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Energy-Efficient Control of Underground Gas Storage

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Abstract:

Purpose: *The work devoted to the development of the system of effective dispatching control of the process of operation of underground gas storage facilities. This method connects the parameters of the filtration processes in the reservoirs (with concentrated sources) with the parameters of the discrete-continuous processes that take place in multi-shop compressor stations.*

Design/Methodology/Approach: *The authors formulated the necessary direct and inverse problems for planning the operating modes of the UGS according to effective optimality criteria. To solve them, they developed fast-converging methods and algorithms of minimal complexity, in particular, they applied/developed combinatorial optimization methods for forming optimal operational and predictive operation modes of UGS. This made it possible to achieve the necessary level of efficiency in decision-making by dispatch services. The authors paid special attention to the developed methods of solving problems of flow analysis in a network-type system with various boundary conditions.*

Findings: *The APC was funded by AGH University of Krakow*

Practical Implications: *This approach made it possible to combine various models of processes of different physical nature into a single thermo-hydraulic complex. The article provides examples of solving problems of analysis and planning of UGS operating modes. The qualitative and quantitative characteristics of the developed and tested mathematical models in the real conditions of its operation are described.*

Originality/Value: *The article presents methods and tools for the effective management of underground gas storage facilities.*

Keywords: *Underground gas storage, optimization methods, mathematical methods and software, adaptation, planning of work modes, process management.*

JEL codes: *L71, L95, C6, C88, C93*

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Abbreviations: *Underground gas storage (UGS), gas transmission system (GTS), gas distribution network (GDN), gas gathering station (GGS), compressor station (CS), gas pumping unit (GPU), piping and instrumentation diagram (P&ID), post-compressor station (PCS).*

1. Introduction

Underground gas storage is a complex technological facility. In this research object (as a complex dynamic system), distributed gas-dynamic and filtration processes take place at significant spatial and temporal time intervals. Transient processes of continuous and discrete types in system facilities change over time – from seconds to several months. A set of basic parameters that characterize a separate storage or a group of technologically connected storages is significant.

The main integral operational characteristics of UGS are their "peaking" and optimality according to the fuel-energy criterion. Among the possible options, there are the most effective options for the operation of UGS according to integral indicators, which take into account fuel and energy costs and operational reliability parameters. Optimizing the operation of the gas transmission system together with UGS encourages the development of optimal strategies for the interaction of GTS and UGS for the maximum use of their joint energy-saving potential. The complexity of modeling processes in UGS facilities consists in:

- non-linearity of gas filtration processes in the bottomhole zones of wells;
- diversity of models according to their mathematical representation;
- certain uncertainty, which is generated by the available metrological support and unclear geophysical data;
- solving incorrect mathematical problems.

Mass transfer processes in UGS facilities are described by gas-dynamic models on network-type structures and filtration models in heterogeneous porous multilayered media with geological faults (equations of mathematical physics).

Taking into account the operation of each facility in the technological connection with other facilities makes it possible to manage processes to achieve the given optimality criteria. Representation of the model of the UGS structure in terms of graph theory provides: modeling of all possible variants of the UGS operation under variable initial and boundary conditions; a sufficient level of automation of the formation of the model for the variable topology of the structure of the facility and the implementation of updated models of the operation of the UGS.

Effective operation of UGS requires:

- constant monitoring of the throughput capacity of its technological facilities;
- activation of the optimal amount of technological equipment in operation mode;
- reliable substantiation of ground equipment operating modes – for given productivity and pressures at inputs and outputs of technological equipment, etc.

The above-mentioned and many other problems require prompt and highly accurate solving of operation problems for their inclusion in decision-making systems – dispatch control systems of both individual UGSs and GTS in general. The maximum realization of the available optimization potential (according to the weighted energy criterion and the mode reliability criterion) is possible only with the use of modeling and optimization software.

Optimum operation of UGS is ensured by: optimal operation of compressor stations, wells and gas collection and preparation system; optimal modes of operation of reservoirs; as well as the optimal organization of the interaction of technologically connected UGSs with each other and with main gas pipelines of the GTS, in particular.

2. Analysis of Literary Data and Formulation of the Problem

This work is a continuation of works (Iwaszczuk *et al.*, 2022a; Iwaszczuk *et al.*, 2022b; Prytula *et al.*, 2017b; Prytula *et al.*, 2017c; Prytula *et al.*, 2022), which are devoted to development of models of gas flows in certain technological facilities of UGS (Prytula *et al.*, 2017c), problems of building integrated mathematical models of gas flows in UGS facilities (Iwaszczuk *et al.*, 2022b), development of methods for control the operation of groups of technologically related UGSs in general, and in peak operation modes in particular (Prytula *et al.*, 2022), methods of calculating optimal modes of operation of UGSs (Prytula *et al.*, 2017b).

In open sources, mainly research-type works are presented. Most of them are tested on simple examples that take place in gas fields. However, the path from individual studies to their implementation is quite long and difficult. And that is why only single modeling complexes are actually operated in Europe. Among them, it is necessary to highlight the developments of the SIMONE (SIMONE, 2022) and Schlumberger (SLB) (Schlumberger, 2022) companies, which are used for simulation (in real time) of control processes, network dispatching, optimization of the gas transmission process, and development of the transmission system.

Publications devoted to the analysis of gas pipeline systems of the network type can be conditionally divided into the following groups:

- analysis of networks with stationary gas flows – optimization problems (Yusta and Beyza, 2021; Lochran, 2021; Rodríguez *et al.,* 2014; Gugat and Herty, 2020; KYPipe, 2023; Gyrya and Zlotnik, 2019; Egger, 2018);
- analysis of unsteady modes of operation of gas pipelines (Osiadacz and Gburzyńska, 2022; Esmaeili *et al.,* 2022; Osiadacz and Chaczykowski, 2020; Su *et al.*, 2021);
- methods of implementing flow models in complex flow systems (Ekhtiari *et al.,* 2019; Domschke *et al.,* 2021; Jurek and Iwanek, 2019; Sprangers, 2020);
- methods of analysis of multi-energy systems (Guerra-Fernandez, 2020; Fokken *et al.,* 2019; Vaccariello *et al.,* 2021);
- strategies for the development of multi-energy network systems taking into account the predicted challenges (Devlin *et al.,* 2017; Zhang *et al.,* 2020; Fokken *et al.,* 2020).

The infrastructure of natural gas transmission and distribution systems plays a key role in the transition to a new energy model. This model assumes a high share of energy from renewable sources, which has a positive effect on energy security, stable operation of electric power systems, and protection of the natural environment. In this context, the authors of the study (Yusta and Beyza, 2021) proposed a cooperative EU model to meet gas demand.

To implement the model, they used mathematical optimization of resources and infrastructure. The model considers the dynamic control of underground gas storages, limiting the daily withdrawal depending on the volume of working gas available in each storage. The model's ability to make quick decisions is illustrated in six examples of gas demand related to the cold winter in Europe in January 2017 and hypothetical supply disruptions.

The problem of energy security is particularly important for energy importing countries. It intensified even more in Europe in February 2022, as a result of Russia's aggression against Ukraine.

The European gas market is becoming increasingly dependent on imports (Lochran, 2021). A set of mathematical models of the European gas network (an exhaustive list of developers is given in (Lochran, 2021)) was created to implement these problems and was traditionally used to evaluate supply disruption scenarios. Most existing European gas network models are not detailed enough to analyze changes in supply and demand dynamics, evaluate proposed infrastructure investments and assess the impact of supply disruption scenarios over projected time intervals.

The gas network optimization model for Europe (GNOME) presented in (Lochran, 2021) is a dynamic linear optimization model of the European natural gas network and its exogenous suppliers with a high degree of detail. GNOME represents supply and demand for all EU-27 member states except Cyprus, Luxembourg and Malta.

Great Britain, Norway, Switzerland, Belarus, Ukraine and Turkey are also included in this model. Russia, Southern Corridor suppliers, Qatar, North Africa, Nigeria and America are modeled as supply regions only. GNOME meets each country's gas demand by generating the minimum cost of own gas production, pipeline flows, liquefied natural gas (LNG) imports and storage usage. If demand cannot be met with existing infrastructure, GNOME will offer a cost-optimal investment strategy for pipelines, LNG regasification and additional gas storage capacity.

The model solves the specified problems every month, starting from 2025 until 2040, with a step of 5 years. The capabilities of the GNOME model are demonstrated by analyzing the consequences of failure to complete the future Nord Stream 2 gas pipeline between Russia and Germany. A complete description of GNOME is provided, including input files, equations, and source code.

The results of research on the coordination of the functioning of natural gas and electric power systems are presented in the report (Guerra-Fernandez *et al.,* 2020). The authors proposed a modeling platform to study the interdependence between natural gas networks and electric power networks based on direct current, as well as an economic dispatching model for power and gas systems, its hydraulic model.

They analyzed the cost of operating electricity and natural gas networks, and also pointed to the importance of considering gas pipeline limitations when analyzing the operation of power systems, in the presence of gas generators and variable renewable energy sources. In systems with a high share of renewable energy, the use of gas balances the variability of wind and solar energy, which increases the overall reliability of the energy system.

The given real examples demonstrate the effectiveness of the proposed approach to the balanced operation of energy and gas systems in the case of variability in the functioning of renewable energy sources (Guerra-Fernandez *et al.,* 2020).

A new method of solving gas flow equations through the network in stationary conditions is proposed in the article (Ekhtiari *et al.,* 2019). The author first presented the model through nonlinear matrix equations, and then divided it into linear and nonlinear components. They solve the nonlinear system approximately. Moreover, the nonlinear equations of the natural gas transmission system include the main variables and characteristics of the gas network, focusing on the pressure drop in it. The authors compare the results obtained using the new method with the results of two numerical methods, the Newton-Raphson solution using MATLAB and SAInt.

An overview of various models for describing gas flow in networks is given in (Domschke *et al.,* 2021). The authors pay particular attention to the detailed description of certain components such as valves and compressors. They also considered classes of network models based on purely algebraic relations and energy Port-Hamiltonian models. In addition, they gave a brief overview of the main numerical methods and concepts for solving hyperbolic balance equations.

The article Jurek and Iwanek (2019) presents a model of a real branched gas network, which was created using the GAZNET software. The authors analyzed the operation of a real gas network. The obtained results indicate significant pressure drops and an increase in the flow rate in certain areas of the model, which not only poses a threat to maintaining the necessary gas parameters in the network, but also makes it impossible to connect new consumers to it.

Based on the obtained results, the authors proposed two options for improving the gas network: (1) expanding the network by building two additional sections of the gas pipeline or (2) increasing the number of gas sources by building an additional pressure reduction and measurement unit. The conducted modeling showed that the network changes proposed in both options improve the hydraulic conditions in the network model.

As part of the research (Sprangers, 2020), a dynamic simulation model of the gas network was developed, which uses various strategies for its operation. Such a model makes it possible to obtain the potential power of green gas injection. With the help of the developed model, it is possible to implement and simulate various configurations of the gas distribution network, simulate various gas demand scenarios and consumer profiles, and regulate the pressure at the city station. It can be used to simulate static and dynamic pressure control, as well as to simulate the injection of excess “green” gas (one of the options is hydrogen obtained from renewable sources) into the storage, and from the storage into the network.

The main part of the publication (Osiadacz and Gburzyńska, 2022) is devoted to the analysis of approaches to the modeling of gas pipeline networks in the case of transient (unsteady) processes of gas flow. The transient gas flow in such networks was described by the authors using a set of simple differential equations or equations with partial derivatives, classified as hyperbolic or parabolic.

An important question when modeling pipeline/network operation is which model to choose, hyperbolic or parabolic. The numerical solution of the parabolic model is much simpler. However, in some cases, parabolic models describe network change less accurately than hyperbolic models. Time, which is often limited, is also important for analysis and operational decision-making. Therefore, for the calculation of non-stationary parameters of gas flows, not only effective models are needed, but also high-speed algorithms for their implementation.

The work of Esmaeili *et al.* (2022) is devoted to the development of an innovative basis for dynamic modeling of natural gas transmission networks. Due to its flexibility, this tool can be used to support the operation of the network. It also provides an opportunity to explore and test new optimized scenarios, introduce new elements to the network itself (from new branches to new machines) and test the use of new gas mixtures. In order to test their own development, the authors used the Italian high-pressure gas transmission network, on the example of which they simulated previous daily operation scenarios. They compared the results with real measurements to confirm the correctness of the adopted approach.

In the article by Osiadacz and Chaczykowski (2020), the authors consider the technical problems associated with maintaining the stable operation of the gas distribution system. They describe models and selected methods for the analysis of steady-state and transient processes of gas networks that are relevant to the current debate in the field of multi-energy systems. The article presents and discusses the problems of modeling a real large-scale gas distribution network.

The paper by Fokken *et al.* (2019) presents a mathematical apparatus that allows the integration of gas and electric networks. The authors describe the dynamics of the gas flow using the Euler isentropic equation, and the power flow equation is used to model the power grid. Appropriate numerical methods are used in the models. The work also presents the results of experimental studies – how the use of gas for electricity production can affect the dynamics of the gas and energy networks, respectively.

Article by Vaccariello (2021) is devoted to the modeling of gas networks for use in an integrated scenario with several energy carriers. The authors used a simplified description of the gas network based on graphs. Models of gas flows in pipelines, compressors and pressure reduction stations, they easily derived based on assumptions about their steady-state operation and isothermal behavior. The obtained results proved that the proposed approach can become a practical tool for gas networks, which can be easily integrated into a joint modeling system.

The authors of the publication Devlin *et al.* (2017) presented the first multi-vector energy analysis of the interconnected energy systems of Great Britain and Ireland. Moreover, both systems use wind energy, and Ireland imports gas from Great Britain. The authors have developed a fully realistic economic dispatching model, which is combined with a model of energy flows in the gas supply network.

To increase the load on the gas supply network, the authors added extreme weather events (which lead to increased gas demand) and low wind speeds. The paper shows that (1) overloading the gas entry node in the Irish system led to increased short-term costs for generators by 40%, (2) gas storage reduces the impact of congestion caused by high demand, providing a 14% reduction in total generation costs over the study period and a reduction in electricity imports from the UK.

The work by Zhang *et al.* (2020) is devoted to operation and coordination of electricity and natural gas networks. For the gas-electric combined system, the authors proposed interval optimization, which is built on the basis of a coordination planning model. At the same time, they took into account the dynamics of the flow of natural gas, the integration of wind energy and the management of the response to the demand for energy resources.

The article also presents the results of a comparison of the used methods of stochastic optimization and robust optimization. The obtained simulation results demonstrate the effectiveness of the proposed interval optimization method.

The work by Rodriguez *et al.* (2014) is devoted to the problems of optimizing the gas network during its operation. Optimization refers to the quantity and quality of natural gas flow, minimizing the energy consumption of transmission, provided that consumers are supplied with gas. To minimize fuel gas consumption by compressors, the authors used the sequential quadratic programming (SQP) algorithm.

In Gugat and Hery (2020), mathematical results from the analysis of gas flows in the network are considered with an emphasis on the issues of their control and stabilization. The main attention is paid to models for spatially one-dimensional flow governed by the hyperbolic balance law. The results of controllability of gas flows, taking into account the inclusion of existing uncertainty in models and numerical methods, are given.

The authors of the publication by Fokken *et al.* (2020) proposed a tool for modeling and optimization of gas pipeline networks connected to power grids and, accordingly, to gas power plants. The proposed model consists of the isentropic Euler equations used to describe the gas flow in combination with the alternating current flow equations. Moreover, the compressor station provides gas pressure control, subject to compliance with certain technological limits.

The authors presented a numerical study that demonstrates the impact on gas pipelines of rapid changes in electricity demand and what actions the operator should take to ensure the optimal operation mode. They model the gas dynamics in each pipeline using Euler's isentropic equations supplemented with appropriate coupling and boundary conditions. They use well-known power flow equations to build a model of power grid sections.

They model the connection between gas and energy networks at gas power plants with (algebraic) demand-dependent gas consumption conditions. To respond to demand-dependent impacts on the gas network, compressor stations are used in the gas network. The purpose of the work is to fulfill the specified pressure limits and at the same time to minimize the consumption of fuel gas or electricity by the compressor stations.

The work KYPipe (2023) provides a description of the KYPipe software complex, which has been constantly updated and maintained for over 40 years and is the most widely used and reliable mechanism for hydraulic analysis in the world. The functionality of KYPipe provides simulation of flows of water, oil, petroleum products, chemicals, refrigerants, as well as flows in low-pressure sewage systems, etc. It can be used to select and size pipes, pumps, valves, tanks and other devices. Calibration tools and pump optimization features help ensure sound simulation.

With the development of gas generators, more and more gas-electric systems generate pressure or flow rate fluctuations in gas pipeline networks. Such fluctuations are the result of jumps in gas consumption by gas generators.

In Su *et al.* (2021), a hybrid approach to the modeling of gas networks is proposed. This approach can make full use of physical laws and measurement data to characterize the transient behavior of gas flow in pipeline networks. Taking into account the fact that the number of model parameters increases with the size of the gas network, the authors developed a decentralized algorithm for identifying model parameters, which increases the efficiency of calculations.

They divided the general network into a certain number of non-overlapping subnetworks, and for each of them, parameter identification was performed as a nonlinear optimization problem, solved using the Levenberg-Marquardt iterative algorithm. They compare the simulation results with the commercial simulation software Apros™. The obtained results showed that the proposed method significantly improves the accuracy of modeling transient processes of the gas network.

The paper by Gyrya and Zlotnik (2019) presents a numerical model of natural gas transmission in pipeline networks. Such a model guarantees that the mass conservation condition is fulfilled exactly. The authors formulated the mathematical model in terms of density, pressure and mass flow variables, which made it possible to use the general equation of state to determine the relationship between gas density and pressure for a given temperature. They compared the proposed checkerboard method with the explicit operator splitting method and lumped element scheme.

They described numerical experiments they performed to test the order of convergence of the new approach as a function of discretization. The authors also investigated the influence of the non-ideal equation of state and temperature models on the modeling of the pipeline with boundary conditions at different time and spatial scales.

Egger (2018) considered the numerical approximation of the compressible flow in the pipeline network. They formulated the appropriate conditions that make it possible to derive the variational characteristic of solutions and to prove the laws of global balance for the conservation of mass and energy in the entire network.

He subjected this variational principle (which became the basis of further research) to the appropriate Galerkin approximation by mixed finite elements. The author analyzed the correctness of the completely discrete scheme of the gas flow model and proposed an iterative method for solving nonlinear systems of equations that arise at each individual time step. He presented some computational results to illustrate the theoretical conclusions and to demonstrate the reliability and accuracy of the new method.

3. Formulation of the Problem

Decarbonization of energy production, which has become a global trend, has led to the creation of multi-energy systems. Such systems, in turn, require the development of network-type modeling systems with flows of different physical nature. In the stationary case, the flow distribution in such networks is governed by Kirchhoff's laws.

And that is why the operation and development of underground gas storages, transport and distribution gas and energy systems are closely related to each other. In such systems, similar problems related to energy accumulation arise. One of the economically feasible ways to solve them is the use of underground gas storages and depleted hydrocarbon deposits.

And that is why it is urgent to develop integrated modeling and optimization systems of complex network-type systems that will ensure balanced development and efficient operation of multi-energy systems.

4. The Purpose and Objectives of the Research

The purpose of this study is to develop convergent methods and algorithms of minimum complexity for calculating the parameters of gas flows in reservoir and gas gathering systems, which is presented in the form of a hydraulic distribution network. This will make it possible to model and plan the operation of the UGS in the modes of gas injection and withdrawal.

To achieve the goal, we formulated the following problems:

- to develop methods for solving problems of flow analysis in a network-type system (“bottomhole zones – wells – gas gathering systems”) with various boundary conditions that connect the parameters of filtration processes in reservoirs with concentrated sources (wells);
- to develop methods of operational and predictive solving of direct and inverse problems for dispatching control of gas injection and withdrawal processes.

5. Materials and Methods of Implementing Models, their Adaptation and Research

The object of the research is underground gas storage facilities as part of the gas transmission system. The subject of the research is the development of methods and algorithms, as well as (based on them) software tools for the effective operation of underground gas storage facilities as part of the gas transmission system.

In parallel, we built models of gas flows in UGS facilities and searched for methods of their implementation. We adapted the models based on the results of measuring the parameters of gas flows in facilities – at their inputs and outputs. In order to build an integrated model of the UGS, we tried to build algorithms for matching flow parameters at the outputs of some facilities with the inputs to others. It turned out that the process of coordinating flow parameters between the gas collection system and the wells (there were more than 300 of them) was significantly unstable.

The accuracy of matching flows, flow and pressure, was beyond the accuracy of their measurement. In this case, we had the only possible way out – to combine the gas collection system models and well models, as well as add nonlinear models of gas inflows to the wells and present them as a gas distribution network (GDN). The model developed in this way complicated the process of developing a method for its implementation.

It was necessary to develop a method for solving a system with various types of nonlinear equations with the following boundary conditions – gas pressure or flow rate at the gas collection station and reservoir pressures on well supply circuits or well flow rates. To develop a method of hydraulic calculation of the obtained GDN, we used the experience of developing methods for calculating the parameters of gas flows in the GTS with compressor stations (Prytula *et al.*, 2022).

The developed method made it possible to formulate a set of optimization problems for the calculation of direct and reverse operational dispatching problems and to solve them effectively.

Incoming data:

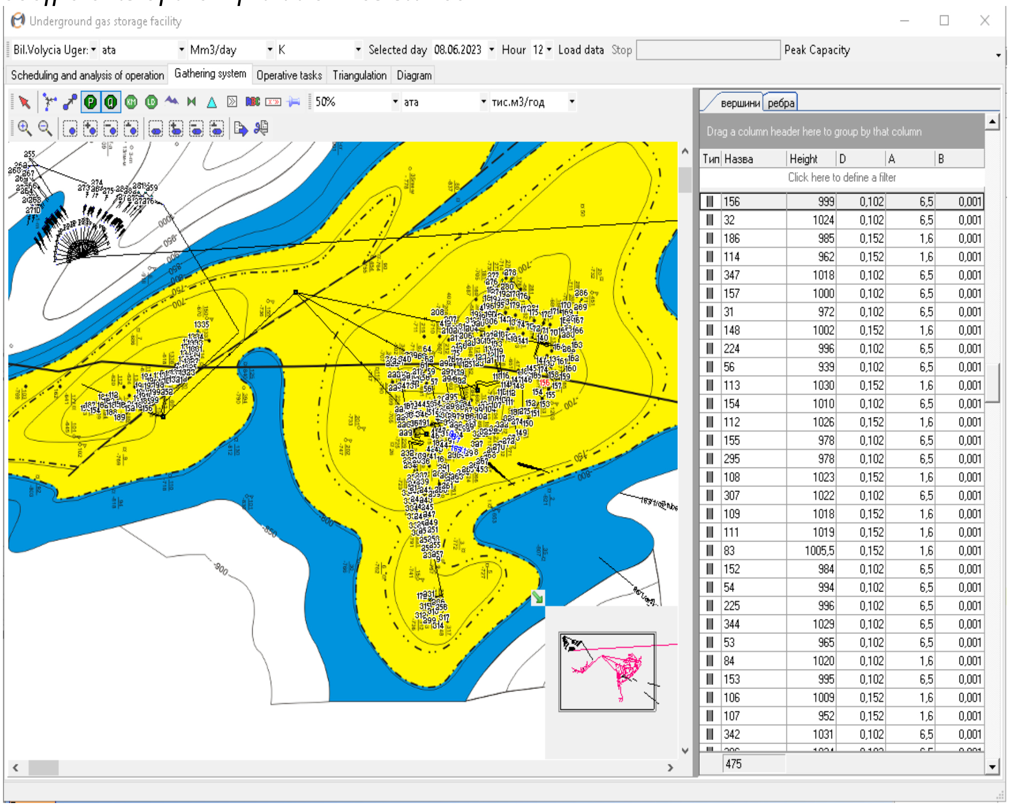
- results of studies of wells in gas injection and withdrawal modes for ten years (reports of geological and technological operation of underground gas storages);
- information bases of measured data (accounting and analytical system of the gas transmission enterprise).

We received additional input data as a result of numerical experiments using GIMS and GTS Calculation software (Iwaszczuk *et al.*, 2022a; Prytula *et al.*, 2017b), which were already involved in the process of solving mode-technological problems for planning operational modes of UGS operation. Realized models of filtration and gas dynamic processes are given in (Iwaszczuk *et al.*, 2022a; Prytula *et al.*, 2017c). They provide the solution of a set of direct and inverse problems for the calculation and analysis of UGS operating modes.

6. Hydraulic Calculation of the System "Bottomhole Zone – Well – Gas Gathering System"

The system "bottomhole zone – well – gas gathering system" is presented in the form of a gas distribution network, in which the bottomhole zones of wells (in the shape of a circle) are uniform in permeability and are represented by an edge – a radius vector. This type of edges are described by parameters: length (radius of the gas inflow area) and permeability, or filtration resistance coefficients. The vertices of the edge connect the reservoir and the well bottomhole zone (one of the vertices is characterized by pressure on the contour of the bottomhole zone, and the other by pressure on the top of the bottomhole zone). Figure 1 shows a fragment of the gas gathering system.

Figure 1. *Structure of the gas gathering system, coordinates of well placement and coefficients of their filtration resistance*



6.1 Calculation of the "Well-Reservoir" System

Most well research methods assume the isotropicity of the reservoirs of their bottomhole zones and do not take into account the heterogeneity of the reservoirs in terms of porosity, permeability and the structure of the porous space. The perceptible influence of many factors manifests itself after dozens of days of operation. Their change during the day is within the accuracy of measurement by the metrological system. The supply area of each well is assumed to be circular. The radius of such a circle is approximately equal to half the distance to the nearest well. The circular zone, in the center of which is a well and in which Darcy's linear law is violated (Sanchez, 1980), is called a bottomhole zone.

All hydrodynamic methods of research, which are used to determine the filtration parameters of the reservoir and the well, are divided into two main groups:

1) methods based on the study under a stable filtration operation mode;
2) well research methods in non-stationary filtration operation modes (processing of curves: – restoration of bottomhole pressure after stopping the well; – stabilization of pressure and flow rate after starting the well).

The first group is based on the measurements of bottomhole pressures and flow rates during the steady process of gas filtration in reservoirs. The essence of this method is to establish the relationship between the constant flow rate of the well and the value of its bottomhole pressure. This dependence determines such important characteristics as the productivity coefficient of the well and the hydraulic conductivity coefficient of the formation.

The pressure recovery method belongs to the second group of hydrodynamic research methods (Iwaszczuk *et al.*, 2022b). This method is based on monitoring the bottomhole pressure and flow rate of a working well that is suddenly stopped. As a result of research using this method, three graphs are obtained: the curve of the flow rate change of the well during its operation; the curve of the flow rate change of the well after its stoppage; the recovery curve of the bottomhole pressure after stopping.

Based on the analysis of the obtained curves, not only the average values of the filtration characteristics in some area of the reservoir are determined, but also their changes at some distance from the well, namely the distance to the place of a sharp change in the filtration characteristics of the reservoir. This makes it possible to specify various types of inhomogeneities of the reservoir, to detect the presence of impermeable and permeable boundaries of the reservoir, places of flow between reservoir layers and probable impermeable screens, the condition of the bottomhole zone, etc.

It is still necessary to distinguish between analytical and approximation methods. Each of these methods has its own advantages and disadvantages. The second method is most often used to study wells. To use the first method, it is necessary to specify the factors affecting the flow rate of the wells in advance. The main influencing factor is the vortex drag, which has not yet been sufficiently investigated. Various variants of its presentation are known, mainly for gas fields (Prytula *et al.*, 2017c).

The following key problems arise during the operation of UGS reservoirs:

- identification of factors affecting the flow rate of wells;
- study of the nature of the impact of some factors on the flow rate and depression (repression) of pressure in the area of wells' bottomhole zones, during changes in their operation mode;
- development of well research methods that are adapted to the operation of UGS reservoirs;
- a meaningful interpretation of the filtration resistance coefficients of the bottomhole zones of the wells during the operation of the wells;
- establishment of a functional relationship between filtration resistance, flow rate of wells, reservoir pressure and depression (repression) in the bottomhole zone of wells.

To assess the complexity of the solutions to the listed problems, we will conduct numerical experiments with the aim of preliminary analysis of the behavior of reservoir pressure in the working zone, flow rate and gas pressure at the gas gathering point.

It is the established functional relationship of the specified parameters that makes it possible to calculate the pressure on the gas gathering station (GGS) with the required accuracy. This pressure is one of the boundary conditions for calculating the operation modes of compressor stations.

We will consider that the adaptation of the reservoir – GGS model is successful if, based on the calculated reservoir pressure in the working zone of the wells, it is possible to calculate (with satisfactory accuracy) the pressure at the gas gathering station.

The results of adaptation are shown in Figure 2 where:

NW_{ells} – the number of wells in operation;
$PG_{qs}Cl_c$ – measured average reservoir pressure of gas in the area of operation of the wells, ata;
PR_{sr} – measured average reservoir pressure of gas in the area of operation of the wells, ata;
PG_{qs} – measured gas pressure at the gas gathering station, ata;
$Pl_{qs}M$ – calculated gas pressure at the gas gathering station, ata;
Q – flow rate of gas injection / extraction, mln m^3/day;
$A_{ctive}G_{as}$ – volumes of active gas, mln m^3.

Adaptation of the reservoir-GGS model was carried out based on the findings $p_i(t) = f(q_i(t,k), t, p_i^c)$. As we can see, under the condition of significant changes in the flow rate of gas withdrawal, the measured PG_{qs} and calculated pressures $PG_{qs}Cl_c$ at the gas gathering station are close over a significant interval of the UGS operation time.

The main stages of calculating the pressure at the gas gathering station based on the flow rate of gas withdrawal/injection:

1) calculation of the pressure distribution $p(x,y)$ in the gas reservoir $(x,y) \in \Omega$;
2) calculation of the pressure $p_i(t)$ in the bottomhole zone as a function $p_i(t) = f(q_i(t,k), t, p_i^c)$;
3) $(i = 1, \dots, n)$, where $q_i(t,k)$ is the flow rate of the ith well based on the permeability of the bottomhole zone of the well k, p_i^c – is the reservoir pressure on the contour of the region of gas inflow to the i-th well;
4) pressure calculation $p_i(t), (i = 1, \dots, n)$ calculation of gas flow parameters in the distribution network formed by wells and the gas gathering station. One of the vertices of such a network is a gas gathering station (Figure 3).

Figure 2. *Calculated and measured operational data of the underground gas storage.*

The coincidence of the measured and calculated pressures on the GGS is slightly different at the initial time interval of the simulation of the gas filtration process. This is due to the fact that the initial input data are averaged (not distributed over the entire area of the reservoir). In the process of simulation, the system independently reproduces the reservoir pressure distribution and, after a certain time, is ready to calculate the predicted modes of operation of the UGS.

In order to ensure the speed of calculation of pressures on the GGS, in the process of adapting the reservoir-GGS model, we integrally took into account the main factors affecting the flow rate of wells. Among the main influencing factors, we included:

– mechanical skin effect is a general skin effect, or Darcy skin (Kondrat *et al.*, 2020);
– dynamic skin effect, which depends on the productivity of fluid inflow to the bottomhole (Kondrat *et al.*, 2020);
– type of well termination and opening of its bottomhole;
– mutual influence of wells (interference of the distribution of pressures in the bottomhole zones);
– the anisotropy of bottomhole zones in terms of permeability and porosity,
– variability of gas withdrawal/injection flow rates;
– the structure of the porous medium around the well zone of the reservoir;
– the nature of the connection between the flow rate and filtering resistances, etc.

The calculation of the filtration resistance coefficients of the bottomhole zones of wells is affected by their evaluation method and the incomplete consideration of the factors influencing the flow rate, in particular, the skin factor. The given models of facilities (Prytula *et al.*, 2017c) make it possible to analyze the influence of factors on the flow rate of wells. Among the main factors, it is necessary to highlight the following:

o well supply circuit,
o permeability of the near and far supply zones of the wells,
o vortex in the bottomhole zone,
o anisotropy of the bottomhole zone and spatial anisotropy of the reservoir,
o skin effect,
o mutual influence of wells,
o density and parameters of perforation channels, etc.

The possibility of evaluating the factors is affected by a high degree of uncertainty and a complex hydraulic connection between the influencing factors, which confirm the results of the wells we studied (Table 1).

In Tables 1, 3 and 4, coefficients A and B are the coefficients of filtration resistances, which are included in the equation of the binomial formula of gas inflow to the gas well bottomhole (Iwaszczuk *et al.*, 2022a), which characterizes the dependence of reservoir energy losses on flow rate. Aq – corresponds to pressure losses caused by viscous forces, Bq^2 – corresponds to pressure losses caused by inertial forces, and q – flow rate of the well.

Table 1. *Flow rate of wells for filtration resistance coefficients established from measured data in different years of UGS operation (2014-2015)*

Well number	**Coefficients of filtration resistance A/B**	**Flow rate of wells (m^3/s) at different reservoir pressures and pressures at the GGS (kgf/cm^2)**					
		25		**35**		**45**	
		20	**25**	**20**	**30**	**30**	**40**
172	0.1849/0.0024	0.942	0	2.012	1.075	2.330	1.136
	0.2796/0.0051	0.884	0	2.893	1.002	2.193	1.059
147	0.1718/0.0023	0.953	0	2.019	1.079	2.337	1.140
	0.3724/0.0064	0.849	0	1.833	0.963	2.126	1.018

Source: *Own study.*

The analysis of the results (Table 1) showed that, on average, the area of uncertainty in the flow rate of wells is 7-12%. Therefore, the filtration resistance coefficients are not stable and reliable characteristics of the bottomhole zones of wells in underground gas storages.

The flow rate of the wells significantly depends on the area of the open area of gas filtration to the wells. There are several ways to increase such an area – by drilling wells and perforating the casing column. There is a nominal value of the perforation density, after which increasing the perforation density becomes impractical.

Cumulative perforators, which create holes in the casing column and rock, with the help of focused jets of gases arising during the explosion of cumulative charges, have become the most common. However, there is no guarantee of full disclosure of the near-bottomhole zone, and therefore additional perforation is often performed. During numerical experiments, we approximately established the density of perforation channels for certain wells in the Bilche-Volitsky UGS.

On the basis of the developed models, we proposed to increase the perforation density to a justified value, which was implemented during the repair work on the wells. In Tables 2-4, we presented the results of the influence of changes in the density of additional perforation on the flow rate of wells (Prytula *et al.*, 2017c).

The analysis of the simulation results showed that there is an economically justified value of the perforation density, after increasing which the flow rate of the wells does not change.

Table 2. *Flow rate of wells (m^3/s) for different values of additional perforation density*

Well number	**0**	**5**	**10**	**20**	**30**	**40**
201	0.917	1.053	1.131	1.209	1.268	1.307
202	0.917	1.034	1.112	1.190	1.248	1.288
203	1.014	1.131	1.209	1.288	1.346	1.366
204	1.658	1.658	1.658	1.658	1.678	1.678
206	1.268	1.288	1.288	1.288	1.307	1.307
208	1.444	1.483	1.522	1.522	1.541	1.561
209	0.897	1.014	1.092	1.170	1.229	1.248
210	1.209	1.248	1.307	1.346	1.385	1.405

Source: *Own study.*

Table 3. *Coefficient of filtration resistance (A) for different values of the density of additional perforation*

Well number	**N_1**	**N_2**					
		0	**5**	**10**	**20**	**30**	**40**
202	20	0.7734	0.7126	0.6784	0.636	0.6084	0.5878
203	20	0.6961	0.6409	0.6097	0.5712	0.5461	0.5274
204	20	0.2798	0.2555	0.2418	0.2249	0.2138	0.2056
206	20	0.3798	0.3479	0.3298	0.3075	0.293	0.2822
208	20	0.3588	0.3284	0.3113	0.2901	0.2763	0.2660
210	20	0.5624	0.5237	0.5001	0.4697	0.4494	0.4341

Source: *Own study.*

Table 4. *Coefficient of filtration resistance (B) for different values of the density of additional perforation*

Well number	**N_1**	**N_2**					
		0	**5**	**10**	**20**	**30**	**40**
202	20	0.0273	0.0128	0.0084	0.005	0.0035	0.0027
203	20	0.0225	0.0106	0.0069	0.0041	0.0029	0.0023
204	20	0.0044	0.0021	0.0013	0.0008	0.0006	0.0004
206	20	0.0076	0.0036	0.0023	0.0014	0.001	0.0008
208	20	0.0068	0.0032	0.0021	0.0012	0.0009	0.0007
210	20	0.0124	0.0067	0.0045	0.0028	0.0023	0.002

Source: *Own study.*

In addition, we carried out numerical experiments on the influence of the radius of the drilled bottomhole of wells on the flow rate of a single well, as well as on its flow rate in a group of working wells. If the flow rate for a single well is increased by 2.5 times in wells with a drilled bottomhole (we conducted research for radii up to 40 cm), then its flow rate during operation as part of all working wells will increase by only 25%. This result confirms that in complex systems, the productive potential of some facilities and the influence of some factors on this potential can be assessed only as part of the entire system (in such cases, a synergistic effect is manifested).

6.2 Calculation of the Technological Chain "Gas Gathering Station – Reservoir" as a Single Hydraulic Complex

The piping and instrumentation diagram (P&ID) of the gas gathering system, wells, and bottomhole zones of wells are presented in terms of graph theory – a graph $G(V,E)$.

The graph $G(V,E)$ includes the following types of edges:

- edge-radius of the circular region of gas inflow to the well,
- edge-well,
- edge-hydraulic equivalent,
- edge-pipe of the gas gathering system.

The main vertices of the graph: the gas gathering station and the vertices on the contour of the inflow area – reservoir pressure or flow rate of the wells (diagram in Figure 3).

At the vertices of the graph diagram $G(V,E)$, gas flows satisfy the conditions of Kirchhoff's first law:

mass flow balance equation

$$\sum_i m_{ij} + \sum_k m_{jk} = 0, j \in V, \tag{1}$$

heat balance equation

$$T_J \sum_k q_{jk} - \sum_j q_{ij} T_i = 0, j \in V. \tag{2}$$

We believe that the mathematical models of gas flows in facilities are updated. As initial conditions, you can take:

1) measured average reservoir pressure in the working area of the wells;
2) consumption at a gas gathering station;
3) gas pressure at the gas gathering station.

If the gas consumption calculated (according to the pressure at the gas gathering station) coincides (with the necessary accuracy) with the measured one, then the system is generally updated and ready for solving operational problems.

The identification of the parameters of the integrated model can be ensured in two ways:

- the measured gas parameters can be taken as input data – the pressure at the gas gathering station and the average reservoir pressure in the working area of the wells;
- if the reservoir filtration model is up-to-date, then it is worth using the calculated pressure on the contours of the bottomhole zones of the wells.

Calculation of the pressure at the gas gathering station based on the variable gas flow rate (with the required accuracy) guarantees the accuracy of the calculation of the operation modes of the CS. In addition, it makes it possible to form the optimal operation of the compressor station. In this way, the efficiency of managing the flow rate of gas withdrawal (injection) over time is ensured.

In a mathematical sense, the calculation of flow parameters in $G(V, E)$ (in the general case) comes down to the need to solve systems of nonlinear equations. Usually, it is not possible to find their solutions by analytical methods, even for significantly simplified schemes.

For the use of numerical methods to be effective, we must conduct a preliminary analysis of the input information regarding the influence of their reliability and accuracy on the convergence of the used computational procedures. Today, there are no convincing numerical methods for solving systems of nonlinear algebraic equations. Often the instability of the iterative process is influenced by:

- linearization of the original nonlinear equations (possible appearance of ill-conditioned matrices);
- machine error;
- impossibility of establishing the nature of changes in model parameters;
- quality of input data measurement in terms of accuracy, frequency, synchronicity, etc.

We can apply two approaches to determining the flow distribution parameters in the technological chain of the system $G(V, E)$:

1) construct a system of balance equations;
2) construct a system of contour equations.

Both of these approaches are based on the use of Kirchhoff's first and second laws. Let's consider them in turn.

6.2.1 Construction of a system of balance equations

The hydraulic calculation of the gas distribution network can be carried out using several methods. One of them requires the construction of a system of balance equations in the form (Khymko *et al.* 2023):

$$
\begin{aligned}
&c_{11}q_1 + c_{12}q_2 + \cdots + c_{1n}q_n = q^1 \\
&c_{21}q_1 + c_{22}q_2 + \cdots + c_{2n}q_n = q^2 \\
&\ldots \\
&c_{m1}q_1 + c_{m2}q_2 + \cdots + c_{mn}q_n = q^m \\
&c_{(m+1)1}S_1|q_1|q_1 + c_{(m+1)2}S_2|q_2|q_2 + \cdots + c_{(m+1)n}S_n|q_n|q_n = \Delta p_1 \\
&c_{(m+2)1}S_1|q_1|q_1 + c_{(m+2)2}S_2|q_2|q_2 + \cdots + c_{(m+2)n}S_n|q_n|q_n = \Delta p_2 \\
&\ldots \\
&c_{n1}S_1|q_1|q_1 + c_{n2}S_2|q_2|q_2 + \cdots + c_{nn}S_n|q_n|q_n = \Delta p_n
\end{aligned} \tag{3}
$$

where

m – the number of vertices;

n – the number of sections in the network graph;

q_i – consumption in the pipe i;

$c_{ij} = 1, -1, 0$ is determined taking into account the orientation of gas flow and the orientation of the pipeline;

q^i – inflow or outflow to the top i;

Δp_i – pressure difference on the way to the i-th vertex;

S_i – equivalent resistance.

For the method of successive approximations, we present system (1) in the form $C(x)x = B$. System (1) has a unique solution. Methods of successive approximations with inertia are used to solve this system. The solution is found as the limit of iterations

$$C\left(\sum_{j=i-k}^{i} \alpha^{(j)} q^{(j)}\right) q^{(i+1)} = B, k \geq 0,$$

where $q^{(j)}$ is the approximation of the solution at the j-th step, $\sum_j \alpha^j = 1, \alpha^{(j)} \geq 0$.

Additional inertia terms make it possible to significantly expand the area of convergence of the method. For a certain selection α or sets of α, the method can be equivalent to one of the Newton-type methods (Atkinson, 1989).

6.2.2 Algorithm for constructing a system of contour equations for a network with active facilities

We set ourselves the problem of developing algorithmic support for automating the process of building a model (equation system) of gas pipelines (distribution, gas transmission with active facilities, etc.) with variable network topology. For this, the following algorithms were developed:

- an algorithm for analyzing the topological properties of the structure of P&IDs and dividing it into sub-diagrams according to a given feature;
- an algorithm for the synthesis of P&IDs or its modification;

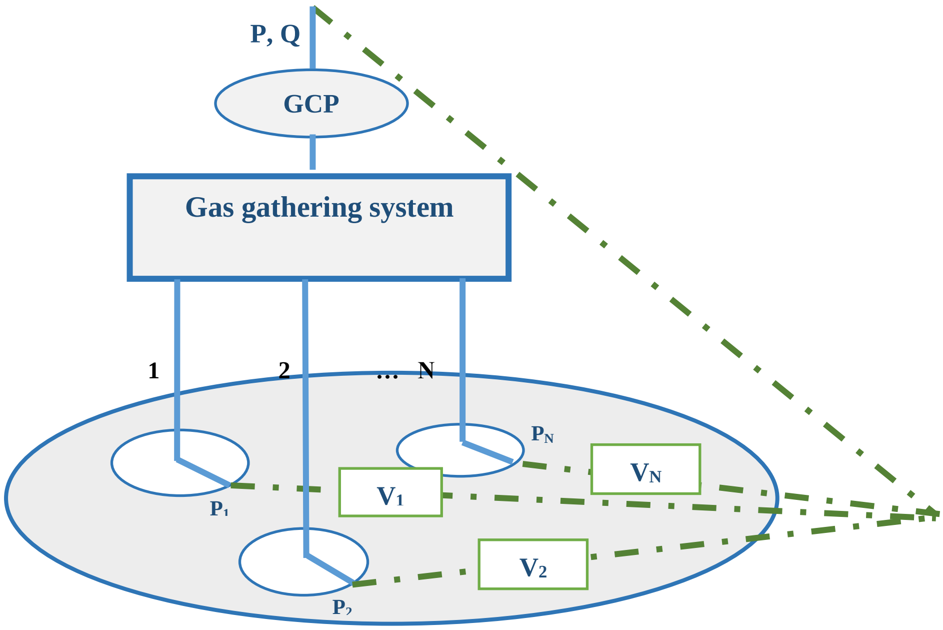
– an algorithm for analyzing the correctness of the input data specification (among the set of specifications), which ensure the fulfillment of the conditions for a single solution of the system of equations.

To implement the task, we developed the following algorithms on graph diagrams:

– the algorithm for establishing graph connectivity and selecting all connected components;
– algorithm for establishing isomorphism of graphs;
– minimal spanning tree construction algorithm;
– the algorithm for finding the shortest paths according to the given criteria;
– the algorithm for developing the minimum number of fundamental contours in the graph;
– the algorithm for selecting subgraphs based on a given feature (belonging to, presence of this or that parameter, etc.);
– algorithm of static and dynamic interpretation of modeling results.

To calculate the GGS-GL gas network, we used the method developed for planning the operating modes of gas transmission systems with active objects (compressor stations) (Prytula et al., 2017a). We presented the diagram of the technological chain "reservoir – gas gathering station" in the form of a gas network with fictitious facilities (highlighted in green) in the following way (see Figure 3).

Figure 3. *Scheme for the method of hydraulic calculation of the "reservoir – gas gathering station" system.*



Source: *Own study.*

To use the proposed method of hydraulic calculation of the GCP-GL network (Prytula, 2009), it is necessary to add fictitious edges of zero length (indicated by dashed line) and virtual active facilities $V_1, V_2, \dots, V_N$ to the GCP-GL P&ID. It is known (Prytula, 2009) that the gas transmission system with active facilites has a single solution for the following input data:

- one of the data – inlet pressure, outlet pressure, flow or gas compression ratio (inlet and outlet pressure ratio) for each active facility;
- gas consumption at gas inflows and withdrawals and gas pressure at one of the vertices.

For the hydraulic calculation of such a diagram, the possible options for specifying input data are:

- gas flow rate or pressure (P, Q) on the gas reservoir and reservoir pressures $(V_1, V_2, \dots, V_N)$ on the contours of the gas inflow areas to the wells;
- gas flow rate or pressure at the GGS and flow rates of wells or pressures on the contour of the area of gas inflow to the wells.

The task of calculating the distribution of steady-state flows in gas networks with circuits is reduced to finding such distribution of flow rates in all sections of the network, which ensure the fulfillment of Kirchhoff's first law for all vertices of the network and the fulfillment of Kirchhoff's second law for all independent circuits (Prytula *et al.*, 2017a).

Among the various possible sets of independent contours, it is necessary to select the set for which the methods of calculating the flow distribution parameters would be as fast as possible. The basis of the algorithms for the allocation of the system of contours is the construction (according to a certain criterion) of the minimum tree of connection between gas inflows and withdrawals (the skeleton of the graph) and finding the preliminary distribution of flows in such a network.

As an option, the connection tree can be built in the following way – all flows from the vertices with inflows should be sent to the vertices with outflows by the shortest path, or paths with the smallest total hydraulic losses.

For the spanning tree of the graph constructed (according to one or another criterion), there is a single distribution of flows between inflows and outflows of gas, for each vertex of which Kirchhoff's first law is fulfilled. Then a complete system of linearly independent contours is built $C_1, C_2, \dots, C_k$. To fulfill Kirchhoff's second law, an unknown flow $q_1, q_2, \dots, q_k$ is introduced into each circuit $C_1, C_2, \dots, C_k$ (an unknown flow is added to the already known gas flow on each edge of the circuit). It is easy to see that in this case Kirchhoff's first law will be fulfilled. For each contour $C_s, s = \overline{1, n}$, we find the flow q_s and add it (with a certain sign) to the previous distribution of flows in the edges of the contour C_s.

The idea of the flow rate $q_1, q_2, \dots, q_k$ finding algorithm is given below. On the condition that there are no edges of the compressor station (CS) type on the contour with m the vertices, we form equations according to Kirchhoff's second law (the total pressure loss on the contour is zero) in the following way. Consider one of the contours with vertices numbered $1, 2, \dots, m$. For each contour edge, we write down the gas flow equation (Prytula, 2009). Let's get a set of equations

$$\begin{cases} P_2 = f_1(P_1, q_{10} + \sum \pm q_{1i}) \\ P_3 = f_2(P_2, q_{20} + \sum \pm q_{2i}) \\ \dots \\ P_m = f_{m-1}\left(P_{m-1}, q_{m-1,0} + \sum \pm q_{m-1,i}\right) \end{cases} \quad (4)$$

Where:

P_i – unknown gas pressure in i vertex;
q_{j0} – the initial gas consumption on j edge;
$\sum \pm q_{1i}, \sum \pm q_{2i}, \dots, \sum \pm q_{mi}$ is the sum of flow rates for all adjacent contours that contain the corresponding edge.

In addition, we find a path from the nearest vertex, with a given pressure (such a vertex exists, because its task involves input data), to one of the contour vertices and write down the system of equations:

$$\begin{cases} P_1' = f_1'(P_0, q_{00}' + \sum \pm q_{1i}') \\ P_2' = f_1'(P_1, q_{10}' + \sum \pm q_{1i}') \\ P_3' = f_2'(P_2, q_{20}' + \sum \pm q_{2i}') \\ \dots \\ P_l' = f_{l-1}'\left(P_{l-1}, q_{l-1,0}' + \sum \pm q_{l-1,i}'\right) \end{cases} \quad (5)$$

Where:

q_{i0}' – the initial flow rate of consumption for i–th edge;
f_i' – a function that establishes the hydraulic connection of the gas parameters at the inlet and at the outlet of the edge;
$\sum \pm q_{ji}'$ is the amount of consumption from adjacent contours for the corresponding edges.

Using the method of excluding pressures, the set of equations (4) and (5) can be easily reduced to an equation of the form:

$$F\left(q_{i_1}, q_{i_2}, \dots, q_{i_m}, P_0\right) = 0, \quad (6)$$

Where:

$q_{i_1}, q_{i_2}, \dots, q_{i_m}$ – unknown flow rates for binding from the relevant circuits;
P_0 is the given pressure value.

If one of the edges on the contour has a CS type with a given pressure (at the inlet or outlet), then in this case there is another vertex with a given pressure. Then we have the following set of equations:

$$\begin{cases} P_1 = f_0\left(P_0, q_{00} + \sum \pm q_{0i}\right) \\ P_2 = f_1\left(P_1, q_{10} + \sum \pm q_{1i}\right) \\ \dots \\ P_s = f_{s-1}\left(P_{s-1}, q_{s-1,0} + \sum \pm q_{s-1,i}\right) \end{cases} \quad (7)$$

where P_0, P_s are the given pressures.

Finally, we will get system (4) of the following form:

$$0 = F\left(q_{i_1}, q_{i_2}, \dots, q_{i_m}, P_0, P_s\right),$$

Where:

$q_{i_1}, q_{i_2}, \dots, q_{i_m}$ – unknown flow rates on the relevant circuits;
P_0, P_s – given pressures.

The formed system of equations is solved by an iterative method. Accordingly, there is a flow rate distribution $q_{i,j}$ for all sections (i, j) of the system. After that, the absolute pressure value P_i is calculated for each i vertex, and the average compressibility coefficient $\bar{z}_{ij}$, average temperature $\bar{T}_{ij}$, and average density $\bar{\rho}_{ij}$ are calculated for each section (i, j).

The iterative process is completed under the condition $|P_{i+1} - P_i| < \varepsilon$ that for each i – vertex where the pressures are found P_{i+1}, P_i at $i + 1$ and i iterations, respectively.

7. Solving Problems of Dispatch Planning of UGS Operating Modes

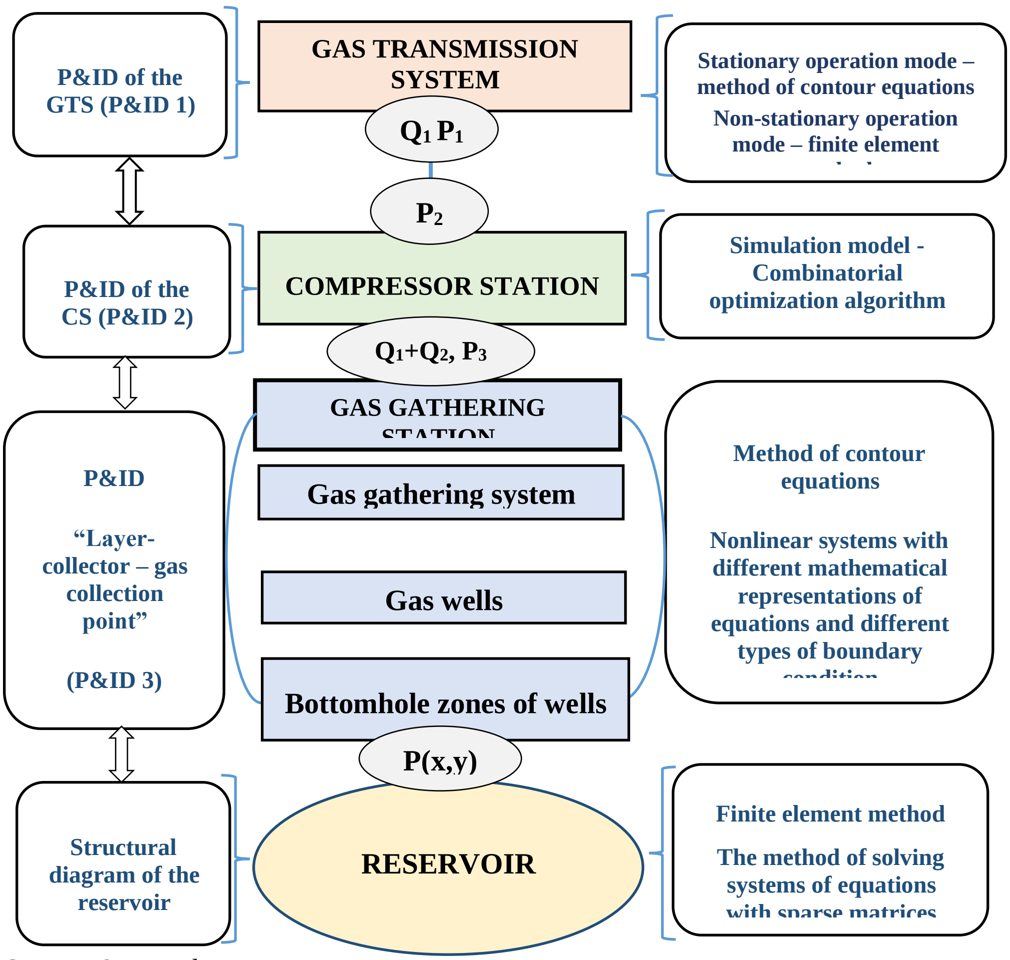
7.1 Algorithm for Calculating the UGS Operating Mode

The mathematical methods developed by us automates the following processes:

- construction of the UGS model (system of equations);
- analysis of the properties of the structure of P&IDs of UGS;
- division of the diagram into sub-diagrams according to a given feature;
- synthesis of P&IDs;
- diagram modifications;
- the selection of those input data (among the set of given ones) that provide the necessary conditions for the existence of a solution to the system of equations.

The general diagram of the technological chain "reservoir – gas pipeline-outlet" is shown in Figure 4 (Prytula *et al.*, 2022).

Figure 4. *Diagram of the technological chain „reservoir – gas pipeline withdrawal"*



Source: *Own study.*

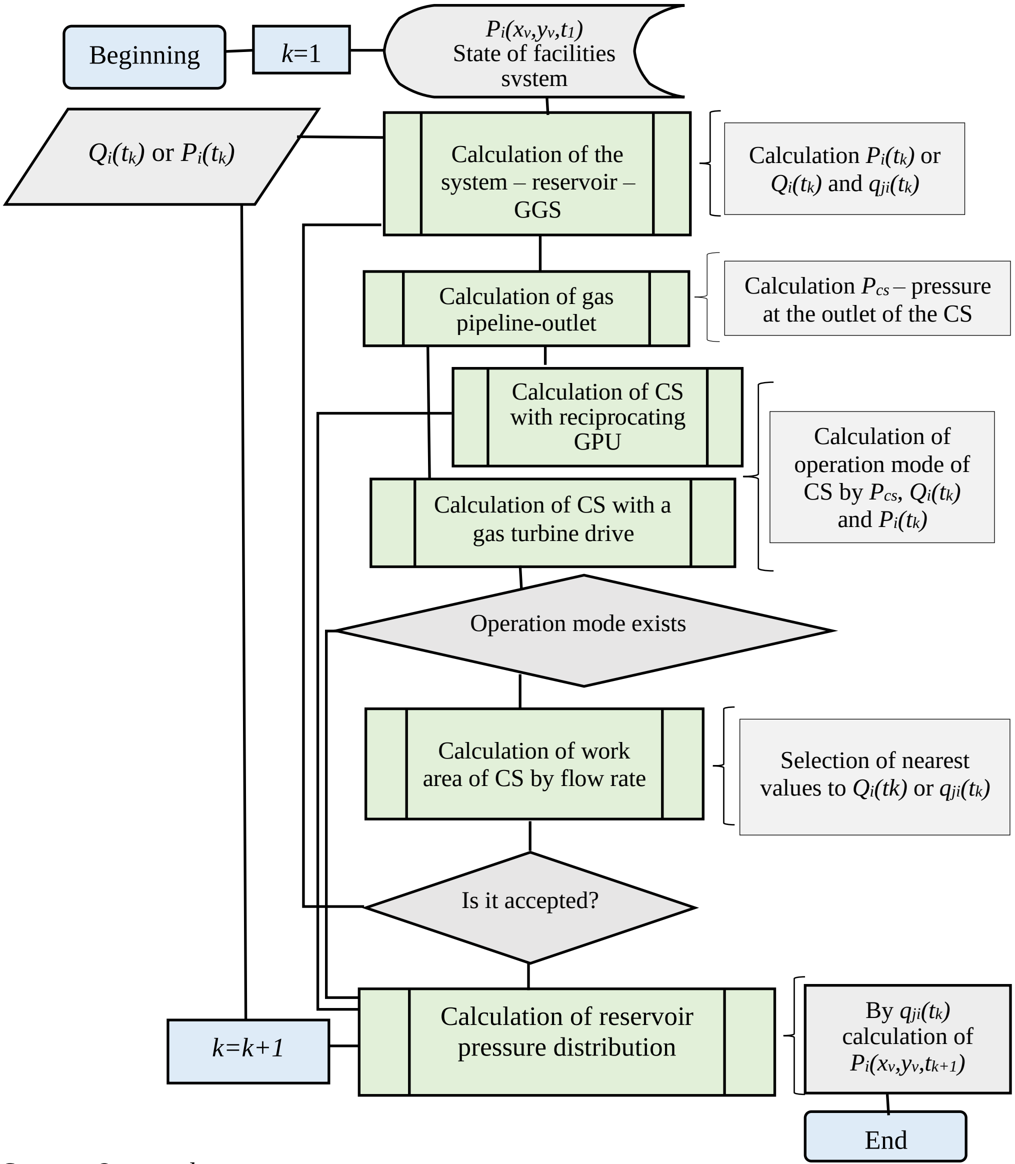
The complete diagram of the algorithm for calculating the operation mode of the UGS (technological chain „reservoir – gas pipeline-outlet") is shown in Fig. 5. Let's enter the notation:

$Q_i(t_k), P_i(t_k)$ – gas flow rate and pressure at the gas gathering station for the i-th UGS at the moment of time t_k;

$q_{ij}(t_k)$ – flow rate of i-th UGS wells;

$P_i(x_v, y_v, t_k)$ is the reservoir pressure of gas at the vertices of the triangles with coordinates (x_v, y_v) – (elements of reservoir division).

Figure 5. *Diagram of the algorithm for calculating the operation modes of the UGS*



Source: *Own study.*

7.2 The Structure of the Problems of Operative Dispatch Planning of UGS Operation Modes

The software developed by us provides the following basic functions:

- calculation and analysis of the current distribution of reservoir pressure, hydraulic losses in technological facilities, control of compliance of actual and estimated data;
- identification of parameters of gas flows and establishment of restrictions on parameters of flows taking into account hydrodynamic studies of wells and technological limitations;
- carrying out and analyzing operational forecast calculations of gas injection / gas extraction modes for choosing the optimal operation modes of UGS;
- calculation of the maximum possible withdrawal/injection at the current time;
- maintenance of the modeling archive database, which stores retrospective information necessary for the operation of the mode control system, as well as options for predictive calculations.

The structure of problems for operational planning of operation modes is given in Table 5, in which it is indicated:

P_{RA} – reservoir pressure in the working zone of wells, specified or calculated;
P_{GGS} – pressure at the gas gathering station, given;
Q_{GGS} – gas consumption;
P_{PIPE} – measured or calculated gas pressure in the outlet gas pipeline;
M_{CS} – mode of operation of the boosting compressor station;
M_{UGS} – mode of operation of the underground gas storage;
N_{NW} – number of working wells;
Q_{MFR} – maximum gas flow rate.

Table 5. *Operational operation problems*

Problems	**Incoming data (specified and measured)**	**Result**	**Setting additional data**	**Result**	**Comment**
1	P_{RA}, P_{GGS}	Q_{GGS}	P_{PIPE}	M_{CS}, M_{UGS}	By setting the minimum Pggs at the entrance of the BCS, we will get the maximum throughput of the system – reservoir-GGS, by choosing the Pggs, we will get the maximum productivity of the UGS (BCS)
2	P_{RA}, Q_{GGS}	P_{GGS}	P_{PIPE}	M_{CS}, M_{UGS}	Not every Pggs has an Mcs mode. If necessary, it is possible to estimate the limits of the Qggs change, for which there is a UGS operating mode
3	P_{GGS}, Q_{GGS}	P_{RA}	–	–	–
4	P_{GGS}, P_{PIPE}, Q_{GGS}	N_{NW}	–	–	It is possible to select wells according to the established criteria

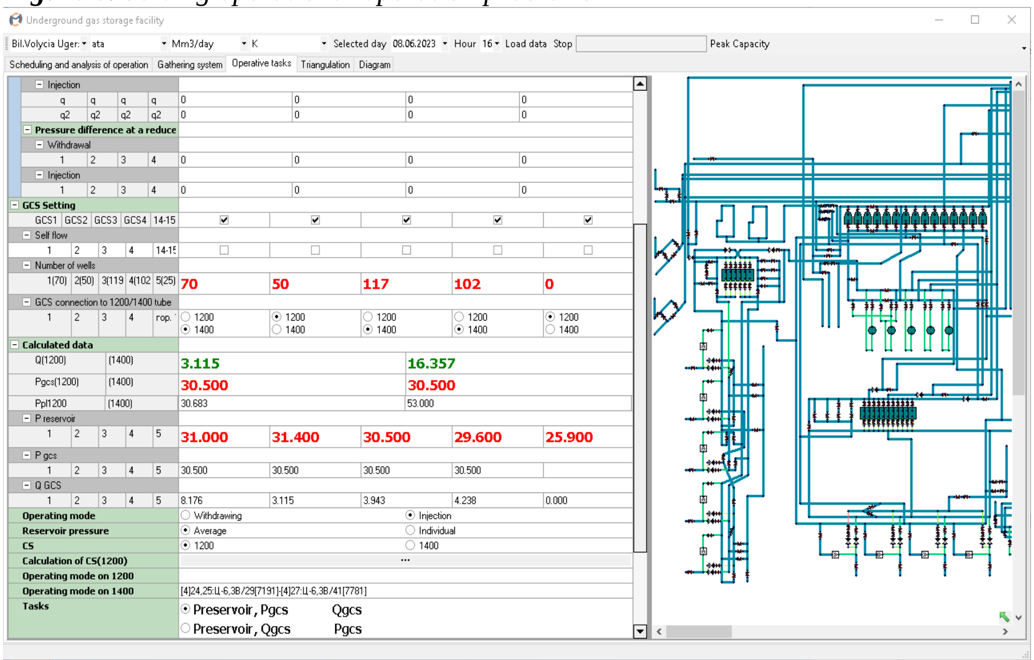
5	P_{RA}, P_{PIPE}	P_{GGS}, Q_{GGS}	–	M_{CS}, M_{UGS}	Among the obtained operation modes, it is possible to distinguish the modes according to the energy and stability criteria
6	P_{RA}, P_{PIPE}	Q_{MFR}	–	M_{CS}, M_{UGS}	Among the obtained operation modes, it is possible to distinguish the modes according to the energy and stability criteria

Source: *Own study.*

7.3 Numerical Experiments

The software module for forming the parameters of the operating modes (including extreme ones) of UGS facities is integrated with the database of measured data and therefore it is constantly updated (Figure 6).

Figure 6. *Solving operational operation problems*



It is possible to study the system response as a result of adjusting input data – technical limitations on process parameters, current coefficients of filtration resistance, distribution of gas flows, etc.

The software module – CS UGS, which is integrated with the software module – reservoir-GGS, and ensures the correct „stitching" of filtration, gas dynamic and gas compression processes of UGS into a single thermo-hydraulic process is shown in Figures 7-9.

Figure 7. *Calculation of the operation modes of the CS (window on the right) coordinated with the operation modes of the technological chain – reservoir – gas gathering station.*

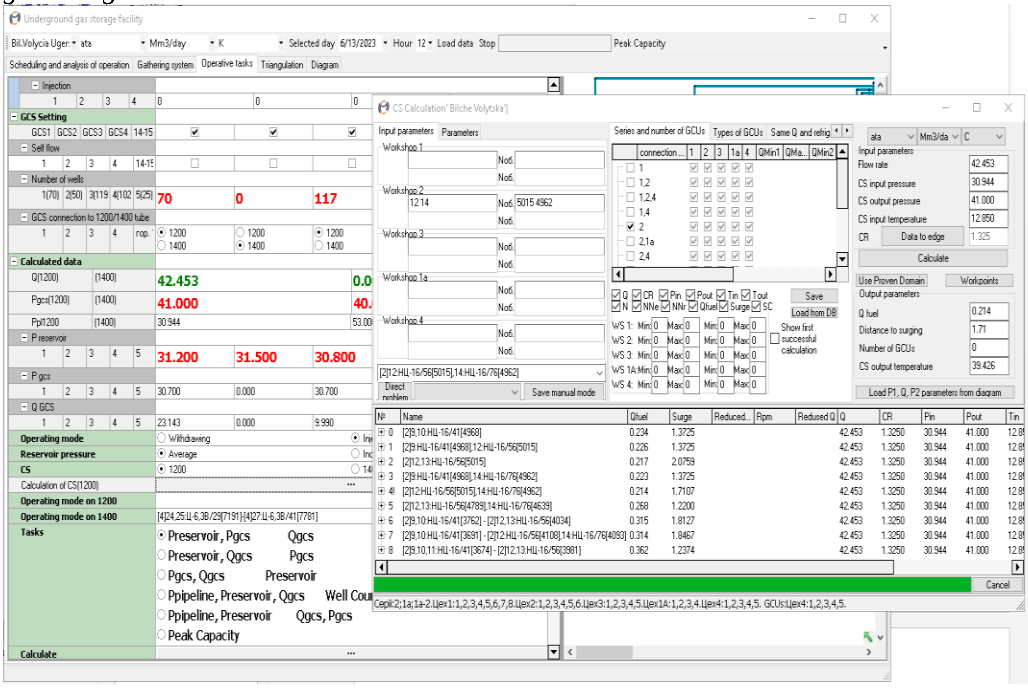


Figure 8. *Setting the operational parameters of the CS gas pumping units*

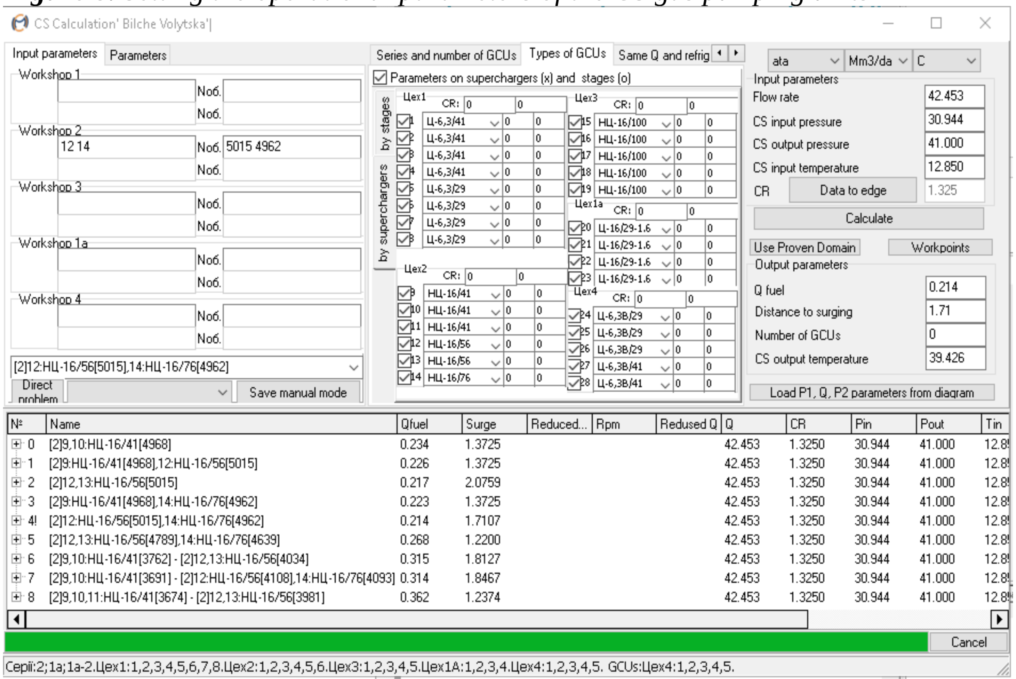
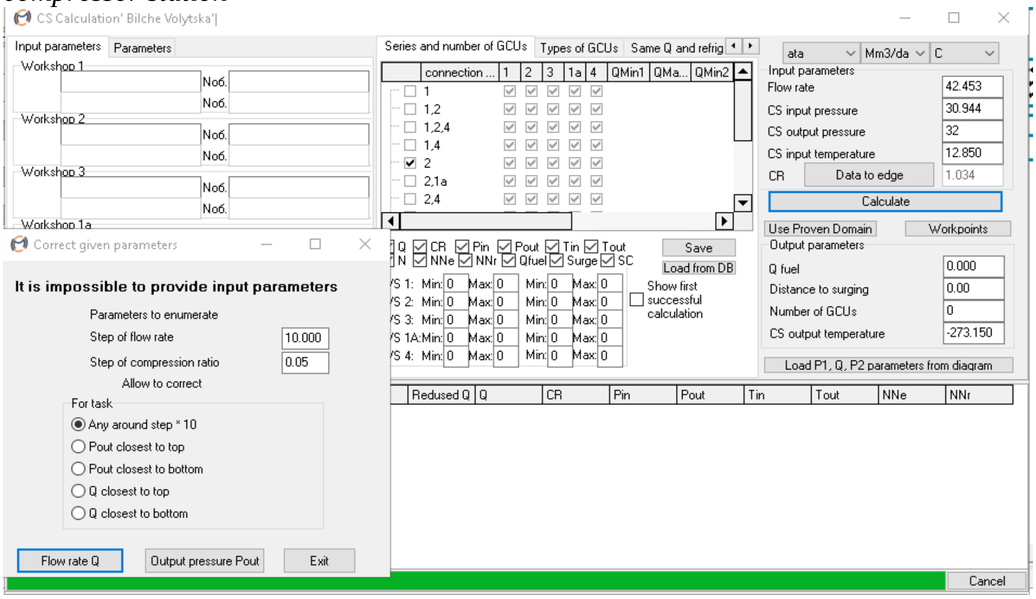


Figure 9. *Clarification of the area of change of operational parameters of the compressor station*



7.4 The Structure of the Main Implemented Operation-Technological Problems

The formulation of problems and their effective solution were facilitated by our proposed approaches to building a model of a complex facility:

- the model of the UGS structure is presented in terms of graph theory and includes gas flow models of various types of continuous and discrete physical processes and their various mathematical representations;
- the developed model of the system takes into account thermo-hydraulic interaction in all facilities of the system in the technological chain “reservoir – pipeline-outlet”;
- automation of the process of building the system model is ensured in the event of a change in its P&ID;
- developed software tools for adapting system models to the real state of facilities, the topology of the P&ID and changing the set of technological equipment.

Other requirements apply to the implementation of models:

- fast modeling of filtration processes in heterogeneous reservoir systems is provided;

- methods of discrete optimization of processes are proposed, which made it possible to calculate the optimal parameters of nonlinear discrete-continuous processes in the entire range of input data changes;
- implemented the possibility of conducting computing experiments as quickly as possible to find optimal options for work and reconstruction;
- the calculation of the extreme mode parameters of UGS operation under the conditions of variable modes of operation of the GTS is ensured.

The analysis of the conducted numerical experiments and real modes of operation of the UGS confirmed the sufficient accuracy of the calculated modes. The developed algorithms and software made it possible to obtain the optimal operating parameters of the UGS in an acceptable time.

The integration of the software with dispatch information databases ensured minimal intervention in the process of setting input data and the process of optimizing the system.

The developed mathematical methods was tested during its long-term operation at real facilities of the gas industry of Ukraine. The accuracy of the simulation of the operation of the UGS depends on the accuracy of the identification of the parameters of the models of its objects. The accuracy of the average formation pressure calculation is proportional to the accuracy of its measurement.

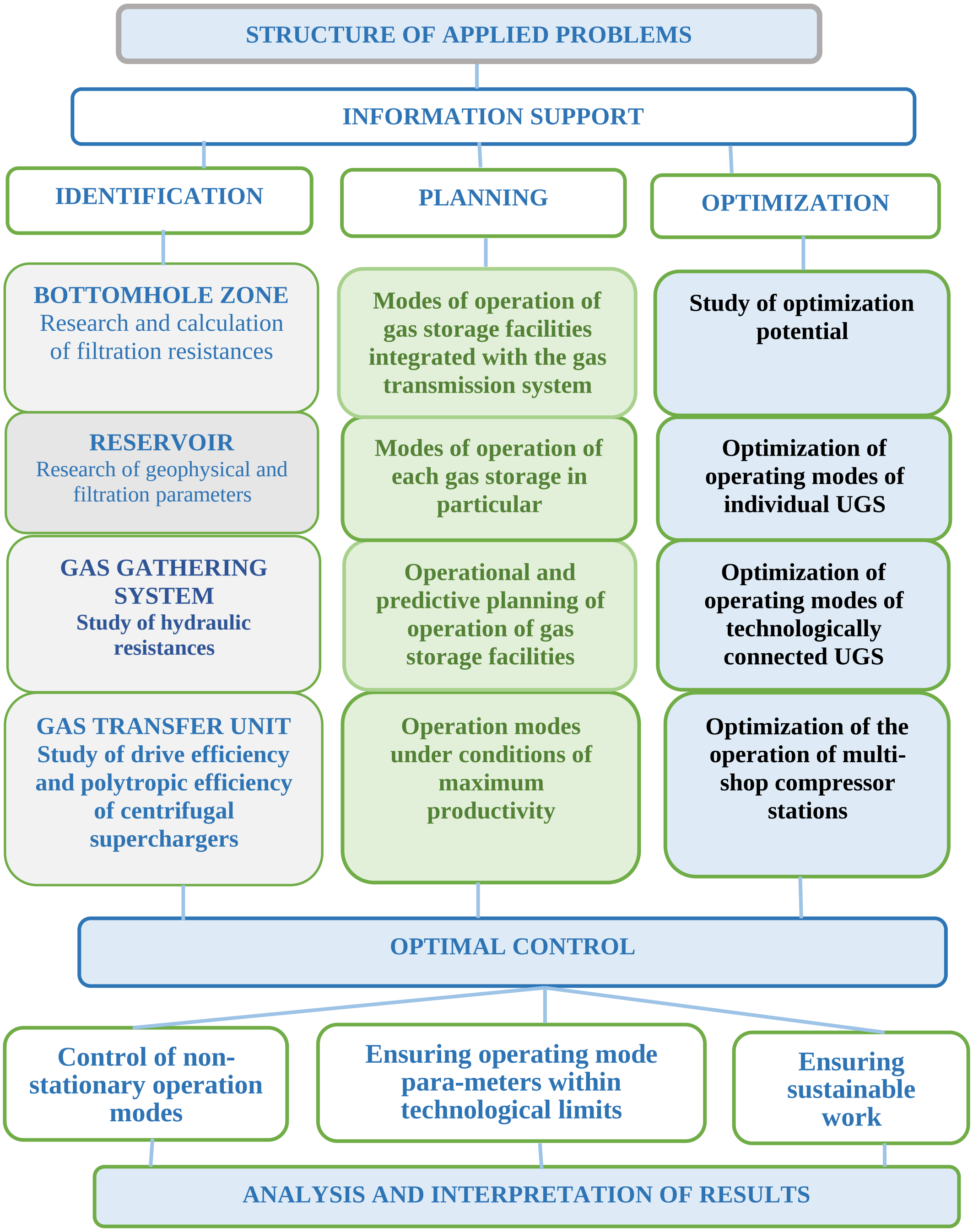
In real conditions, the accuracy of CS modeling is achievable within one to two percent. The modeling complex supports the high accuracy of the calculation of mode indicators during many years of modeling processes in UGS without specifying the identification parameters of facility models.

The impact of casing column perforation parameters, filtration resistance coefficients on the operation of some wells and UGS in general, as well as the influence of hydrodynamic parameters on the operation of the "UGS reservoir- gas pipeline-outlet" system in general, was investigated. Non-stationary models and methods of analysis of UGS facilities made it possible to conduct gas-hydrodynamic studies of UGS reservoirs and wells. Figure 10 shows the structure of operation mode problems.

A brief description of the developed software modules is shown in Table 6. The developed software is multi-functional and therefore it required the development of such mathematical methods that would guarantee sufficient accuracy of modeling under the existing metrological support, fast and guaranteed convergence of iterative processes, minimal complexity of algorithms, etc.

The lack of measured data of the necessary quality in terms of set, frequency, and synchronicity required the construction of various detailed P&IDs, the construction of empirical dependencies based on long-term operational data, hydraulic equivalents, etc. By the way, greater detail of P&IDs does not always ensure greater modeling accuracy.

Figure 10. *Structure of operation-technological problems*



This is due to the fact that the existing metrological support does not allow the identification of hydraulic parameters of all facilites, without exception, which are components of detailed P&IDs.

And what is more, on detailed three-dimensional diagrams, there is a significant amount of local resistance – branches, turns, etc. All of them create additional hydraulic resistance.

Taking them into account in full significantly complicates the model and, accordingly, reduces the speed of solving operation mode problems. The developed software allows you to calculate the parameters of gas flows and on detailed P&IDs.

If the distances between the inlets and outlets of the vehicle are small, then it is advisable to build the corresponding hydraulic equivalents based on the measured data for the entire subsystems of the P&ID.

This makes it possible to quickly solve dispatching problems using the combinatorial optimization method, which require the calculation and analysis of many possible options for implementing the required operation modes of the UGS.

Table 6. *Brief characteristics of the developed software modules*

UGS facilities	**Mathematical representation options**	**Characteristics of the method**	**Notes**
Single and multi-layer reservoir with concentrated sources	Equations of unsteady filtration in heterogeneous porous media with concentrated sources	Finite element method	A fast method for solving systems of equations with sparse matrixes of coefficients. The proposed model provides a quick result in the process of modeling filtration processes at significant time intervals. It is possible to take into account the existing anisotropy of the reservoir by spatial coordinates
Compressor station	Computational algorithms for solving operation-technological problems for given P&ID of the CS	Purposeful enumeration of options taking into account dependence (time, accuracy). Combinatorial optimization method	It is possible to adjust the parameters of models, technological limitations, form models of new gas pumping unit (GPU) and edit the P&ID. Calculation of parameters of gas flows on detailed P&ID
Gas gathering system	Detailed P&ID in terms of graph theory with boundary conditions of various types	Iterative method of non-gradient type. The convergence of the method does not depend on the given initial data	The software for hydraulic calculation of gas distribution networks is used

The bottomhole zone	Possible options: filtering supports: permeability of the bottomhole zone; permeability of the near and far zones of gas filtration	Analytical and iterative numerical methods	Possible options: open filtering area; parameters of perforation channels. Taking into account anisotropy in the bottomhole zone
Shut-off and regulating valves	Anticipated models of various types of devices for narrowing gas flows	Adiabatic process of gas outflow through the hole	We implement available in the P&ID of UGS
Units for air cooling and gas prepa-ration	Empirical models	Multi-parametric approximation of operational data	The temperature of the cooled gas is calculated according to the constructed empirical formula
UGS + gas transmis-sion system (GTS)	Detailed P&ID. Access to all parameters that affect the topology of the GTS+UGS.	Iterative process of coordination by pressure, flow rate and gas temperature.	The operation mode of the UGS is formed at the stage of planning the optimal modes of the GTS
A group of technologically connected PSGs	Combinatorial optimization methods	Non-classical iterative methods for solving optimization problems	Solved problems of optimal operation of UGSs at the stages of planning gas injection and withdrawal

7.5 Analysis of Influencing Factors on the Quality of UGS Operation: Numerical Experiments

The optimal operation of the UGS is evaluated according to the energy criterion, under the condition of maintaining the given stability of the GPU CS operation mode.

There are many options for implementing the specified operating modes of UGS on multi-shop CS with various types of GPU. Versatility is ensured by variable flow parts, change in rotations of centrifugal superchargers within 70-105% of nominal, etc.

In the operation mode of injection specified volumes of gas, the gas pressure at the well bottomholes will change over time, which affects the volumes of fuel gas (Tables 7-10).

Table 7. *Optimum operating modes of UGS in gas injection mode, according to fuel gas consumption*

Flow rates, mln m³/day	**Fuel flow rates, mln m³/day**	**Reduced fuel flow rates**	**Reduced coefficient of fuel flow rate**
10.0	0.058	0.0058	1.318
14.7	0.065	0.0044	1.000
21.0	0.116	0.0055	1.250
27.7	0.127	0.0045	1.022

Over a longer time interval, the fuel gas consumption coefficients will increase due to an increase in the pressure at the well bottomholes and, accordingly, at the gas gathering station and the outlet of the CS.

The relationship between withdrawal flow rate and fuel gas is not linear. And therefore, in the presence of several UGSs, there is an opportunity to redistribute the necessary flow rates of withdrawal between UGSs in such a way as to minimize the combined total consumption of fuel gas.

Table 8. *Dependence of the reduced consumption of fuel gas on the flow rate of gas withdrawal*

Operation mode number	**Withdrawal flow rate, mln m³/day – Q_1**	**Fuel flow rate, mln m³/day – Q_2**	**Reduced fuel flow rate – Q_2/Q_1**
1	15.92	0.128	0.00804
2	14.93	0.114	0.00763
3	12.64	0.058	0.00458
4	9.71	0.039	0.00402

In the process of analysis given in the table 8 of the first and last operation modes, we get that the combined energy costs in the case of an increase of 6.21 mln m^3/day in the flow rate of gas withdrawal, the consumption of fuel gas will increase twofold.

Table 9. *Dependence of combined fuel gas consumption on gas withdrawal volumes and pressure in the gas pipeline-outlet*

Pressure in the pipeline-outlet, kgf/cm²	**Withdrawal flow rate 15.915, mln m³/day**		**Withdrawal flow rate 18.232, mln m³/day**	
	Fuel flow rate, mln m³/day	**Coefficient magnification costs, %**	**Fuel flow rate, mln m³/day**	**Coefficient magnification costs, %**
35.0	0.080		0.087	
37.0	0.090	12.5	0.098	12.6
40.0	0.128	60.0	0.150	72.4
45.0	0.140	75.0	0.166	83.9

The simulation results are shown in table 9 show a non-linear relationship between fuel gas consumption and the pressure in the gas pipeline. A change in the pressure in the gas pipeline within the range of 2-3 kgf/cm^2 significantly affects fuel gas consumption – 12-60%.

Table 10. *Maximum flow rate of gas withdrawal, under constant pressure in the working zone of the wells*

Pressure in the working zone, kgf/cm^2	**Pressure in the pipeline-outlet, kgf/cm^2**	**Maximal withdrawal flow rate, mln m^3/day**	**The maximum throughput capacity of the reservoir-GGS at a pressure of 16 kgf/cm^2 on the GGS, mln m^3/day**	**Fuel flow rate, mln m^3/day**	**Coefficient fuel flow rate**
25	35	13.6	16.6	0.213	0.01566
	40	12.2		0.201	0.01647
	45	8.3		0.101	0.01216
35	35	21.5	25.3	0.251	0.01167
	40	20.7		0.259	0.01251
	45	19.7		0.227	0.01152
45	35	30.6	35.0	0.211	0.00689
	40	28.8		0.229	0.00795
	45	26.2		0.251	0.00958

With an increase in reservoir pressure (Table 10), the maximum flow rate of gas withdrawal depend less significantly on the pressure in the gas pipeline-outlet. In such cases, the main factor affecting the flow rate of gas withdrawal is the throughput capacity of facilities and the available capacity of the post-compressor station (PCS).

8. Discussion

Graph-diagram of the technological chain "reservoir – bottomhole zones of wells – wells – gas collection diagram – gas collection station" at underground gas storages have different graphical representations.

The simplest representation of such a diagram is in the case of a simple gas collection system. In such cases, the hydraulic calculation of graph diagrams is not a problem.

More complicated is the hydraulic calculation of graph-diagrams, which are formed for the tree system of gas collection. In this case, the difficulty lies in the fact that the graph components have models of gas flows with different mathematical representations and the structure of the graphs can be variable.

And that is why we proposed to present such a diagram in the form of a distribution network, which would cover all possible variants of diagrams, including diagrams with contours. In existing distribution networks, gas flow models (mainly) have the same type of mathematical representation.

In the paper, we proposed a method of non-gradient type (with guaranteed convergence from zero initial conditions) of hydraulic calculation of complex distribution networks, in the general case. And therefore it can be used (without additional modifications) to calculate arbitrary hydraulic networks (including networks with active facilites) both at the stage of their design and operation.

The proposed method (together with a combinatorial algorithm of minimum complexity for calculating multi-shop CS) made it possible to formulate and solve a complete set of direct and inverse control mode problems.

The conducted research (compared to similar ones known at this time) has advantages due to:

- presentation of the gas flow model of UGS in the form of three integrated models: 1) filtration model of reservoir; 2) the network model of the technological chain "bottomhole zones of wells – wells – gas collection system"; 3) compressor station models;
- proposed universal methods and algorithms for implementing these models.

It is this approach that closes the problematic part defined in section 2.

Among the advantages of this study (compared to similar ones known at this time) can also be counted: the speed of calculation of UGS operating modes; multifunctionality; ease of adaptation and operation. The implemented studies provide:

- conducting an analysis of the effectiveness of the UGS in the real conditions of their operation;
- research of optimal strategies for the joint operation of several UGSs, as well as UGSs and GTS;
- assessment of gas losses in case of depressurization of UGS facilities;
- evaluation of the effectiveness of design solutions and plans for the reconstruction of the UGS.

The process of applying the results of the work in practice (as well as in further theoretical research) requires constant updating of gas flow models in UGS technological facilities. This especially applies to the models of the bottomhole zones of wells.

The model of gas inflow to wells is sensitive to changes in gas composition (relative to the amount of heavy hydrocarbons or water), which significantly affects the permeability of the bottomhole zone of the wells and, accordingly, the magnitude of its depression/repression, especially at the gas injection stage.

It should also be taken into account the fact that the existing methods of researching wells in stationary and non-stationary operating modes of UGS do not ensure the construction of a model of gas inflow to wells with the necessary accuracy, taking into account the influence of all available factors.

The difficulty lies in the fact that the influence of individual factors on the change in depression/repression for several days cannot be measured – their influence is beyond the accuracy of pressure measurements.

A noticeable effect is observed as a result of the operation of UGS for a month or more. This should be taken into account in the case of researching options for the operation of the UGS during the seasons of gas withdrawal or injection.

9. Conclusions

A method of solving problems for the analysis of gas flows in a network-type system (with different initial and boundary conditions) has been developed. This method links the parameters of the filtration processes in the reservoir (reservoir pressure on the supply circuit of wells) with concentrated sources (wells) and discrete and continuous processes at multi-shop compressor stations. The method is fast-converging in the entire range of changes in input data – initial and boundary conditions.

The developed methods of solving direct and inverse problems for operational and predictive dispatching control of gas injection and withdrawal processes ensured:

- operational and as fast as possible conducting computer experiments to find optimal options for the operation of the UGS, options for the reconstruction of the UGS, analysis of emergency situations during the depressurization of the UGS;
- the time efficiency of the software according to the criterion of obtaining the result in an acceptable time and the ability to solve the problems of optimizing the operating mode using the available minimum amounts of resources (time and information).

The process of calculating UGS operating modes is maximally automated and requires only minimal user intervention. The developed software has been tested over many years of operation and has been used for operational and predictive planning in dispatching decision-making systems.

Regarding the prospects for the development of research. The implemented functionality will make it possible to build effective systems for:

- energy audit of UGS operation with the aim of identifying the available optimization potential and ensuring its maximum implementation;

- analysis of the effectiveness of the developed, implemented, operational and transitional modes of operation of the UGS;
- automation of the process of supporting dispatcher decision-making.

References:

Atkinson, K.E. 1989. An Introduction to Numerical Analysis. John Wiley & Sons, Inc.

Devlin, J., Li, K., Higgins, P., Foley, A. 2017. A multi vector energy analysis for interconnected power and gas systems. Applied Energy, 192, 315-328. https://doi.org/10.1016/j.apenergy.2016.08.040.

Domschke, P., Hiller, B., Lang, J., Mehrmann, V., Morandin, R., Tischendorf, C. 2021. Gas network modeling: An overview.

Egger, H. 2018. A robust conservative mixed finite element method for isentropic compressible flow on pipe networks. SIAM Journal on Scientific Computing, 40(1), A108-A129. https://doi.org/10.1137/16M1094373

Ekhtiari, A., Dassios, I., Liu, M., Syron, E. 2019. A novel approach to model a gas network. Applied Sciences, 9(6), 1047. https://doi.org/10.3390/app9061047

Esmaeili, M., Moradi, M. R., Afshoun, H. R. 2022. A new empirical model and neural network-based approach for evaluation of isobaric heat capacity of natural gas. Journal of Natural Gas Science and Engineering, 102, 104575.

Fokken, E., Göttlich, S., Kolb, O. 2019. Modeling and simulation of gas networks coupled to power grids. Journal of Engineering Mathematics, 119(1), 217-239.

Fokken, E., Göttlich, S., Kolb, O. 2020. Optimal control of compressor stations in a coupled gas-to-power network. Advances in Energy System Optimization, 67.

Guerra-Fernandez, O.J., Sergi, B., Hodge, B., Craig, M., Pambour, K.A., Sopgwi, R.T., Brancucci, C. 2020. Electric Power Grid and Natural Gas Network Operations and Coordination (No. NREL/TP-6A50-77096). National Renewable Energy Lab. (NREL), Golden, CO (United States). https://doi.org/10.2172/1665862

Gugat, M., Herty, M. 2020. Modeling, control and numerics of gas networks. arXiv preprint arXiv:2010.02743

Gyrya, V., Zlotnik, A. 2019. An explicit staggered-grid method for numerical simulation of large-scale natural gas pipeline networks. Applied Mathematical Modelling, 65, 34-51.

Iwaszczuk, N., Prytula, M., Prytula, N., Pyanylo, Y., Iwaszczuk, A. 2022a. Modeling of Gas Flows in Underground Gas Storage Facilities. Energies, 15(19), 7216. https://doi.org/10.3390/en15197216

Iwaszczuk, N., Zapukhliak, I., Iwaszczuk, A., Dzoba, O., Romashko, O. 2022b. Underground Gas Storage Facilities in Ukraine: Current State and Future Prospects. Energies, 15(18), 6604. https://doi.org/10.3390/en15186604

Jurek, T., Iwanek, M. 2019. Gas network improvement proposal using numerical simulation. In IOP Conference Series: Materials Science and Engineering (Vol. 710, No. 1, p. 012005). IOP Publishing.

Khymko, O., Prytula, M., Prytula, N., Prytula, Z. 2023. Methods of Optimal Development and Modernization of Existing Distribution Networks for Gas-Hydrogen Mixtures. In: Blikharskyy, Z. (eds.) Proceedings of EcoComfort 2022. EcoComfort 2022. Lecture Notes in Civil Engineering, vol. 290. Springer, Cham. https://doi.org/10.1007/978-3-031-14141-6_15

Kondrat, R.M., Shchepanskyi, M.I., Haydarova, L.I. 2020. Study of the impact of contamination of the bottomhole zone of the reservoir and parameters of perforation channels on the productivity of gas wells.Prospecting and Development of Oil and Gas Fields, (3 (76)), 23-32. (in Ukrainian)

KYPipe – Software for pipe network analysis. Available online: https://dl.kypipe.com, (accessed on 14.02.2023)

Lochran, S. 2021. GNOME: A Dynamic Dispatch and Investment Optimisation Model of the European Natural Gas Network and Its Suppliers. In Operations Research Forum (Vol. 2, No. 4, pp. 1-44). Springer International Publishing. https://doi.org/10.1007/s43069-021-00109-5

Osiadacz, A.J., Chaczykowski, M. 2020. Modeling and simulation of gas distribution networks in a multienergy system environment. Proceedings of the IEEE, 108(9), 1580-1595.

Osiadacz, A.J., Gburzyńska, M. 2022. Selected Mathematical Models Describing Flow in Gas Pipelines. Energies, 15(2), 478.

Prytula, M., Prytula, N., Pyanylo, Y., Prytula, Z., Khymko, O. 2022. Planning optimal operating modes of underground gas storage facilities as part of the gas transmission system. Eastern-European Journal of Enterprise Technologies, 3(2 (117), 76–91. https://doi.org/10.15587/1729-4061.2022.258953

Prytula N.M. 2009. Mathematical modeling and numerical analysis of gas transmission system operating modes. (in Ukrainian)

Prytula, N., Frolov, V., Prytula, M. 2017a. Optimal scheduling of operating modes of the gas transmission system. Math. Model. Comput, 4(1), 78-86.

Prytula, N., Prytula, M., Boyko, R. 2017b. Development of software for analysis and optimization of operating modes of underground gas stores. Technology Audit and Production Reserves, 2(3(40), 17–25. https://doi.org/10.15587/2312-8372.2018.128574

Prytula, N., Prytula, M., Boyko, R. 2017c. Mathematical modeling of operating modes of underground gas storage facilities. Technology Audit and Production Reserves, 4(1(36), 35–42. https://doi.org/10.15587/2312-8372.2017.109084

Rodríguez, T., Sarabia, D., Valbuena, M., de Prada, C. 2014. Modelling and optimization of natural gas networks. In Computer Aided Chemical Engineering, 33, 367-372.

Sanchez, P. 1980. Non-homogeneous media and vibration theory. https://doi.org/10.1007/3-540-10000-8

Schlumberger. Available online: https://www.software.slb.com/products, (accessed on 20.12.2022)

SIMONE. Available online: https://www.simone.eu/simone-company-about.asp, (accessed on 20.12.2022)

Sprangers, W. 2020. Modelling, simulation and analysis of green gas injection into the gas distribution network.

Su, L., Zhao, J., Wang, W. 2021. Hybrid physical and data driven transient modeling for natural gas networks. Journal of Natural Gas Science and Engineering, 95, 104146.

Yusta, J.M., Beyza, J. 2021. Optimal cooperative model for the security of gas supply on European gas networks. Energy Strategy Reviews, 38, 100706. https://doi.org/10.1016/j.esr.2021.100706.

Vaccariello, E., Leone, P., Canavero, F.G., Stievano, I.S. 2021. Topological modelling of gas networks for co-simulation applications in multi-energy systems. Mathematics and Computers in Simulation, 183, 244-253.

Zhang, Y., Huang, Z., Zheng, F., Zhou, R., An, X., Li, Y. 2020. Interval optimization based coordination scheduling of gas–electricity coupled system considering wind power uncertainty, dynamic process of natural gas flow and demand response management. Energy Reports, 6, 216-227.