Answer:

m^2

Task 1.4 - Sectors



The Gate of Provence

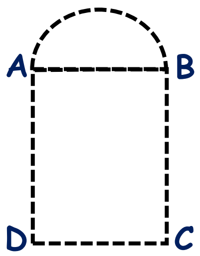
The Main Gate, also called the Gate of Provence, is the third gate that actually leads one into the city.

The Knights of Provence (French) were responsible for its defence.

Works on the three gates were recommended by the French military engineering mission led by L.F. de Tigne' and C.F. de Mondion in an effort to improve the defence system. These works were completed in the times of Grand Master Manoel de Vilhena.

The **semi-circular arch** (marked in yellow in the diagram above) forms a **semi-circle** on top of a **rectangle**.

a. If you are told that ABCD is a **rectangle** (see diagram below), is AB equal to CD? Why? Justify your answer using the properties of a rectangle.



Answer:

b. Now, use your tape measure to find the **diameter (d)** of this semi-circle. Give your answer correct to the **nearest cm**.

Answer:

cm

c. So, what is the **radius (r)** of this semicircle? Give your answer correct to **1 decimal place**.

Answer:

cm

The semi-circle can be divided into **sectors** as shown in blue below. Assuming that all sectors are **equal**:

d. What is the **total number** of sectors?

Answer:

sectors

e. Given that the angle of a semi-circle is 180°, calculate the **angle (θ)** of each sector. Give your answer correct to **2 decimal places**.

Answer:

°

f. Use your calculator to find the **length of arc** of each sector. Give your answer correct to the **nearest cm**.

Answer:

cm

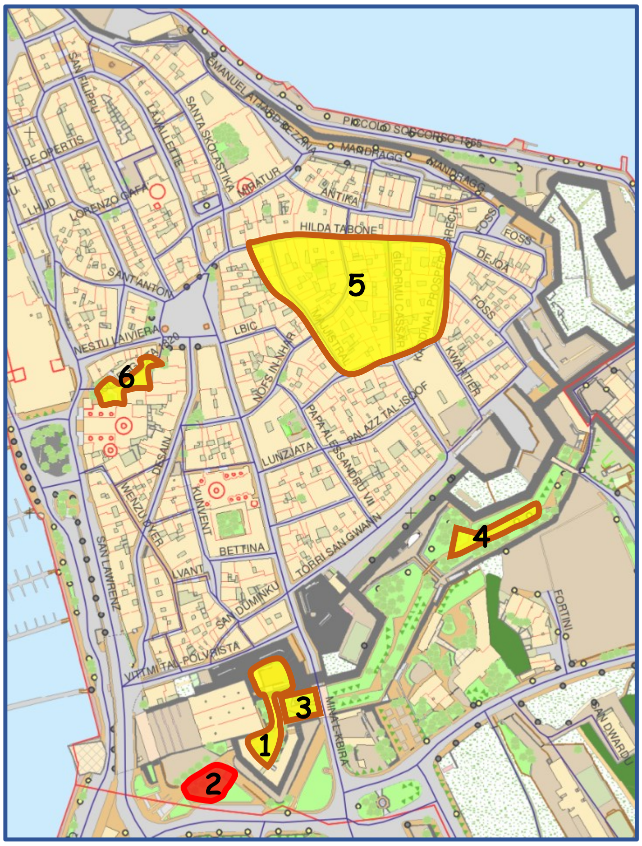
g. Use your calculator to find the **area of each sector**. Give your answer correct to the **nearest cm^2**.

Answer:

cm^2

Station 2 – Couvre Porte Belvedere

Numerical Calculations (Proportions; Money) and Measures (Time)



Learning Objectives

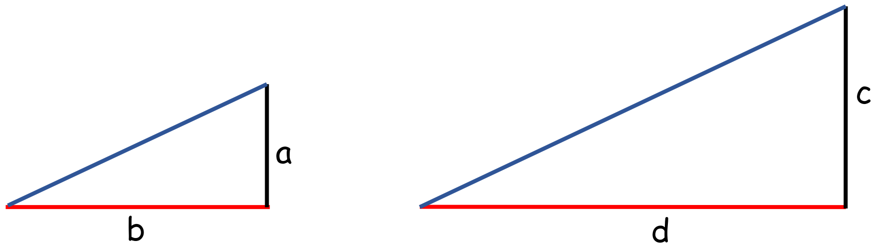
- I can work through situations that involve direct proportion.
- I can measure tall items using shadows.
- I can recognize and extend number sequences.
- I can use algebraic expressions to describe the nth term of a linear sequence.
- I can read and use a timetable and a timeline.
- I can work through simple situations involving personal finance.

Couvre Porte Belvedere

Task 2.1 – Measuring Heights using Shadows

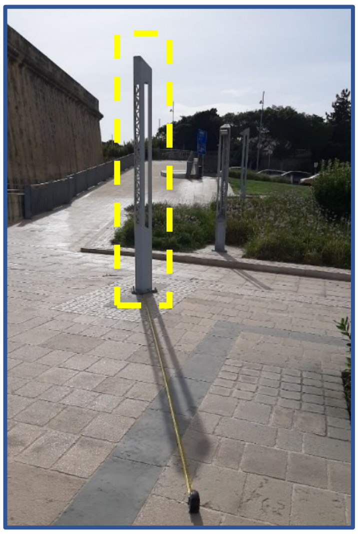
At a given time of day, objects create **shadows** that are **proportional** to one another.

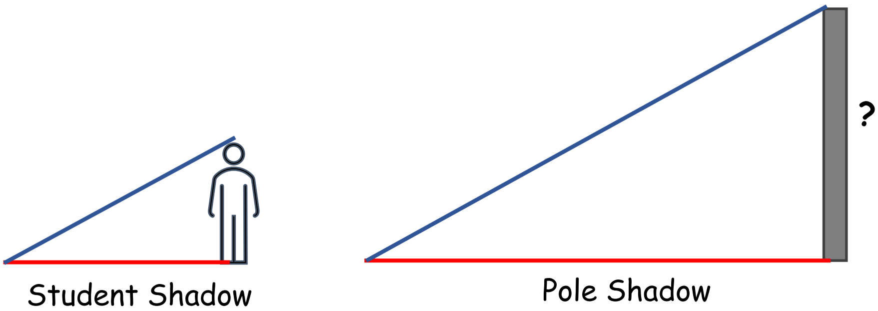
Thus, when measurements are taken **at the same time of the day**, the **ratio** of the **height of an object to the length of its shadow** is the **same** for all objects.



$$\frac{a}{b} = \frac{c}{d}$$

You can use this fact to calculate the height of the light poles (marked in yellow below) at Couvre Porte Belvedere.





$$\frac{\text{Student Height}}{\text{Student Shadow}} = \frac{\text{Pole Height}}{\text{Pole Shadow}}$$

a. Select **one student** from your group. Find a **position** where the student's **shadow** can be **clearly** seen. The student needs to remain **still** in this position whilst measurements are being taken.

The rest of the group needs to use the tape measure to find to the **nearest cm** the:

- **student's body height**

Answer:

cm

- **length of student's shadow**

Answer:

cm

b. Choose **one of the light poles** for which the shadow can be **clearly** seen. Use the tape measure to find the **length of the pole's shadow**, correct to the **nearest cm**.

Answer:

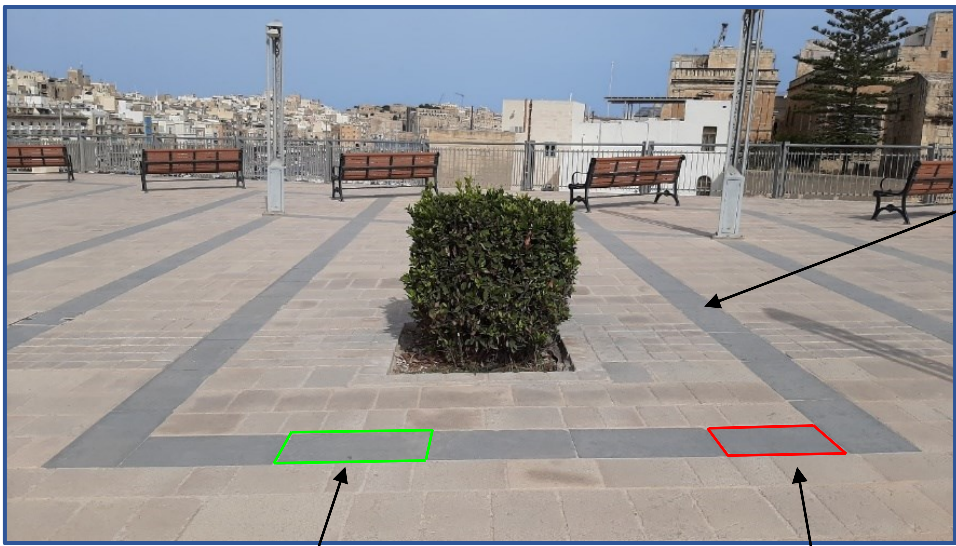
cm

c. Now calculate the **pole's height** using the formula below and give your answer correct to the **nearest cm**.

Answer:

cm

Task 2.2 – Sequences



Grey Tiles

Whole tile

Not a whole tile because part of it has been cut off.

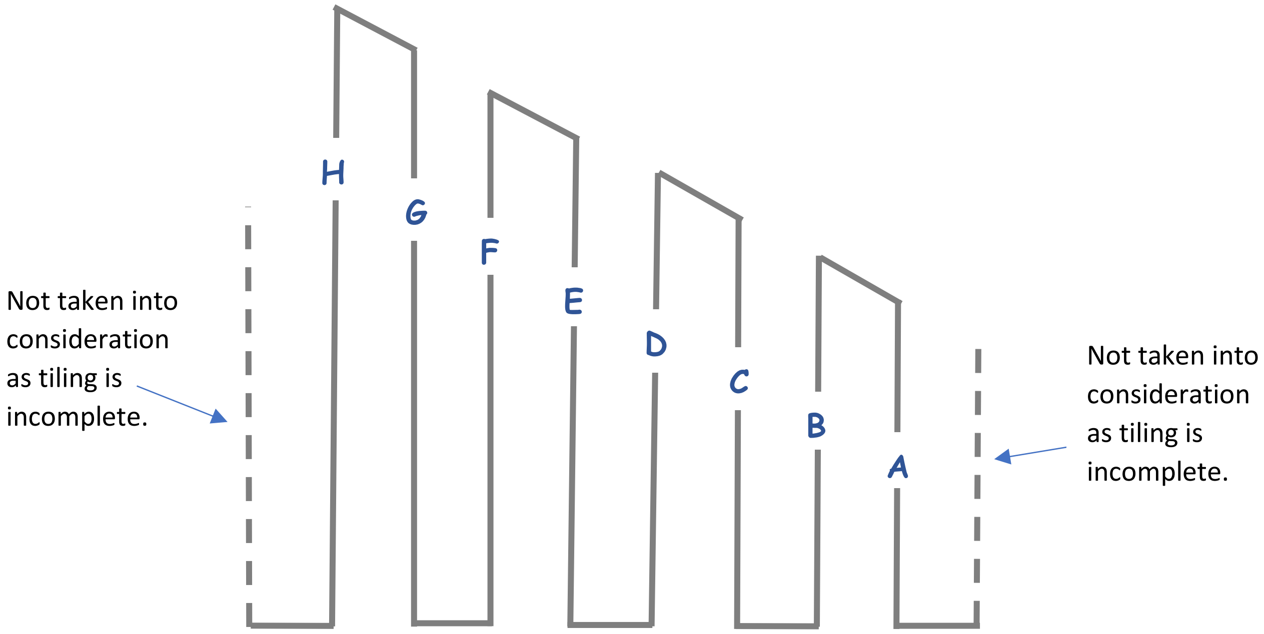
a. Observe the **grey tiles**. Focus on a tile that is a **whole one** and not on a tile that has a part cut off. Use your tape measure to find the **length and width of the tile**. Tick (✓) the correct answer.

- ☐ 60 x 60cm
- ☐ 30 x 60cm
- ☐ 60 x 90cm

Part of the grey tiling follows a particular **sequence**, highlighted in yellow below:

Aerial View of Couvre Porte Belvedere

This sequence is represented in the following diagram.



b. (i) Use your tape measure to find the **lengths of the vertical strips** A, B, C, D, E, F, G and H, correct to the **nearest cm**. Write your answers in the empty boxes below. Some of them have been already measured.

(ii) Divide these lengths by 60cm to find the number of **whole tiles.**

For example, 1789 ÷ 60 = **29**.817.

This means that there are **29 whole tiles** (ignore the decimal part).

Write your answers in the empty boxes below. Some of them have been already worked out.

Vertical Strip	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Length (cm)	900					1538		1789
No of whole tiles	15		19	21		25		29

c. Considering the number of whole tiles for each vertical strip, **is this a linear sequence**? Why do you think so?

Answer:

d. The following table shows the number of whole tiles per vertical strip according to their position 'n' in the sequence. For instance, vertical strip A has 15 whole tiles. Complete the following table.

Vertical Strip	A	B	C	D	E	F	G	H
n	1	2	3					8
nth term	15		19	21		25		29

e. Find the **nth term** of this sequence **using an algebraic expression**. You can use:

either:

$\underline{+}an\underline{+}b$, where

a is the difference

b is the adjustment

or:

$a + (n-1)d$, where

a is the 1st term

d is the common difference

Answer:

Task 2.3 – Time and Money

Dockyard Creek

Dockyard Creek (originally called Galley Creek) is a very sheltered area in the Grand Harbour. In the Middle Ages, the Spanish and Sicilians used it for their commercial ships. The Knights used it to berth and repair their galleys. The British used it for Royal Navy ships. It used to be the location for Dock No.1. Nowadays it is the Vittoriosa Yacht Marina and also has a small ferry quay.

FERRY SCHEDULE

VALLETTA - 3 CITIES - VALLETTA

Winter 01/11 - 31/05

Cospicua Dept.	Valletta Dept.
06.30	06.45
07.00	07.15
07.30	07.45
08.00	08.15
08.30	08.45
09.00	09.15
09.30	09.45
10.00	10.15
10.30	10.45
11.00	11.15
11.30	11.45
12.00	12.15
12.30	12.45
13.00	13.15
13.30	13.45
14.00	14.15
14.30	14.45
15.00	15.15
15.30	15.45
16.00	16.15
16.30	16.45
17.00	17.15
17.30	17.45
18.00	18.15
18.30	18.45
19.00	19.15

DAILY SCHEDULE

SUNDAYS & PUBLIC HOLIDAYS SCHEDULE

MARSAMXETTO STEAMFERRY SERVICES LTD

VALLETTA FERRY SERVICES

Adults:
Day - Single: € 1.50
Day - Return: € 2.80

Children:
Single: € 0.50
Return: € 0.90

60+ ID Card Holders & Blue Badge Holders:
Single: € 0.50
Return: € 0.90

Weekly Pass:
Valid for seven (7) consecutive days
Unrestricted use: € 10.00

Frequent Traveller:
3 Months, 6 Months & Yearly tickets also available on request.

Night Service Commences @ 19.45
Night - Single: € 1.75
Night - Return: € 3.30

Summer 01/06 - 31/10

Cospicua Dept.	Valletta Dept.
06.30	06.45
07.00	07.15
07.30	07.45
08.00	08.15
08.30	08.45
09.00	09.15
09.30	09.45
10.00	10.15
10.30	10.45
11.00	11.15
11.30	11.45
12.00	12.15
12.30	12.45
13.00	13.15
13.30	13.45
14.00	14.15
14.30	14.45
15.00	15.15
15.30	15.45
16.00	16.15
16.30	16.45
17.00	17.15
17.30	17.45
18.00	18.15
18.30	18.45
19.00	19.15
19.45	20.00
20.35	20.50
21.20	22.00
23.00	23.15
23.30	00.00

SUNDAYS & PUBLIC HOLIDAYS SCHEDULE

DAILY SCHEDULE

This is the **Valletta - Three Cities ferry schedule**.

Refer to this schedule for tasks in this section.

Lawrence lives in Birgu and travels to and from Valletta every day during **weekdays** for work. He starts work at **09:00**.

a. If Lawrence leaves his house at **08:10** and arrives near the Three Cities ferry terminal at **08:23**:

i. Calculate the **amount of time** he spends walking from his house to the ferry terminal.

Answer:

minutes

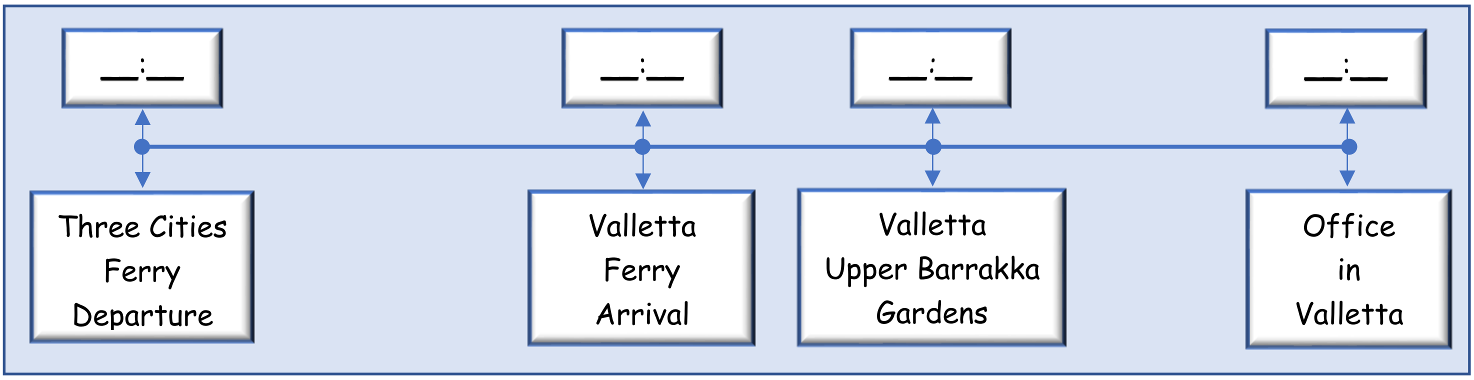
ii. How long does he have to **wait** to catch the **next ferry**?

Answer:

minutes

b. Lawrence manages to catch the **08:30** ferry. The duration of the ferry trip is **12 minutes**. If Lawrence takes **7 minutes** to get from the Valletta ferry terminal to the Upper Barrakka Gardens using the Barrakka Lift, and then has a **10-minute** walk to his workplace:

(i) Fill in the following **timeline** to represent Lawrence's journey from the Three Cities ferry terminal to his office in Valletta.



(ii) Does he manage to arrive to work **on time**?

Answer:

c. Lawrence finishes work at **17:00**. He takes the same amount of time to reach the Valletta ferry terminal from his office as it takes him in the morning.

(i) Which is the **next ferry** from Valletta to the Three Cities that he can catch?

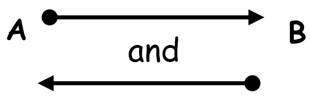
Answer:

(ii) Give the equivalent answer in **words**.

Answer: quarter to

Single ticket means **one-way** (either from A to B or from B to A)

Return ticket means **two-way** (from A to B and from B back to A)



d. What is the **cost** of the Adults Day-Return ticket for **one day**?

Answer:

€

e. What is the **weekly travelling cost** for Lawrence to go to work if he buys the Adults Day-Return tickets?

Answer:

€

f. How much will he **save** if he buys the Weekly Pass instead of the Adults Day-Return tickets?

Answer:

€

g. If for a particular week he has 2 days off work, which is the **cheapest option**?

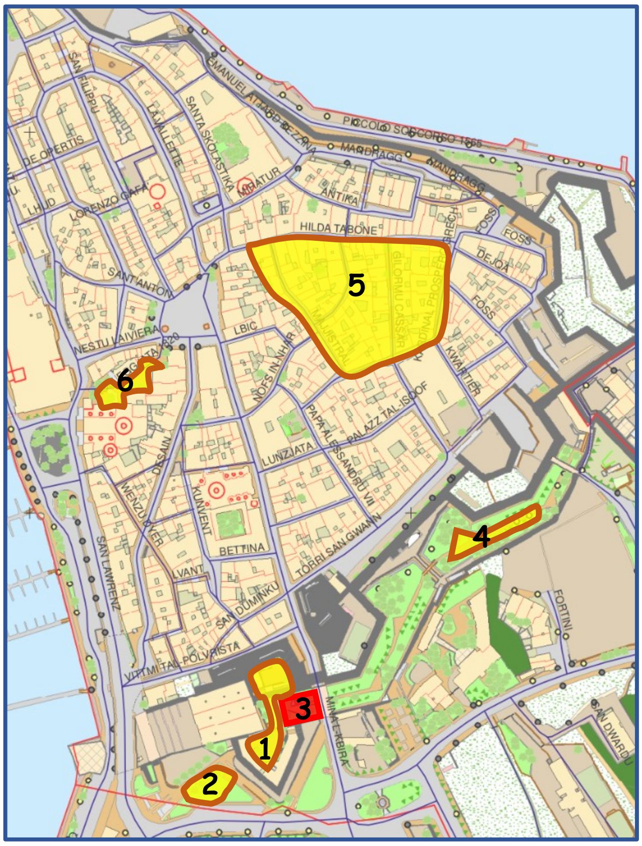
- ☐ Adults Day-Return ticket
- ☐ Weekly Pass

Tick (✓) the correct answer. Show your working to justify your answer.

Answer:

Station 3 – Birgu Ditch (Couvre Porte Area)

Euclidean Geometry (Triangles); Shapes, Space and Measures (Trigonometry)



Learning Objectives

- I know the properties of different types of triangles.
- I can use resources at hand to identify and distinguish between acute, obtuse right angles, and to distinguish between equal and non-equal lengths.
- I can measure angles of elevation.
- I can measure tall objects using the properties of an isosceles triangle.
- I can deduce that the Tangent ratio is the same for all triangles for a given angle.
- I can interpret and use Pythagoras' Theorem.

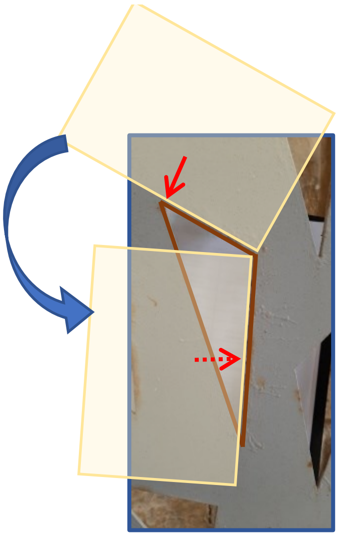
Birgu Ditch (Couvre Porte Area)

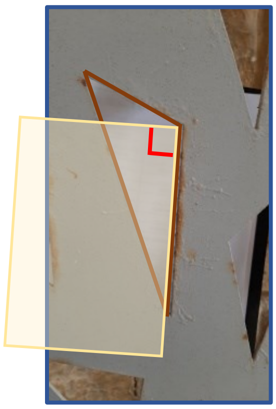
Task 3.1 – Types of Triangles

Before the Knights came to Malta, Birgu had a demarcation line on its land front, referred to as 'tagliata'. This 'tagliata' probably is the same location where, following the Great Siege, the Knights excavated a dry ditch to cut off the city from mainland and hence protect the city from an attack. Works on it by the Knights continued until the 1720s.

The ditch, nowadays referred to as 'il-Foss tal-Birgu', underwent restoration works that were completed in 2016.

To **trace** a triangle:
- clip a plain paper to the clipboard file (or any other hard backing).
- position the paper at the back of the iron panel.
- use a pencil and draw the chosen triangle by stencilling accurately inside it.





To find out **if two lengths are equal**, use one side of an A4 paper to mark the first length. Then compare this marking to the second length.

To find out **if an angle is equal to, greater than, or less than 90°**, use the corner of an A4 paper to estimate if the angle is =, > or < 90 °.

Choose **one of the iron panels** showing different types of triangles.

a. Use an A4 paper to identify the following triangles:

- Right-angled
- Isosceles
- Obtuse
- Acute
- Scalene

b. **Each member** in the group selects **one of these triangles**. Make sure to choose a triangle that fits on an A4 paper. Each member then needs to:

Take a **photo** of the selected triangle.

Trace the selected triangle on a blank A4 paper.

Attach the triangle traced in the step above onto the following page.

Use a **ruler** to measure the sides and a **protractor** to measure the angles of the selected triangle.

Mark these measurements on the A4 paper and **name** the triangle accordingly.

Selected triangle:

Task 3.2 – Measuring Heights using Isosceles Triangles

a. Select **one student** from your group. The student needs to measure the **angle of elevation** using the clinometer as follows:

- Student needs to stand and face the vertical strip highlighted in yellow.
- With the help of the group, the student needs to move slowly backwards till the angle of elevation from the ditch to the top of the bridge is **45°**.

b. The student **needs to remain** in the position where the angle of elevation is 45°. The rest of the group needs to use the tape measure to find:

(i) the **horizontal distance** between the student and the bridge (BC). Give your answer correct to the **nearest cm**.

Note: The ground level gives you the most accurate horizontal measurement.

Answer:

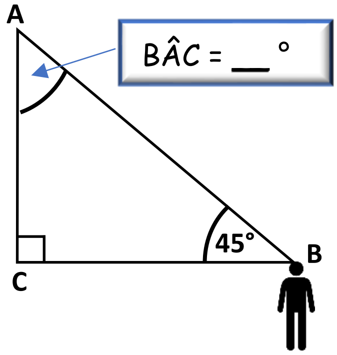
cm

(ii) the **height** of the student that is using the clinometer. Give your answer correct to the **nearest cm**.

Answer:

cm

c. Calculate the **missing angles** in the following diagram:



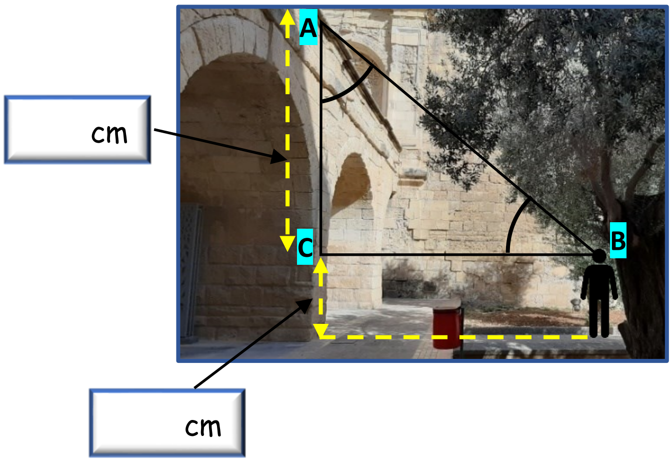
d. What is this **triangle** called? Tick (✓) the correct answer.

- ☐ Equilateral
- ☐ Isosceles
- ☐ Scalene

e. Using the **properties of this triangle**, what is height AC?

Answer: ______ cm

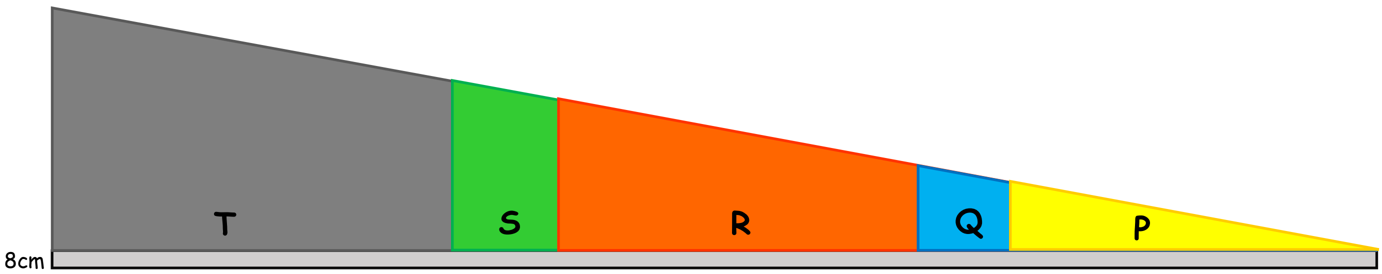
f. Fill in your measurements in the following diagram and calculate the **total height of the bridge**.



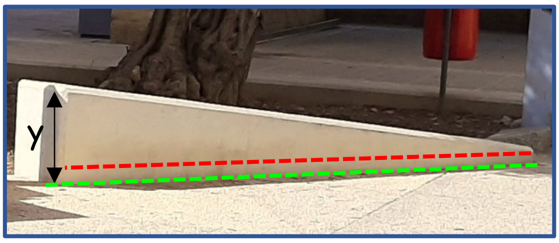
Answer: ______ cm

Task 3.3 – The Tangent Ratio

The structure shown above is made up of **nested triangles** of various sizes. These triangles can be depicted as follows:

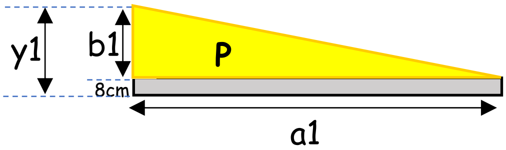


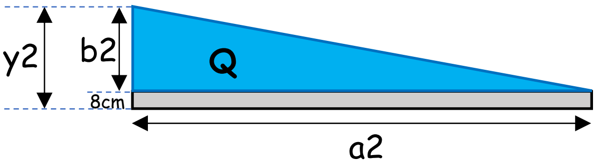
Measuring Guide

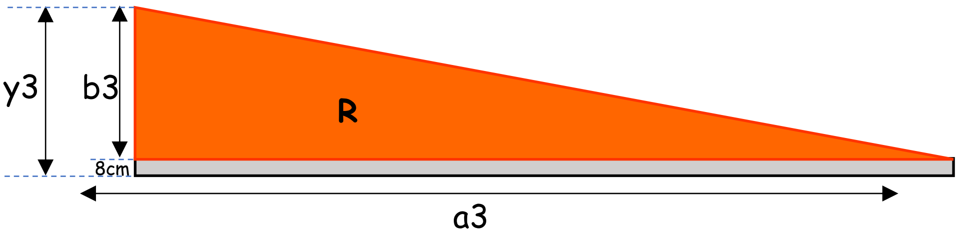


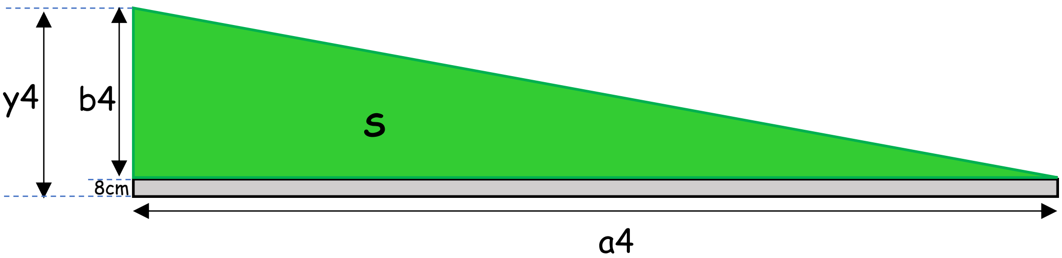
When measuring, use the **inner edges** of the structure.

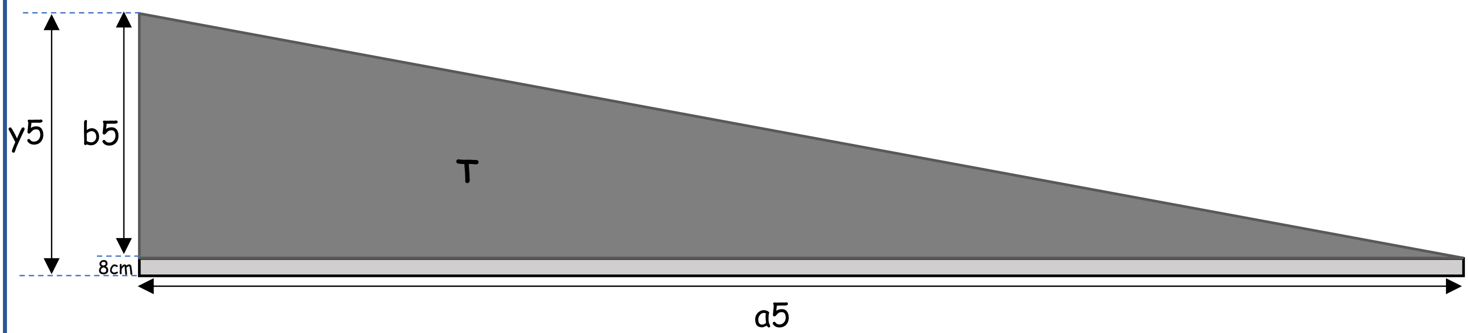
The triangle structure is 8cm above ground level, marked in red above. To measure heights (y), **measure till ground level**, i.e. up to the green line. You will then be asked to deduct the extra 8cm in the task itself.











a. Use the tape measure to find to the **nearest cm**: a1, a2, a3, a4, a5, y1, y2, y3 and y4, y5 and write them in the table below. Find the **'b' values by subtracting 8** from the 'y' values, for example b1 = y1 – 8. Some of the measurements have been filled in already.

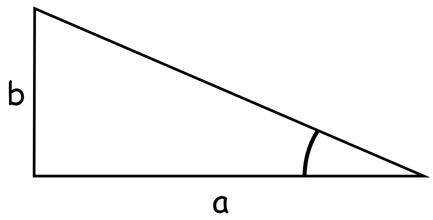
Triangle	y (cm)	b (cm) = y - 8	a (cm)
P (1)	49	41	
Q (2)			
R (3)	93	85	714
S (4)			
T (5)	137	129	1087

b. Use your calculator to find the **ratio of 'b' to 'a' for each triangle** and write it in the table below. Give your answers correct to **2 decimal places**.

Triangle	$\frac{b}{a}$
P (1)	
Q (2)	
R (3)	
S (4)	
T (5)	

c. For **a given angle**, the **ratio** of the **opposite side** to the **adjacent side** is always the **same**, irrespective of the length of the sides.

What is this ratio called? Tick (✓) the correct answer.

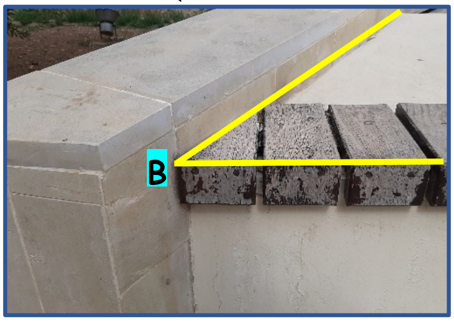


- ☐ Tangent ratio
- ☐ Sine ratio
- ☐ Cosine ratio

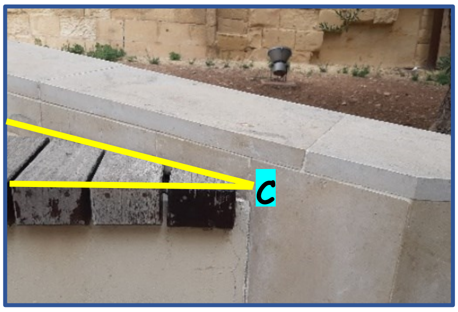
d. Are triangles P, Q, R, S and T **similar**? Why do you think this is so?

Answer:

Task 3.4 – Pythagoras' Theorem



A close-up photo of part B above.



A close-up photo of part C above.

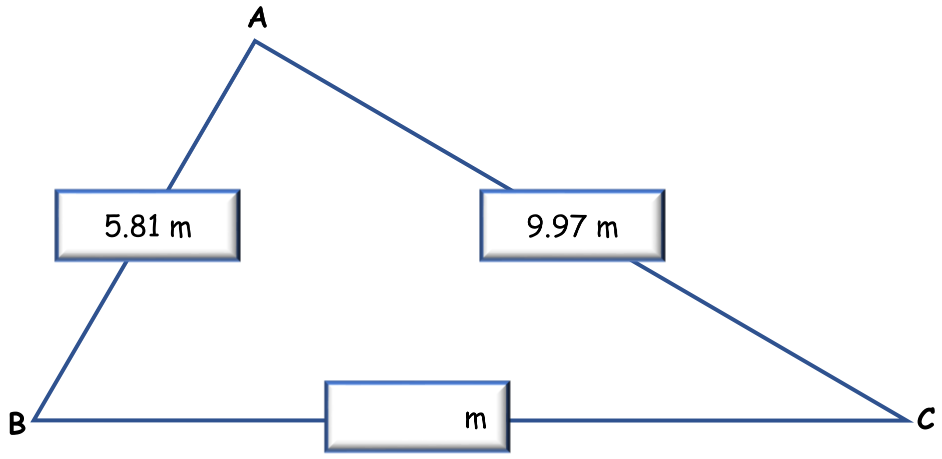
This is a triangular shaped fountain. Sides AB and AC were measured using the **inner edge** of the triangle as shown above.

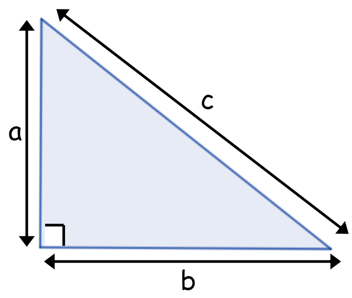
a. Use the tape measure to find BC, correct to the **nearest cm**. Make sure to measure the length from point B to point C as shown in yellow in the close-up diagrams above.

Answer:

cm

b. Fill in the missing length in this diagram:





The converse of **Pythagoras' Theorem** states that if $c^2 = a^2 + b^2$, then:

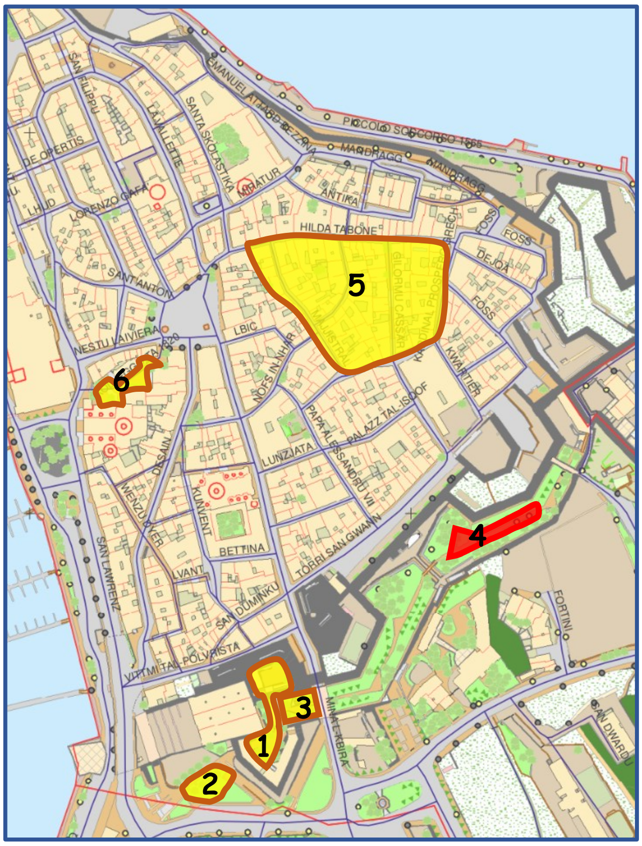
- the triangle is a right-angled triangle
- the right-angle is directly opposite the hypotenuse

c. The converse of Pythagoras' Theorem allows us to check if the fountain, shown as triangle ABC above, is a **right-angled triangle**. We can do so by calculating CB^2, BA^2, and AC^2. Is CB^2 approximately equal to BA^2 and AC^2? So, what can you say about triangle ABC above?

Answer:

Station 4 – Birgu Ditch (St. James Bastion Area)

Shapes, Space and Measures (Cube/Cuboids); Euclidean Geometry; Algebra; Data Handling (Statistics)

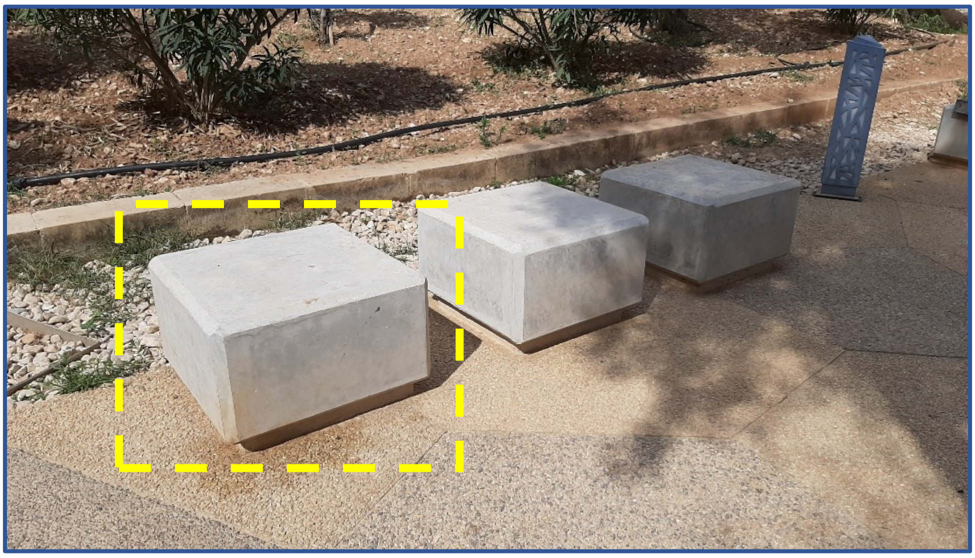


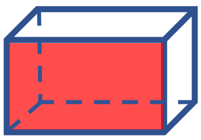
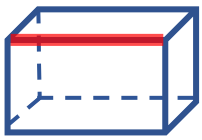
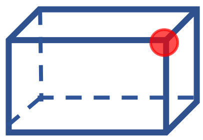
Learning Objectives

- I can use formulae to calculate the volume of cubes and cuboids.
- I can identify nets that are possible or not possible for a cuboid.
- I can use the density of a material to find the mass of an object.
- I can calculate the sum of the interior angles of irregular polygons.
- I can measure the angles at a point.
- I can measure time and distance to calculate the average speed.
- I can convert m/s to km/h or vice-versa.
- I can calculate the mean, mode and median of a data set and identify which is the most robust in case of outliers in the data set.

Birgu Ditch (St. James Bastion Area)

Task 4.1 – Volume and 3D-shapes



Face	Edge	Vertex
		
A **face** of a 3D shape is a flat surface on the outside of a shape.	An **edge** of a 3D shape is a straight line segment where two faces meet.	A **vertex** of a 3D shape is a point where two or more edges meet.

The three structures shown in photo above are **cuboids**. Consider the **one** marked in yellow:

a. How many **faces** does this cuboid have?

Make sure to count all the faces, and not only those shown in the photo.

Answer:

faces

b. How many **edges** does this cuboid have?

Make sure to count all the edges, and not only those shown in the photo.

Answer:

edges

c. How many **vertices** does this cuboid have?

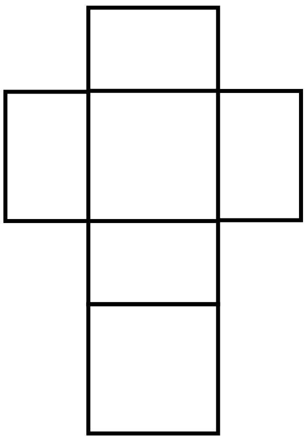
Make sure to count all the vertices, and not only those shown in the photo.

Answer:

vertices

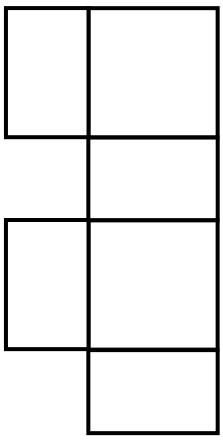
d. Which of the following diagrams represent the **net of a cuboid**? Tick (✓) the correct answer for each net.

(i)



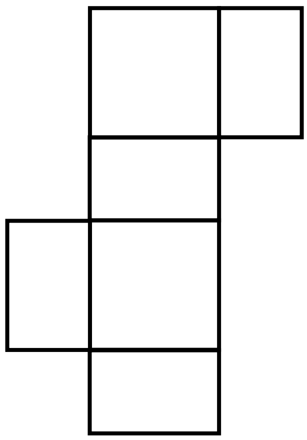
☐ Yes ☐ No

(ii)



☐ Yes ☐ No

(iii)



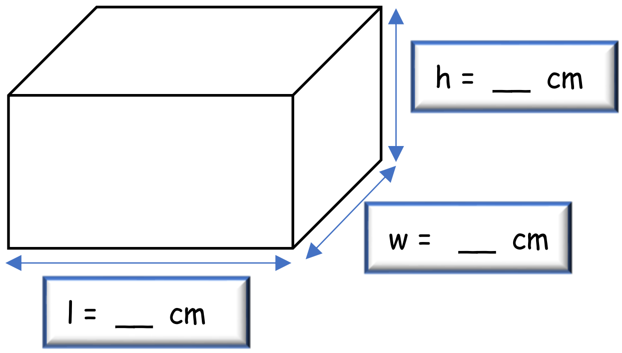
☐ Yes ☐ No

Exclude this base

When measuring:

- use the **inner edges** of the structure, as shown red in this photo.
- focus only on the cuboid, i.e. **exclude the base**.

e. Use your tape measure to find the **sides of the cuboid** (l, w, h) and fill in the measurements in the diagram below. Give your answer correct to the **nearest cm.**



f. Now, calculate the **volume of the cuboid**. Give your answer correct to 3 significant figures.

Answer:

cm^3

Density is how much mass there is in a particular volume. The denser an object is, the heavier it is.

The density of an object is found by dividing the mass of the object by its volume.

The formula for density is:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

g. The concrete used for these structures has a density of **$2.4g/cm^3$**. Use the result of Task 4.1f to find the **mass of one cuboid**. Give your answer **in kg**, correct to the **nearest ten.**

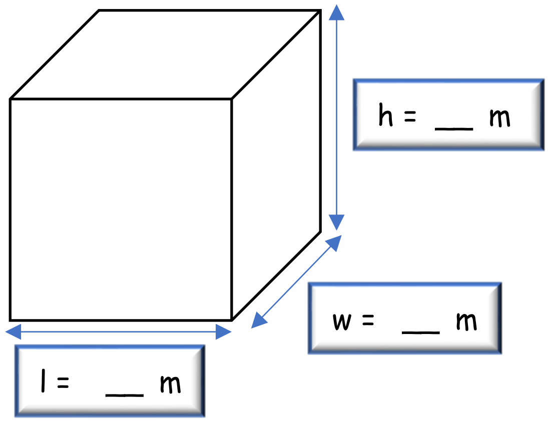
Answer:

kg

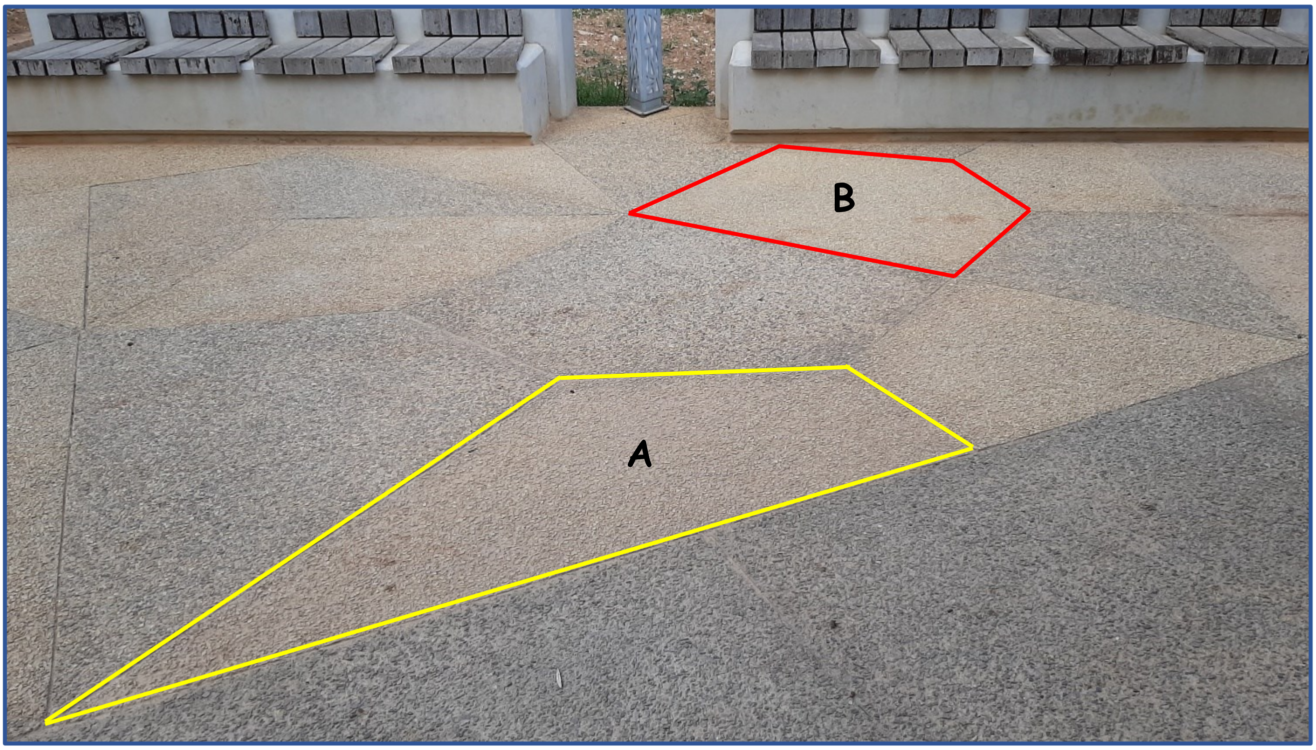
h. Which of the following statements about **cubes are correct**? Write True or False for each statement.

Statement	True or False
A cube is a special type of cuboid.	
A cube has 15 edges.	
A cube is a square.	
A cube has 6 equal faces.	
A cube is a prism.	

i. If the **same amount of concrete** as that found in Task 4.1f is used to create a cube instead of a cuboid, what will the dimensions of the **cube** be? Give your answer in metres, correct to the **1 decimal place**.



Task 4.2 – Polygons and Angles at a Point



This tiling is made up of different polygons of various sizes.

a. Which of the following describes **shape A** (marked in yellow in the diagram above)?

Tick (✓) the correct answers.

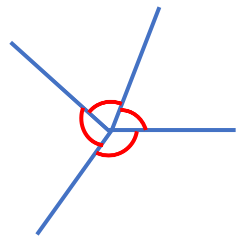
- Quadrilateral ☐ Yes ☐ No
- Pentagon ☐ Yes ☐ No
- Hexagon ☐ Yes ☐ No
- Irregular polygon ☐ Yes ☐ No

b. Which of the following describes **shape B** (marked in red in the diagram above)?

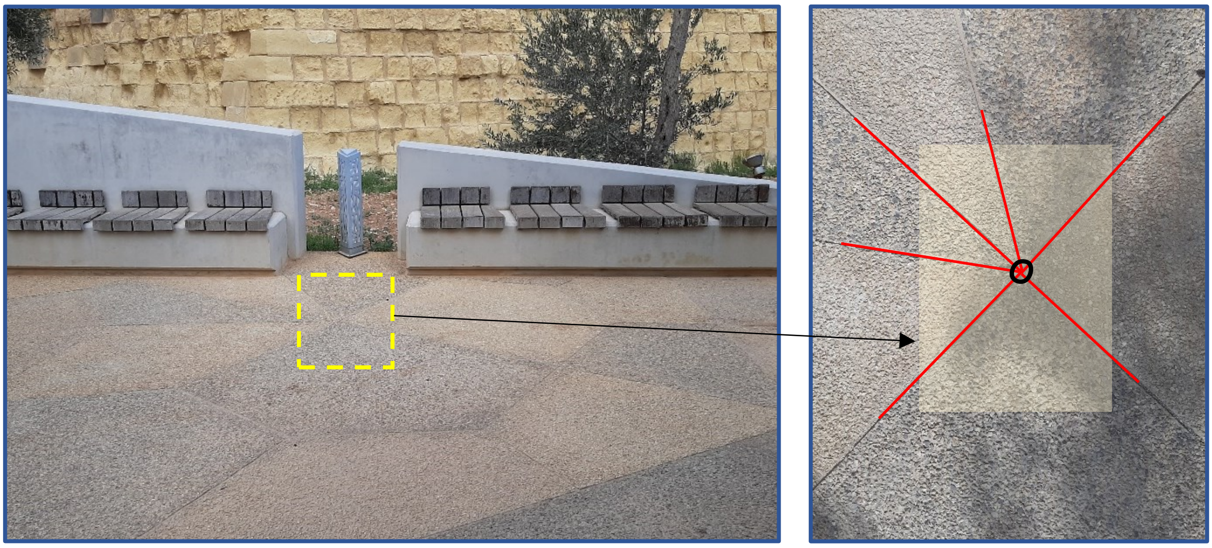
Tick (✓) the correct answers.

- Quadrilateral ☐ Yes ☐ No
- Pentagon ☐ Yes ☐ No
- Hexagon ☐ Yes ☐ No
- Irregular polygon ☐ Yes ☐ No

The angles formed by several rays having a common initial point are called **angles at a point**.

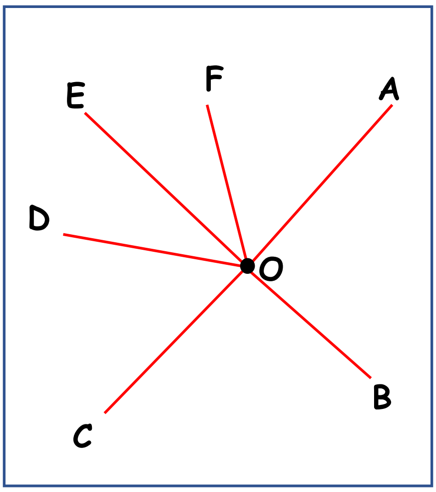


Focus on the part of tiling marked in yellow below:



c. Use a tracing paper to **trace the rays at point O**. Label the traced diagram as follows:

- O is common initial point
- A, B, C, D, E and F are the rays



d. Use your protractor to measure the following **angles**. Write your measurements in the table below, correct to the **nearest degree**. Then, work out the sum of these angles.

	Angle
Angle AÔB	°
Angle BÔC	°
Angle CÔD	°
Angle DÔE	°
Angle EÔF	°
Angle FÔA	°
Total	°

e. So, what is the **sum of angles** at point O?

Answer: °

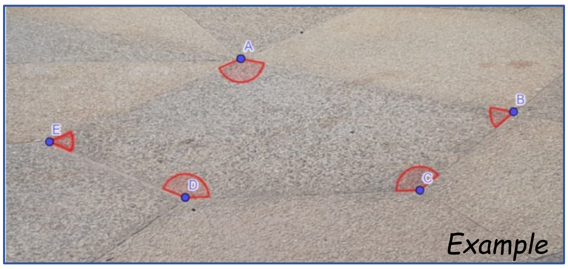
f. **Angles at a point** always add up to °.

g. **Each member in the group** needs to select **one quadrilateral or one pentagon** from the tiling. Then, each member needs to:

Take a **photo** of the selected polygon. Position the camera such that the photo is taken directly from **above** the polygon (like bird's eye view). Make sure that all of the polygon **fits** in the photo.

Import the photo on **GeoGebra** and:

- Use Angle to measure all of the interior angles. Measurement is to be clearly marked on your GeoGebra file.



- Record these interior angles in the table below.
- Work out the sum of these interior angles and write your answer in the 'Total' box below.

If you selected a quadrilateral, use this:

	Quadrilateral
Angle A	°
Angle B	°
Angle C	°
Angle D	°
Total	°

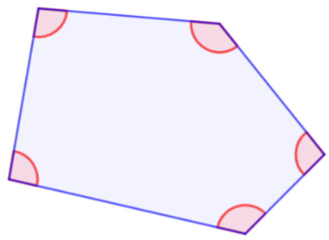
If you selected a pentagon, use this:

	Pentagon
Angle A	°
Angle B	°
Angle C	°
Angle D	°
Angle E	°
Total	°

The **sum of interior angles** of a polygon can be found using the formula:

$$(n-2) \times 180^{\circ}$$

where n is the number of sides



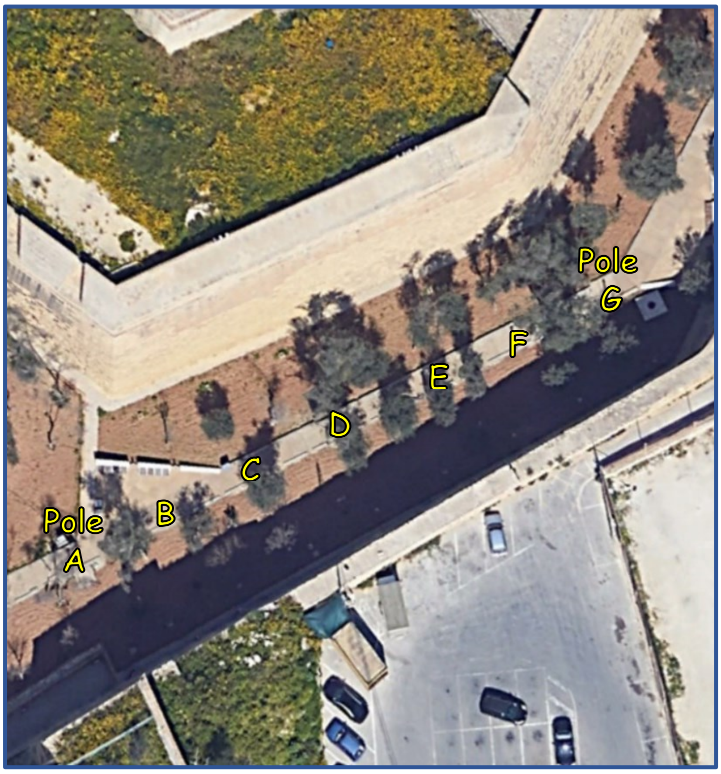
h. Find the **sum of interior angles** of your polygon using this formula.

Answer:

i. Compare the result of Task 4.2g to the answer of Task 4.2h. What can you notice?

Answer:

Task 4.3 – Statistics



Aerial View of Birgu Ditch (St. James Bastion Area)

In the following activity you are going to take it in turns so that **each student** runs from **Pole A to Pole G, and back to Pole A**. In each round there needs to be a student who is a:

- '**Pole A keeper**' situated next to Pole A responsible for the start and finish line;
- '**Time keeper**' situated next to Pole A responsible to measure the time taken for the 'runner' to run from the start line to the finish line;
- '**Pole G keeper**' situated next to Pole G responsible to ensure that the 'runner' runs up to Pole G before heading back to Pole A;
- '**Runner**' to run from Pole A to Pole G and back to Pole A.

a. Use your tape measure to find the **distance** between Pole A and Pole G as marked in the diagrams above. Use this measurement to find the **total distance travelled** from the start line to the finish line. Give you answer in **metres**.

Answer: ______ m

b. Record the time on your stopwatch whenever a student runs **from the start line to the finish line**.

Write these measurements in the table below.

	Running Time	Running Time in seconds
Student 1	__ : __ . __	
Student 2	__ : __ . __	
Student 3	__ : __ . __	
Student 4	__ : __ . __	

Speed is a measure of how fast an object is moving.

When objects do not move at a constant speed, we measure the **average speed**. The formula for average speed is:

$$\text{Average Speed} = \frac{\textbf{Total Distance Travelled}}{\textbf{Total Time Taken}}$$

c. Calculate the average speed of **the fastest student** in your group.

Answer:

m/s

d. Which of the following statements is correct. Tick (✓) the correct answer. Show your working to justify your answer.

- ☐ 4m/s is less than 14.4km/h
- ☐ 4m/s is equal to 14.4km/h
- ☐ 4m/s is greater than 14.4km/h

Answer:

Mean, median and mode

Mean is the **average** value.

- To find the mean, add all numbers together and then divide by the number of values.

Median is the **middle** value.

- To find the median, order the numbers lowest to highest and see which value is in the middle of the list. If there are two middle values, the median is the value halfway between them.

Mode is the number that appears the **most**.

- To find the mode, order the numbers lowest to highest and see which value appears the most often.

e. A group of 10 students carried out a similar run. The table below shows their running times.

Student	A	B	C	D	E	F	G	H	I	J
Running Time (seconds)	28	26	25	30	100	32	27	29	35	28

Calculate the **mean**, the **median** and the **mode** of this data set. Give your answers correct to **one decimal place**.

Mean: s

Median: s

Mode: s

f. As you can notice, the running time of Student E is much larger than the rest of the group. This was recorded incorrectly because of an issue with the timer. We say that this measurement is an '**outlier**' to the rest of the data set.

Eliminate the record of student E from the data set as follows:

Student	A	B	C	D	F	G	H	I	J
Running Time (seconds)	28	26	25	30	32	27	29	35	28

Re-calculate the **mean**, the **median** and the **mode**. Give your answers correct to **one decimal place**.

Mean: s

Median: s

Mode: s

g. Compare the results of the **mean** in Task 4.3e and Task 4.3f. What can you observe?

Answer:

Compare the results of the **median** in Task 4.3e and Task 4.3f. What can you observe?

Answer:

Compare the results of the **mode** in Task 4.3e and Task 4.3f. What can you observe?

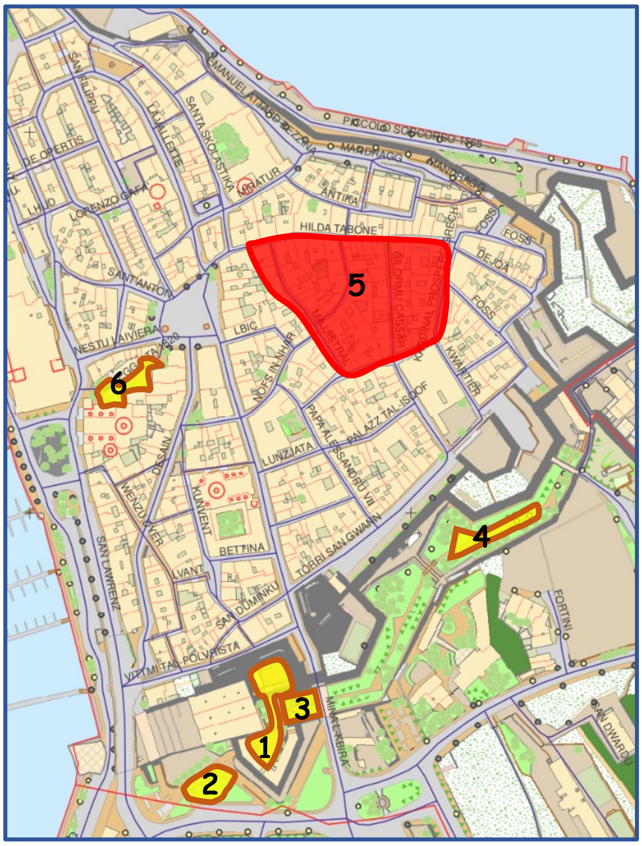
Answer:

h. Which of these calculations do you think is **the most distorted** by the outlier?

Answer:

Station 5 – Il Collacchio

Statistics (Data Handling); Number (Numerical Calculations); Algebra



Learning Objectives

- I can collect data and use this data to construct a frequency table.
- I can construct pie charts and bar charts with the collected data.
- I can interpret data from frequency tables, bar charts and pie charts.
- I can identify even, odd, square, triangular, and prime numbers.
- I can work through situations leading to the solution of a linear equation to find an unknown value.
- I can use a navigation compass to measure bearings.
- I can draw scale drawings, including bearings.

Triq Hilda Tabone – a main street in the Collacchio area

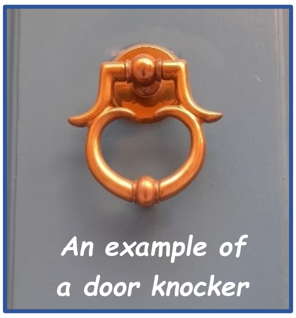
Task 5.1 – Collecting Data

Aerial View of Il Collacchio Area

In 1562, Grand Master La Valette, established the boundary within which the Knights' palaces and houses had to be built and where the Knights were ordered to live. The boundary wall was marked by a number of stone bollards, like low columns, one of which can be seen in Victory Square. The Collacchio has some of the oldest buildings in Birgu and one can still find the Auberge de France and the Auberge d'Angleterre still in good condition. An auberge was the residence of a group of knights of the same nationality.

Door Knockers ('Ħabbata')

Door knockers are a feature on most of the traditional Maltese doors. It was a way for a visitor to alert the house owner of his presence before electronic doorbells and intercoms were invented. Door knockers can be found in numerous shapes and sizes. Typically made out of metal or brass, they often feature the Maltese Cross or maritime motifs such as dolphins, seahorses and fish, amongst others. In the past, elaborate door knockers indicated families that had more status or wealth.



An example of a door knocker

Coat of arms of Malta from 1964 to 1975

The dolphin door knocker became popular post-independence as the dolphin was depicted in Malta's first coat of arms.

Note that a doorknob ('pum') is not a door knocker. A doorknob is used to push or pull the door whereas the door knocker is used instead of a doorbell.

An example of a door knob

In this task you are going to walk from one end of **Triq Hilda Tabone** (start) to the other end (finish), as shown in yellow below. As you walk along this street, you are going to:

- Record the **door numbers** as shown in table in Task 5.1a
- Record the **door colours** as shown in table in Task 5.1b
- Record the **door knockers** as shown in table in Task 5.1c

a. When recording the **door numbers** in the grid below, consider **only** the door numbers written on the Local Council's ceramic number plate, as shown.

Door Number					

b. When recording the **door knockers**:

- Fill in the table below with appropriate tally marks.
- **Count** the door knockers **in singles** rather than in pairs as some doors might not have two identical door knockers.

Example:

Dolphin = | |

Seahorse = | and Fish = |

- **Do not** take into consideration any doors **without** door knockers.
- Remember to group the tally marks in groups of 5 as 𝍸.

Door Knocker	**Examples**	**Tally**
Seahorse	 	
Fish	 	
Dolphin with trident tail		
Ring Handle	  	

c. Record the door colours of **all** doors as you walk from the Start to the Finish points. When recording the **door colours**, fill in the table below with appropriate tally marks.

Door Colour	Tally
White	
Cream	
Yellow	
Mustard	
Beige	
Brown	
Orange	
Red (Wine)	
Blue (Royal/Light/Dark)	
Green (Light/Dark)	
Grey	
Black	

Task 5.2 – Illustrating Data

Pie Charts and Bar Charts

A **pie chart** is a circular graph divided into different **sectors**, where each sector represents a category. The angle of each sector is a fraction of 360°.

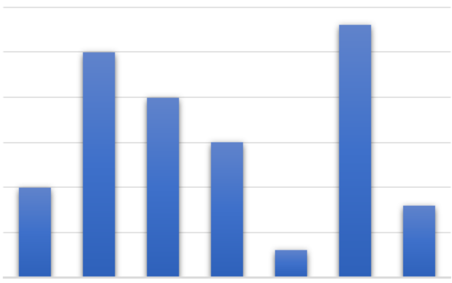
Pros

- A pie chart gives a visual representation of the part-to-whole comparison, i.e. what portion (percentage) of the whole group each category is.

Cons

- A pie chart cannot be used when the number of categories is more than six as it will be difficult to read.

A **bar chart** plots data using rectangular **bars**. It consists of two axes - one represents the category and the other represents the frequency of events occurring i.e. the height of each bar is the frequency.



Pros

- A bar chart gives a visual representation of how each category compares with the others.

Cons

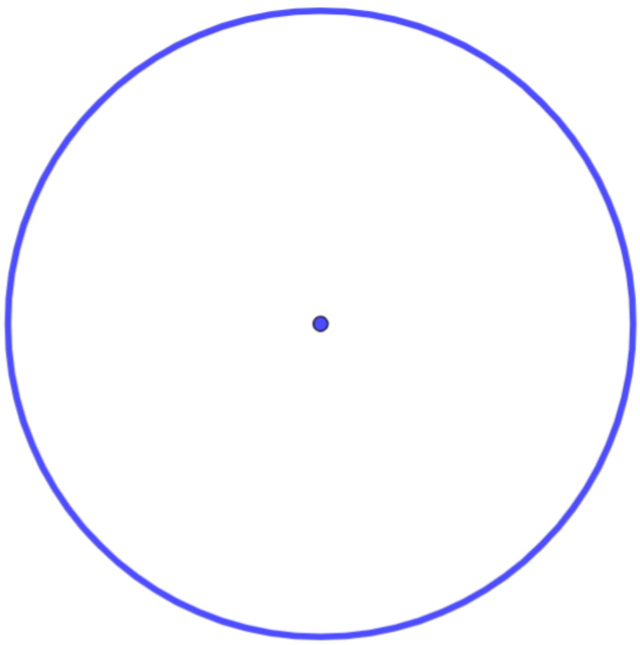
- It is not easy to do a whole-to-part comparison with a bar chart.

a. In this task you need to construct a **pie chart** to represent the **door knockers**.

(i) Refer to the data collected in Task 5.1b to fill in the table below:

Shape	Seahorse	Fish	Dolphin	Ring Handle	Total
Frequency					
Fraction of total					
Angle= Fraction of total $\times 360^\circ$ (correct to the nearest degree)	°	°	°	°	°

(ii) Use your protractor to draw the pie chart below, labelling each category and its corresponding angle on the diagram.



(iii) The total of the angles might not add up to 360°. Why do you think this is so?

Answer:

b. In this task you need to construct a **bar chart** to represent **the door colours**.

(i) Refer to the data collected in Task 5.1c to fill in the table below:

Colour	White	Cream	Yellow	Mustard	Beige	Brown	Orange	Red	Blue	Green	Grey	Black
Frequency												

(ii) Use your ruler to draw a **bar chart** on the squared paper below. Label the axes (title and scale) accordingly.

c. Which is the most frequent door knocker?

Answer:

Which is the most frequent door colour?

Answer:

Task 5.3 – The Number System

a. Refer to the number set of door numbers in Task 5.1a and identify all of the:

Answer:

- even numbers …………
- odd numbers …………
- prime numbers ………
- square numbers ……
- triangular numbers

b. Refer to the table in Task 5.1a and find **3 consecutive numbers**, for example 102,103,104.

(i) Write down these 3 numbers and find their sum.

Answer:

(ii) Is the sum a multiple of the middle number?

Answer:

(iii) Find another set of 3 consecutive numbers from your table in Task 5.1a and check if the sum is also a multiple of the middle number.

Answer:

(iv) Let **n** be the middle number of 3 consecutive numbers. Show that the sum of these 3 numbers is always 3 times the middle number.

Answer:

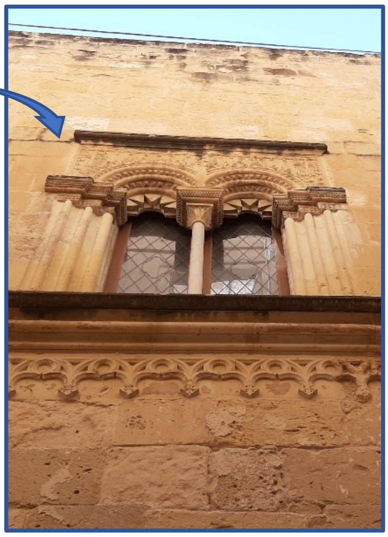
Magic trick with numbers:

Ask a friend to think of 3 consecutive positive whole numbers, add them up, and give their total.

When you have the total, divide it by 3, take note of the middle number, deduct 1 from it and add 1 to it. This will give you the sequence that your friend thought about.

Say the sequence and notice the surprise on your friend's face!

Task 5.4 – Bearings and Scale Drawing



The Norman House

The Norman House is one of the oldest buildings in Birgu. It may have been built in the 13th century and has one of the few medieval windows in Malta. The style in which the window is built is called Siculo-Norman because the Normans built many like it in Sicily.

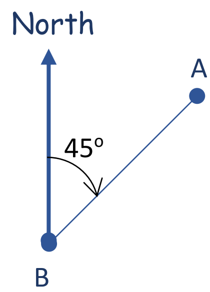
In Malta this style was imported by the wealthy people under the influence of their counterparts in Sicily.

The **bearing to a point** is the **angle** measured in a **clockwise** direction from the **north** line.

A **three-figure bearing** is a bearing given in **three figures**.

When the number of digits in a bearing is less than 3 digits, **add extra zero/s** to make the number of digits equal to 3.

For example, bearing of A from B is 045°.



To use the **navigating compass**:

- Start by **facing** your target location (e.g. a road that you need to follow).
- Hold your navigating compass **flat** with the direction of travel arrow pointing in the direction you are walking.
- If using the mobile app, read the angle given by the app.
- If using a physical navigating compass, rotate the dial so that the 'N' aligns with the red part of the compass needle. Read the angle on the rim.
- This angle is the angle between the North line and your target location.

Lora is in Triq Hilda Tabone and walks around the Collacchio area to find the Norman House. She starts from Point A (Triq Hilda Tabone corner with Triq Ġilormu Cassar), then goes to Point B, C, D, E and then back to Triq Hilda Tabone (Point F).

a. Use your navigating compass to find the following bearings. Give your answers using a **three-figure bearing**. Make sure that you are holding your navigating compass in the '**from**' location.

For example:

- the bearing of Triq Hilda Tabone is 274°
  (bearing of F **from** A)

i) The bearing of Triq Ġilormu Cassar (bearing of B from A)

Answer: ____ °

ii) The bearing of Triq Paċifiku Scicluna (bearing of C from B)

Answer: ____ °

iii) The bearing of Triq il-Majjistral (bearing of D from C)

Answer: ____ °

iv) The bearing of the upper part of Triq it-Tramuntana (bearing of E from D)

Answer: ____ °

v) The bearing of the lower part of Triq it-Tramuntana (bearing of F from E)

Answer: ____ °

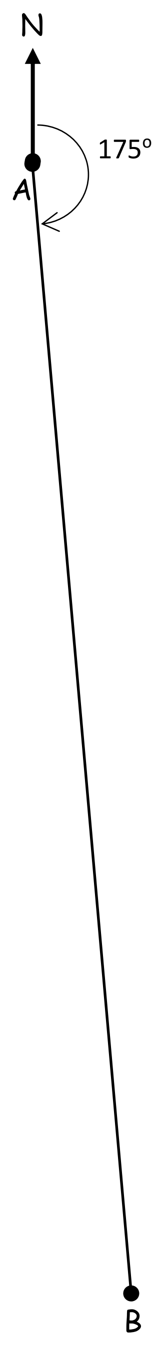
b. The distances between Points A, B, C, D, E and F, as shown on the map above are as follows:

- Point A to Point B = 75m
- Point B to Point C = 35m
- Point C to Point D = 37m
- Point D to Point E = 36m
- Point E to Point F = 30m

These distances were measured using Google Earth.

Draw an **accurate scale drawing** of Lora's route by using your ruler and protractor.

Use a **scale of 1cm to 5m**.



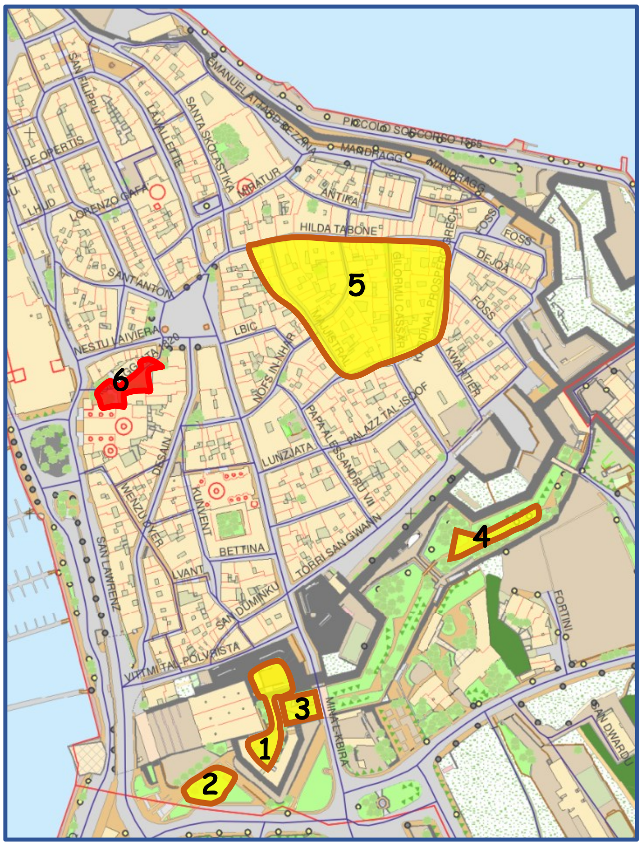
c. Use your diagram to find the **distance** (in metres) Lora needs to walk to go **from Point F to Point A**.

Answer:

m

Station 6 – Wesgħat il-Kolleġġjata

Shapes, Space and Measures (Trigonometry and Euclidean Geometry); Data Handling & Chance (Probability); Numerical Calculations



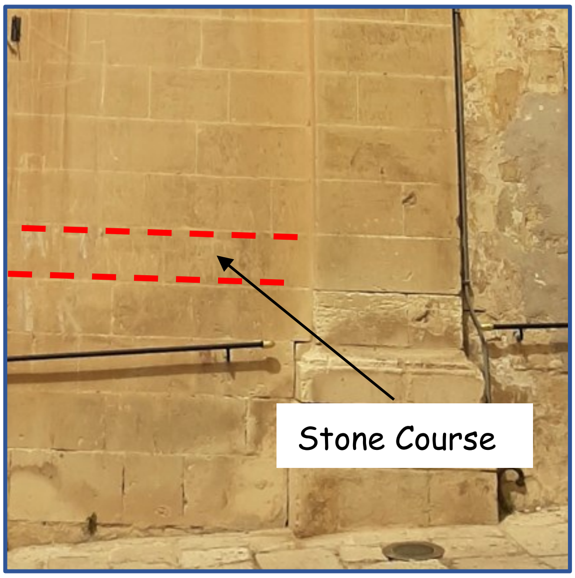
Learning Objectives

- I can use the trigonometric ratios to find unknown lengths in right-angled triangles.
- I can estimate the height of a building using the number of stone courses.
- I know the value of each digit in each number according to its position in the number.
- I can express any integer as a product of its prime factors.
- I can identify the lines of reflective symmetry and the order of rotational symmetry.
- I can construct and use a probability tree diagram to work out the probability of independent events.

Wesgħat il-Kolleġġjata

Task 6.1 – Measuring Heights using Trigonometry

A **stone course** is a layer of stone **running horizontally** in a wall. It can also be defined as a continuous row of any masonry unit such as bricks, stone, etc.



a. Use your tape measure to find the **height** of one stone course ('filata'). Record your answer correct to the **nearest cm**.

Answer:

cm

b. Count the number of stone courses to **estimate** the height AB. Give your answer correct to the **nearest cm**.

Answer:

cm

c. Select **one student** from your group to measure the **angle of elevation** using the clinometer as follows:

- Student needs to find a position **in front of** the vertical column AB and far enough away to see edge at point A through the clinometer.
- Record the **angle of elevation**, **θ**, correct to the **nearest degree**.

Answer:

°

d. The student needs **to remain** in the position where the angle of elevation is measured. The rest of the group needs to use the tape measure to find:

- the **horizontal distance** between the student and the vertical column AB. Give your answer correct to the **nearest cm**.

  Note: The ground level gives you the most accurate horizontal measurement.

Answer:

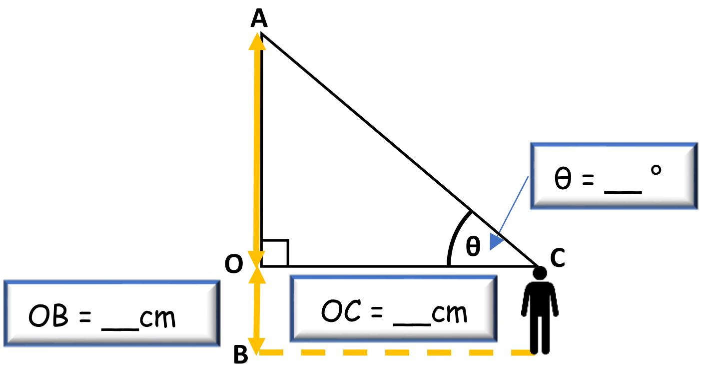
cm

- the **height** of the student that is using the clinometer. Give your answer correct to the **nearest cm**.

Answer:

cm

e. Fill in the empty boxes in the following diagram:



f. Use trigonometry to find the **height AO**, correct to the **nearest cm**.

Answer:

cm

g. Now, calculate the **total height AB**, correct to the **nearest cm**.

Answer:

cm

h. Use the result of Task 7.1b and Task 7.1g to fill in the empty boxes below. What is the **% error** between the estimated height and the measured height? Give your answer correct to **2 decimal places**.

$$\% \text{ Error } = \frac{\textbf{Estimated Height} - \textbf{Measured Height}}{\textbf{Measured Height}} = \frac{\square}{\square} \times 100 = \square\ \%$$

Answer:

%

Task 6.2 – Probability

St. Lawrence Church was founded in 1091 but the one standing now, designed by Lorenzo Gafà, was built between 1681 and 1697. The church is dedicated to St. Lawrence. Among the treasures found in it is a painting of the famous Italian artist Mattia Preti, representing The Martyrdom of St. Lawrence.

The Vittoriosani show a lot of devotion to St. Lawrence. The name Lawrence (or variations of it such as Lorenzo, Lora, Lorenza) is very common in Birgu.

At the side of St. Lawrence's church, you can find a plaque with **names** of World War II Vittoriosani victims, as shown.

a. **Each student** in the group needs to take a photo of the plaque. Make sure that the names on your photo are readable.

b. If a name is selected at random:

(i) What is the probability that the name is of a male?

P (male):

Answer:

(ii) What is the probability that the name is of a female?

P (female):

Answer:

(iii) What is the probability that the name is of a male **and** his name is dedicated to St. Lawrence?

P (male ∩ his name is dedicated to St. Lawrence):

Answer:

(iv) What is the probability that the name is of a male **and** his name is not dedicated to St. Lawrence?

P (male ∩ his name is not dedicated to St. Lawrence):

Answer:

(v) What is the probability that the name is of a female **and** her name is dedicated to St. Lawrence?

P (female ∩ her name is dedicated to St. Lawrence):

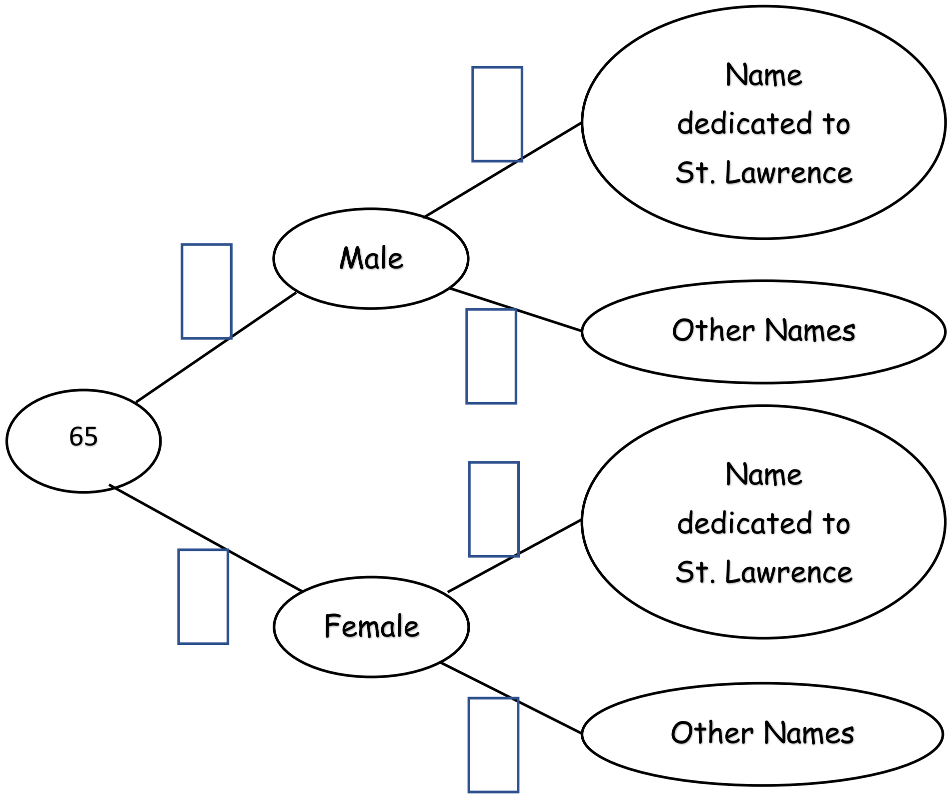
Answer:

(vi) What is the probability that the name is of a female **and** her name is not dedicated to St. Lawrence,

P (female ∩ her name is not dedicated to St. Lawrence):

Answer:

c. Use the answers you obtained in Task 7.2c to complete the following probability tree:



d. Assume that this data is a good indication of the names of the people now living in Birgu. What is the **% probability** that if you meet someone from Birgu, his or her name is dedicated to St. Lawrence? Give your answer correct to the **nearest ten.**

Answer: %

e. Use your answer in Task 7.2d to complete the following statements:

- out of 100, ☐ people are named Lawrence/Lorenz/Lorenza/Lora

- out of 10, ☐ people are named Lawrence/Lorenz/Lorenza/Lora

Task 6.3 – Place Value

Place value is the basis of our number system. Each digit of a number has a place value. In the decimal system, the place values are all powers of 10, including thousands, hundreds, tens, and units.

In every number, each place to the left is 10 times greater and each place to the right is 10 times smaller.

At the side of St. Lawrence Church, you can find the monumental bust of Mons. Dun Ġużepp Caruana, as shown.

a. Fill in the following spaces:

Mons. Dun Ġużepp Caruana was born in the year ____ and passed away in the year ____.

b. How old was Mons. Dun Ġużepp Caruana when he died?

Answer:

years

c. Using the digits in the year that Mons. Dun Ġużepp Caruana was born:

(i) What is the **smallest** possible **4-digit odd number**?

Answer:

(ii) What is **largest** possible **4-digit odd number**?

Answer:

(iii) Is it possible to have a **4-digit even number**? Why do you think this is so?

Answer:

d. Now, consider the year that Mons. Dun Ġużepp Caruana passed away:

(iv) Write this year as a product of its **prime factors**.

Answer:

(v) What is the smallest possible **4-digit multiple of 2**?

Answer:

Task 6.4 – Symmetry

The Oratory of St. Joseph

The Oratory of St. Joseph was built in 1832. It is the place where one can find the hat and sword that La Valette used during the Great Siege. He had left them in the chapel as a thanksgiving to Our Lady of Damascus for the victory over the Turks in the Great Siege.

This chapel was one of the early churches in Birgu that was used by the Greek community who had come from Rhodes with the Knights.

Reflective Symmetry is when a shape is folded (**reflected**) along the line of symmetry (mirror line), and half the shape would fit exactly over the other half. Some shapes can have more than one line of symmetry.

Rotational Symmetry is when a shape is **rotated** about a fixed point to a new position and still looks the same as it did before the rotation. The number of positions in which a shape can be rotated and still looks the same as it did before the rotation is called the **order of rotational symmetry**.

Thus, a shape can have no lines of symmetry, but the order of rotational symmetry is always **at least 1** (as the shape will always look the same as it did before being rotated a full turn).

The following are photos of l-'Oratorju' main door. For each photo you need to:

- **Draw** and **write** the number of lines of symmetry
- **Write** the order of rotational symmetry

a.

i) Draw the line/s of symmetry.

ii) The number of line/s of symmetry is equal to:

iii) The order of rotational symmetry is equal to:

b.

i) Draw the line/s of symmetry.

ii) The number of line/s of symmetry is equal to:

iii) The order of rotational symmetry is equal to:

c.

i) Draw the line/s of symmetry.

ii) The number of line/s of symmetry is equal to:

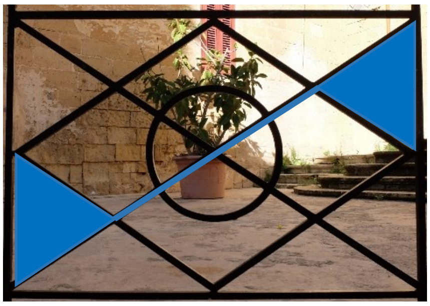
iii) The order of rotational symmetry is equal to:

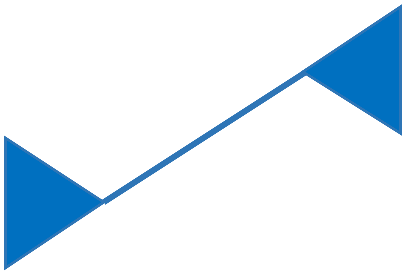
d. Is the order of rotational symmetry **always** equal to the number of lines of symmetry? Justify your answer by giving an example of a quadrilateral.

(Hint: Is this true in the case of a rhombus?)

Answer:

e. Consider the railings next to the stairs. Part of these railings form the shape coloured in blue below:





i) How many **lines of symmetry** does the blue-coloured shape have?

Answer:

ii) What is the **order of rotational symmetry** of this shape?

Answer:

The order of rotational symmetry of a shape is **always** equal to or greater than the number of lines of symmetry.

Trail Sequence for Group A

Activity No.   Path   Stairs   Meeting Point

Trail Sequence for Group B

Activity No. Path Stairs Meeting Point

Trail Sequence for Group C

Activity No. Path Stairs Meeting Point

Trail Sequence for Group D

Activity No. — Path — Stairs — Meeting Point

Trail Sequence for Group E

Activity No. Path Stairs Meeting Point

Trail Sequence for Group F

Activity No. Path Stairs Meeting Point