applied sciences

Communication

A Simple and Easily Implementable Model for the Prediction of Solar Irradiance for All-Sky Conditions: Model Development, Preliminary Evaluation and Application

Alfred Micallef [1,2]

[1] Department of Geosciences, Faculty of Science, University of Malta, MSD 2080 Msida, Malta; alfred.micallef@um.edu.mt; Tel.: +356-2340-3037

[2] Geosciences Observatory, University of Malta, Gozo Campus, Mgarr Road, XWK 9016 Xewkija, Gozo, Malta

Abstract: A simple and easily implementable model and the associated computer code for predicting the solar irradiance at the earth's surface have been developed. The input requirements of the model include Julian day, time, geographical latitude, atmospheric pressure and cloud cover. If the latter is not available, then the program user has the option of entering either the daily global solar flux or the duration of sunshine, from which the average cloud cover is estimated. Preliminary model evaluation has been carried out using data from two meteorological stations situated in rural and semi-rural areas of the East Midlands, United Kingdom. The scatter plots of predicted versus observed solar irradiance gave correlation coefficients approximately equal to 0.6 and slopes in the range of 0.80–0.93. The model is being implemented as a submodule within an urban air quality model. This specific application of the solar radiation model is discussed as a typical implementation. This work shows that by adopting relatively simple 'textbook' material, i.e., basic theory/information, one can achieve reasonably good solar radiation modelling, with outcomes that can be used for applications where accuracy is not a major requirement.

Keywords: solar radiation; solar irradiance; modelling; air quality

Citation: Micallef, A. A Simple and Easily Implementable Model for the Prediction of Solar Irradiance for All-Sky Conditions: Model Development, Preliminary Evaluation and Application. *Appl. Sci.* **2023**, *13*, 12982. https://doi.org/10.3390/app132412982

Academic Editors: Georgios Papadakis and Constantinos A. Balaras

Received: 16 June 2023
Revised: 29 November 2023
Accepted: 30 November 2023
Published: 5 December 2023

1. Introduction

Knowledge of solar radiation data finds application in many fields of science, including agricultural science, biometeorology, micrometeorology and environmental photochemistry. To mention a specific example, information relating to global solar irradiance is very important in estimating equilibrium surface temperature [1], which in turn is important in the study of the urban heat island effect. The latter phenomenon affects urban air quality through an increase in the emission of primary and precursors of secondary air pollutants [2].

A literature survey revealed many solar radiation models, but they all seem to fall into one of two categories. The first category consists of 'detailed' models which require information on constituents of the atmosphere and are generally theoretical in nature. Researchers outside the physical sciences may find these models difficult to implement in their own specific models or even to use them because of their mathematical and computational complexity. Hence, detailed solar radiation models may not be suitable for routine applications. An example of such models is that of Zdunkowski et al. [3]. This is a versatile radiation scheme developed to calculate radiative fluxes and heating rates for the solar and infrared emission spectrum, which incorporates the effects of atmospheric water vapour, carbon dioxide, ozone, pollution gases, aerosol particles and multilayered clouds of arbitrary cloud cover. The model is intended for use within circulation and climate models. Two other detailed models, which are more comprehensible to the non-mathematician, are those of McCullough and Porter [4], and Hatfield et al. [5]. The models of Zdunkowski et al. [3], McCullough and Porter [4], and Hatfield et al. [5] can be used for specific solar spectral regions, and this makes them suitable for applications such as outdoor photosynthetic studies and the effect of ultraviolet radiation on the skin. As mentioned earlier, the only disadvantage is their complexity and, hence, difficulty in implementation.

The second category consists of solar radiation models which require routine meteorological data as input; hence, they may be termed ‘simple’. The models are not necessarily simple in structure; an example of this is the numerical solar radiation model of Atwater and Ball [6]. In general, however, simple models can be very easily implemented into other specific models simulating some aspect of the environment since they normally consist of a small number of regression equations (or parametrisations) and a simple procedure which links the equations together. The only problem with simple models is that most of them are based on statistical analysis of empirical meteorological data collected at one or more stations, making them very site specific. Examples of models of the second category include those of Nielsen et al. [7], Sherry and Justus [8,9], Turner and Mujahid [10], Cerquetti et al. [11], Rangarajan et al. [12] and Topcu and Oney [13].

The model developed and described in this paper has input requirements that consist of routine meteorological data. It has a theoretical basis with assumptions on atmospheric optical characteristics, making it less site specific. Generally speaking, in this work, the approach adopts relatively simple ‘textbook’ material, i.e., basic theory/information, leading to reasonably good solar radiation modelling, with outcomes that can be used for applications where accuracy is not a major requirement.

2. Materials and Methods

The solar irradiance model (dubbed SOLAR) described here is semi-empirical in nature in that it consists of a blend of basic theoretical concepts and empirical results. The input requirements of the model include Julian day, time, geographical latitude and atmospheric pressure. Knowledge of cloud cover is also needed but if this is not available, then it can be estimated from one of two options, namely, the daily global solar flux or daily sunshine hours. The latter provision is made since measurement of daily duration of sunshine is more common amongst meteorological stations as it is easier to measure and requires less accurate instruments [11].

The instantaneous direct solar irradiance at the earth’s surface (S^c_{dir}), in the presence of a turbid atmosphere but clear sky (i.e., no clouds), can be calculated using the following equation given in textbooks on solar radiation theory (see, for example, [14,15]),

$$S^c_{dir} = S_o \beta \cos\psi \tau^m \tag{1}$$

where S_o is the solar constant (1373 W m^{-2}), which is defined as the solar irradiance on a surface perpendicular to the sun’s rays just outside the earth’s atmosphere and at the mean distance of the earth from the sun; β is the square of the ratio of the length of the semi-major axis of the earth’s orbit about the sun to the earth-to-sun separation, given by Equation (2); ψ is the zenith angle, defined as the angle between the direction of the sun and a vertical axis (to the surface of the earth at the place of observation), given by Equation (3); τ is the turbidity coefficient (or atmospheric transmissivity) which quantifies the attenuation of solar radiation by the atmosphere, assumed to be equal to 0.6, which is a reasonable value for cloud-free atmospheres [14] of urban areas, especially those in the East Midlands, United Kingdom [16]; m is the optical air mass defined as the ratio of the path length in the direction of the sun at a given zenith angle to the path length in the vertical direction, i.e., atmospheric depth, given by Equations (4) and (5).

The square of the ratio of the length of the semi-major axis of the earth’s orbit about the sun to the earth-to-sun separation, β, is given by

$$\beta \approx 1 + 2\varepsilon \cos\left(\frac{2\pi}{365} d\right) \tag{2}$$

where ε is the eccentricity of the earth’s orbit about the sun (0.01675); d is the Julian day.

Furthermore, we have the following:

$$\cos\psi = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos h_s \tag{3}$$

where φ is the geographical latitude (in degrees); δ is the solar declination angle, defined as the angle between the orbital (earth round the sun) and earth's equatorial planes, given by Equation (6) (in degrees); h_s is the hour angle, given by Equation (7) (in degrees).

Note that whenever angles are independent variables of sinusoidal functions, then they can be either in radians or degrees; otherwise, they are in radians unless stated. Derivation of Equation (3) is given in standard textbooks on solar radiation theory, but practically all of the derivations are based on the geometrical approach, which makes them cumbersome and algebraically complicated. An easier approach is to use vector analysis by considering the dot (or scalar) product of two vectors corresponding to two (parallel) sun rays, one which impinges on any arbitrary position on the earth's surface and the other such that it is radial to the earth's sphere. When the dot product is equated to zero (due to vectors being parallel), Equation (3) is obtained.

The optical air mass, m, is estimated from the equations hereunder:

$$m = \frac{1}{\cos\psi}\left(\frac{\sqrt{r^2\cos^2\psi + 2rH + H^2} - r\cos\psi}{H}\right) - \frac{2.8}{(90-\psi)^2} \text{ for } 60^\circ < \psi < 80^\circ \tag{4}$$

and

$$m = 0.3885\psi^2 - 63.705\psi + 2618 \text{ for } 80^\circ < \psi < 90^\circ \tag{5}$$

where r is the mean radius of the earth (6.390×10^6 m); H is the mean atmospheric depth (7.991×10^6 m). Equation (5) is the only case where ψ is not an independent variable of a sinusoidal function and expressed in degrees. Equation (4) is based on theory and corrected by empirical data from Kondratyev [17]. Although it can be used for angles greater than 80°, the zenith angle should not be too close to 90° for obvious reasons. Equation (5) was obtained by curve fitting to experimental data of the optical air mass as a function of the zenith angle given by List [18]. Quadratic regression analysis gave the best results with a correlation coefficient of 0.986. The equations are valid for an atmospheric pressure of one bar. Hence, for an atmospheric pressure, p, the optical air mass given by Equations (4) and (5) is corrected by multiplying it by p (in bars) as performed by Kondratyev [17].

The solar declination, δ, is given by

$$\delta = 23.5\cos\left(\frac{2\pi}{365}[172 - d]\right) \tag{6}$$

The latter equation is applicable to the Northern Hemisphere. Note that on 21 June (summer solstice), i.e., Julian day 172, and 22 December (winter solstice), the solar declination angle is at its maximum and minimum, i.e., +23.5° and −23.5°, respectively. δ is a sinusoidal (cosine) wave with an amplitude of 23.5. The expression for the solar declination in Equation (6) does not account for the small precession of the polar axis, which is considered inconsequential for the intents and purposes of the solar radiation model being discussed here.

The hour angle, h_s, is given by

$$h_s = \frac{2\pi}{24}t \tag{7}$$

where t is the time of the day from solar noon, i.e., positive values for time before noon and negative values for time after noon (e.g., 11:00 would be −1, while 13:00 would be +1). The resulting h_s is in radians.

The next step after calculating S^c_{dir} is to calculate the instantaneous diffuse solar irradiance at the earth's surface for a turbid atmosphere and cloudless sky (S^c_{diff}). The latter is expressed in terms of S^c_{dir}, the zenith angle and the turbidity coefficient. The equation has been used previously by Aida and Gotoh [19] and is given by

$$S^c_{diff} = \left(\frac{0.46}{\tau^{\sec\psi}} - 0.5 \right) S^c_{dir} \tag{8}$$

Cloud cover can have a significant effect on the amount of solar (short-wave) radiation reaching the earth's surface [20]. Hence, S^c_{dir} and S^c_{diff} are adjusted to S_{dir} and S_{diff}, respectively, in order to account for cloud cover C, i.e., the fraction of sky covered by clouds (in oktas) as follows:

$$S_{dir} = (1 - 0.75C)S^c_{dir} \tag{9}$$

and

$$S_{diff} = S^c_{diff} + CS_{dir} \tag{10}$$

The adjustment is intuitively obvious. Note that the reason for scaling down the cloud cover C by 25% (to $0.75C$) is that clouds do not have zero transmissivity for short-wave radiation. In this case, a 25% transmissivity is assumed. This measured value is strictly speaking for the Australian continent and is variable [20].

Hence, the instantaneous total (direct plus diffuse) solar irradiance at the earth's surface in the presence of a turbid atmosphere and clouds (S_{total}) is obtained by summing Equations (9) and (10) to give

$$S_{total} = S^c_{diff} + (1 + C)(1 - 0.7C)S^c_{dir} \tag{11}$$

It is worth digressing at this point and stating that one utility of S_{total} is in the determination of the atmospheric stability class. Furthermore, in the calculation of the surface sensible heat flux, the incoming total solar energy minus that reflected by the surface is used. This quantity is equal to S' and given by

$$S' = (1 - A)S_{total} \tag{12}$$

where A is the surface albedo defined as the ratio of incoming to outgoing solar radiation flux.

Hence, Equation (12) can be used jointly with a routine for computing the albedo of the relevant surface, e.g., urban surface, in the calculation of the surface sensible heat flux. Further details are given in Section 4.

Cloud cover is either measured or calculated. There are various ways to estimate cloud cover. Alabiso, Parrini and Sidri [21] use a parameter called cloudiness daily coefficient (C_1). It is defined as the ratio of the measured daily total solar flux (J m^{-2}) to that calculated in the absence of clouds. Alternatively, it can be defined as the ratio of the actual number of sunshine hours to that in the absence of clouds, i.e., day length (C_2). Both definitions give a daily average value of cloud cover (as a decimal). Hence, C is then given by $C = 0.8C_i$, where i stands for 1 or 2, which also correspond to versions 1 (Model 1) and 2 (Model 2) of SOLAR, depending on the method of choice for calculating cloud cover. In the following, it will be explained how the daily total solar flux and sunshine hours can be calculated for clear skies.

The daily total solar flux that would reach the earth's surface in the absence of cloud cover or any other atmospheric attenuation, is calculated as follows:

$$E_{clear} = \frac{86,400}{\pi} S_o \beta (h_s \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin h_s) \tag{13}$$

where the factor 86,400 is the number of seconds in a day. All the other symbols used have been defined earlier on. This result can be found in textbooks on solar radiation theory (see, for example, [14]). It is obtained by time integration, from sunrise to sunset, of Equation (1), with τ equal to one (non-turbid atmosphere, i.e., no attenuation of radiation). Note that the latter approximation is made since, otherwise, Equation (1) cannot be evaluated analytically but numerically, which would have prolonged computation time.

If the above-mentioned time integration is carried out, then the day length (number of sunshine hours in the absence of clouds) needs to be known in order to establish the limits of integration. Also, if the cloud cover is calculated from the number of sunshine hours ($T_{sunshine}$), then the latter can be estimated from the following equation:

$$T_{sunshine} = \frac{24}{\pi} \cos^{-1}(\tan\varphi \tan\delta) \tag{14}$$

This can be easily derived by setting $\psi = 90°$ (corresponding to sunset) in Equation (3), which will give an expression for the angular displacement (in radians) from solar noon to sunset. To convert the displacement to time, it is multiplied by $12/\pi$, which, when multiplied by two, gives the day length.

There are other equally simple methods for calculating cloud cover from routine meteorological data, such as that given by Galinski and Thomson [22].

3. Results

The model has been evaluated using data from two sites, namely Sutton Bonington, a predominantly rural site, and another on the outskirts of the town of Loughborough. Both sites are within the East Midlands region of the United Kingdom.

Figure 1 shows the scatter plots of predicted versus observed solar irradiance data for both sites, for each of the two versions of SOLAR, i.e., Model 1 and Model 2. The correlation coefficient is within the range of 0.58–0.61, while the slope of the resulting linear regression analysis is in the range of 0.80–0.93, implying model overestimation.

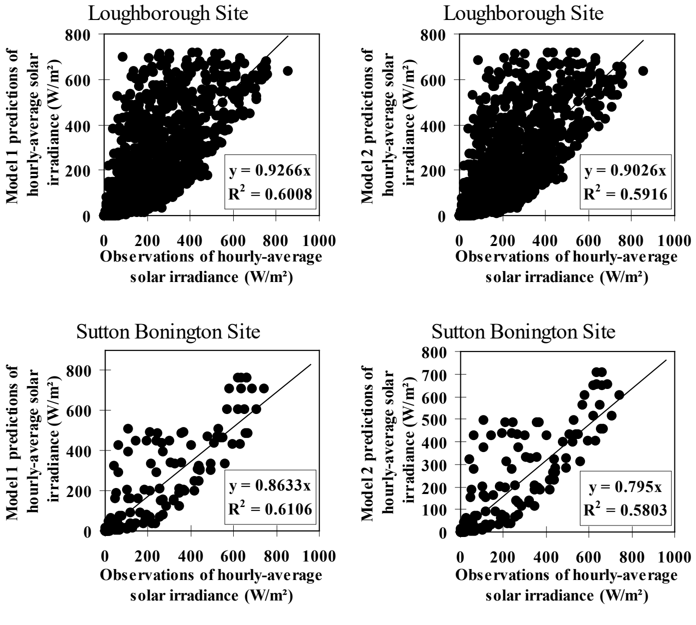


Figure 1. Scatter plots of modelled (predicted) against observed (measured) solar irradiance at Sutton Bonington and Loughborough. Results of linear regression analysis are shown in the graphs. For each site, two sets of modelled results are plotted; one set is generated by the first version of the solar radiation model (Model 1) and the other set by the second version (Model 2).

It is evident from Figure 2 that both versions of the model work better for long sunshine duration and high daily solar flux. This is not surprising since such conditions are equivalent to less cloud cover, reducing the uncertainty in cloud cover estimation. Hence, one can conclude that the model works better for clear-sky conditions.

Figure 3a–c shows a time series of observed and model solar irradiance data for a representative selection of days chosen from different months of the year for each of the two sites. The results can be considered good when one takes into account the simple modelling approach adopted.

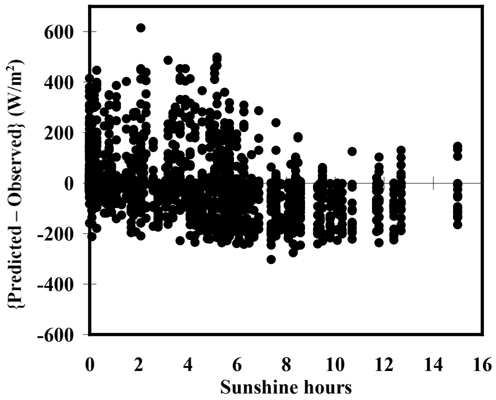
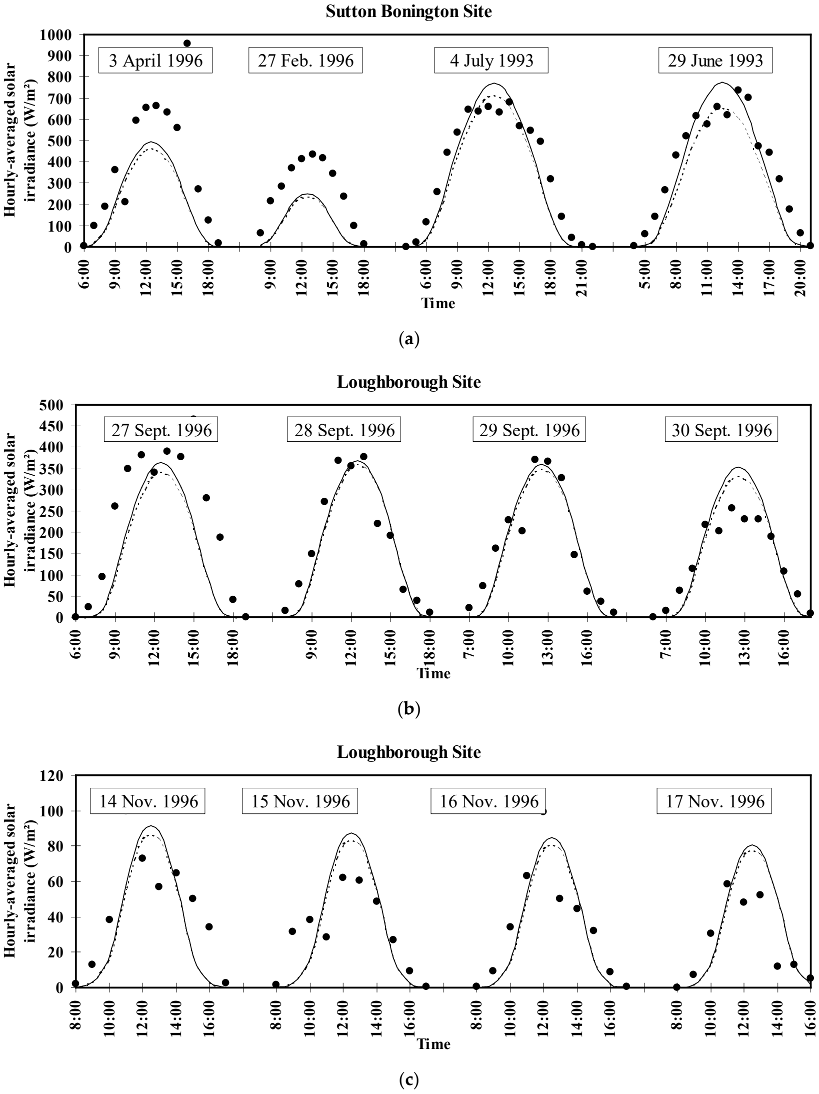


Figure 2. Plot of the difference between modelled (predicted) and observed (measured) solar irradiance against sunshine hours for Sutton Bonington and Loughborough sites taken together.



(**a**)

(**b**)

(**c**)

Figure 3. Time series (on daily basis) of modelled (curves) and measured (data points) solar irradiance at the following: (**a**) Sutton Bonington site on 3 April, 27 February, 4 July and 29 June 1996 (in no particular order); and Loughborough site on (**b**) 27–30 September 1996 and (**c**) 14–17 November 1993. In each case, two sets of modelled results are plotted; one set is generated by the first version of the solar radiation model (Model 1), corresponding to the full-line curves, and the other set by the second version (Model 2), which corresponds to the dotted-line curves.

4. Discussion

The solar radiation model (SOLAR) described in this paper has been implemented within an air pollution model for urban streets. The latter simulates vehicle-derived pollution (specifically, airborne particulate matter) concentration in the confines of street canyons within the urban canopy. The calculated solar irradiance (using SOLAR) is used in two ways in the air pollution model, namely (1) to determine the atmospheric stability class above the urban canopy in order to decide on the use of the correct power-law wind profile and (2) to estimate the surface sensible heat flux which contributes to thermally generated turbulence and mixing. The initial evaluation study of the air pollution model is very promising, as indicative from the linear regression analysis depicted in Figure 4, which clearly justifies the use of SOLAR in the relevant calculations. SOLAR has proved to be very useful in air pollution/quality modelling despite its simplicity.

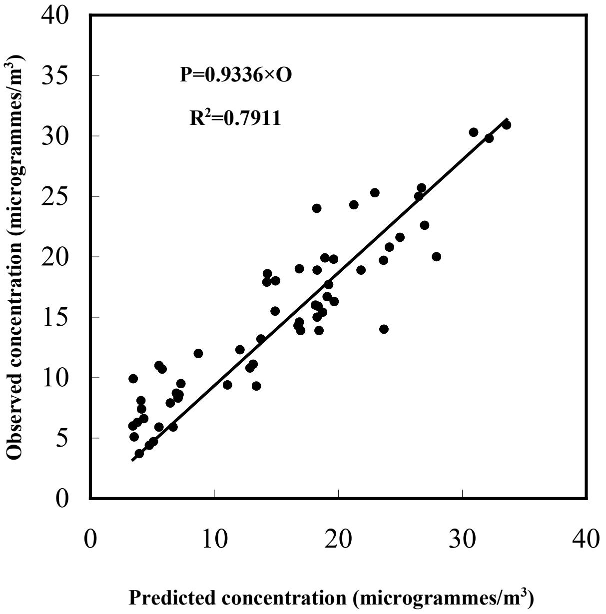


Figure 4. Scatter plot of observed (measured) against predicted (modelled) vehicle-generated airborne particle concentration in a street canyon, including the statistical outcomes of the linear regression analysis.

Furthermore, with reference to the application of SOLAR in the air pollution modelling study, it is worth noting that the use of the solar radiation model is made jointly with a routine for computing the albedo of the street canyon of interest. A limited number of methodologies exist for calculating the albedo of urban locations. Bretz et al. [2] employed a conceptually simple method using estimates of the surface composition of an urban area and the proportion of each land-use category. The method has practical difficulties in that it may require aerial photography. Furthermore, the urban surface is viewed macroscopically, and no attention is given to the altered albedo of different oriented surfaces, such as in the case of street canyons, as evident from the physical model experiments of Aida [23]. Hence, while the method is adequate for large-scale urban studies, it is less appropriate for smaller-scale investigations. Aida and Gotoh [19] describe a two-dimensional model for estimating the urban surface albedo taking into account surface structure. The street canyon radiation model of Arnfield [24,25] has been shown to be capable of producing acceptable estimates of surface albedo for city land-use zones consisting predominantly of street canyons with lengths considerably greater than their width, especially for high irradiance conditions [26]. Another option for estimating the albedo of city street canyons is the numerical scheme of Sievers and Zdunkowski [27].

5. Conclusions

A solar radiation model for the prediction of hourly solar irradiance has been developed. The model is simple in that it utilises a blend of basic theoretical and empirical results and concepts and requires routine meteorological data as input. Preliminary model evaluation has shown that the model works well, especially for long sunshine duration and/or high daily solar radiation flux. The application of the model within an urban air quality model has shown that it is simple to implement and suitable for situations where a high degree of accuracy is not required, and practicality is essential.

In conclusion, it was shown that adopting relatively simple 'textbook' material, i.e., basic theory/information, leads to reasonably good solar radiation modelling, with outcomes that can be used for applications where accuracy is not a major requirement. It may be possible that the paradigm adopted here for solar radiation modelling can be considered for the modelling of other processes, notably those that pertain to the environment, where complexity is a stumbling block, and abstraction is warranted.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All the raw and processed data, as well as the computer code and associated executable version of the program for the model discussed in this paper, are available to anyone for research purposes only. All those interested can contact the author directly via email.

Acknowledgments: The author wishes to thank Loughborough University and the University of Nottingham, both in the United Kingdom, for providing the meteorological data for the Loughborough and Sutton Bonington sites, respectively. The author is grateful and appreciative of the constructive comments made by the reviewers and the editors.

Conflicts of Interest: The author declares no conflict of interest.

References

1. ASHRAE. *Handbook Fundamentals*; American Society of Heating, Refrigerating and Air-Conditioning Engineers: Atlanta, GA, USA, 1993.
2. Bretz, S.; Akbari, H.; Rosenfeld, A. Practical issues for using solar-reflective materials to mitigate urban heat islands. *Atmos. Environ.* **1998**, *32*, 95–101. [CrossRef]
3. Zdunkowski, W.G.; Panhans, W.; Welch, R.M.; Korb, G.J. A radiation scheme for circulation and climate models. *Beitr. Phys. Atmosph.* **1982**, *55*, 215–238.
4. McCullough, E.C.; Porter, W.P. Computing clear day solar radiation spectra for the terrestrial ecological environment. *Ecology* **1971**, *52*, 1008–1015. [CrossRef]
5. Hatfield, J.L.; Giorgis, R.B., Jr.; Flocchini, R.G. A simple solar radiation model for computing direct and diffuse spectral fluxes. *Sol. Energy* **1981**, *27*, 323–329. [CrossRef]
6. Atwater, M.A.; Ball, J.T. A surface solar radiation model for cloudy atmospheres. *Mon. Weather Rev.* **1981**, *109*, 878–888. [CrossRef]
7. Nielsen, L.B.; Prahm, L.P.; Berkowicz, R.; Conradsen, K. Net incoming radiation estimated from hourly global radiation and/or cloud observations. *J. Climatol.* **1981**, *1*, 255–272. [CrossRef]
8. Sherry, J.E.; Justus, C.G. A simple hourly clear-sky solar radiation model based on meteorological parameters. *Sol. Energy* **1983**, *30*, 425–431. [CrossRef]
9. Sherry, J.E.; Justus, C.G. A simple hourly all-sky solar radiation model based on meteorological parameters. *Sol. Energy* **1984**, *32*, 195–204. [CrossRef]
10. Turner, W.D.; Mujahid, A. The estimation of hourly global solar radiation using a cloud cover model developed at Blytheville, Arkansas. *J. Clim. Appl. Meteorol.* **1984**, *23*, 781–786. [CrossRef]
11. Cerquetti, F.; Scuterini, C.; Murri, A. Correlations between total, diffuse and direct radiation and relative duration of sunshine. *Sol. Energy* **1984**, *32*, 557–559. [CrossRef]
12. Rangarajan, S.; Swaminathan, M.S.; Mani, A. Computation of solar radiation from observations of cloud cover. *Sol. Energy* **1984**, *32*, 553–556. [CrossRef]
13. Topcu, S.; Oney, S. The estimation of hourly total irradiation for cloudy sky in Istanbul. *Renew. Energy* **1994**, *4*, 223–226. [CrossRef]
14. Gates, D.M. *Biophysical Ecology*; (Springer Advanced Texts in Life Sciences); Springer: New York, NY, USA, 1980; pp. 104–110.
15. Lunardini, V.J. *Heat Transfer in Cold Climates*; Van Nostrand Reinhold Company: New York, NY, USA, 1981; pp. 212–214.
16. Unsworth, M.H.; Monteith, J.L. Aerosol and solar radiation in Britain. *Q. J. R. Meteorol. Soc.* **1972**, *99*, 778–797. [CrossRef]

17. Kondratyev, K.Y. *Radiation in the Atmosphere*; (International Geophysics Series); Academic Press: New York, NY, USA, 1969; pp. 166–167.
18. List, R.J. *Smithsonian Meteorological Tables*, 6th ed.; Smithsonian Institute Press: Washington, DC, USA, 1971; p. 422.
19. Aida, M.; Gotoh, K. Urban albedo as a function of the urban structure: A two-dimensional numerical simulation. *Bound. Layer Meteorol.* **1982**, *23*, 415–424. [CrossRef]
20. Stafford Smith, D.M.; Noble, I.R.; Jones, G.K. A heat balance model for sheep and its use to predict shade-seeking behaviour in hot conditions. *J. Appl. Ecol.* **1985**, *22*, 753–774. [CrossRef]
21. Alabiso, M.; Parrini, F.; Sidri, R. Estimation of hourly solar radiation on tilted planes from measured daily global radiation on the horizontal surface. In Proceedings of the 2nd International Conference ENVIROSOFT 88, Porto Carras, Greece, 27–29 September 1988.
22. Galinski, A.E.; Thomson, D.J. Comparison of three schemes for predicting surface sensible heat flux. *Bound. Layer Meteorol.* **1995**, *72*, 345–370. [CrossRef]
23. Aida, M. Urban albedo as a function of the urban structure: A model experiment. *Bound. Layer Meteorol.* **1982**, *23*, 405–413. [CrossRef]
24. Arnfield, A.J. Numerical modelling of urban surface radiative parameters. In *Papers in Climatology: The Cam Allen Memorial Volume*; Discussion Paper Number 7; Davies, J.A., Ed.; Department of Geography, McMaster University: Hamilton, ON, Canada, 1976.
25. Arnfield, A.J. An approach to the estimation of the surface radiative properties and radiation budgets of cities. *Phys. Geogr.* **1982**, *3*, 97–122. [CrossRef]
26. Arnfield, A.J. Validation of an estimation model for urban surface albedo. *Phys. Geogr.* **1988**, *9*, 361–372. [CrossRef]
27. Sievers, U.; Zdunkowski, W. A numerical simulation scheme for the albedo of city street canyons. *Bound. Layer Meteorol.* **1985**, *33*, 245–257. [CrossRef]