Iso-Taxi Geometry: A New Approach

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Math Anxiety

"Let no one ignorant of geometry enter here".

These words were inscribed over the doors of Plato's Academy, and such a forbidding perception of the field of Mathematics has permeated society since those days. For more than 5500 years, from Ancient Greece to the beginning of the third millennium, ordinary people have passed on this fear of Mathematics to their children. In ancient times, the public could have been informed clearly about the field of mathematics, the methods of mathematical thoughts, mathematical objects and their properties, and how these relate to nature and society. Unfortunately, however, Mathematics was conveyed as being both difficult and abstract, and thus, it became generally accepted that Mathematics is not for the average mind. As a result, instead of well-understood strategies of investigation, something quite awkward and unattractive appeared. The common widespread assumption that has continued through present day is that people are either good with words or with numbers, not both.

Generally once a negative attitude and an anxiety are formed, it becomes quite difficult to change. These feelings often persist into adult-life with far-reaching consequences in the form of avoidance of mathematics, distress, and interference with conceptual thinking and memory processes. Although mathematics aims at right answers, these answers can be reached through open-ended problems - mathematics being experienced as a series of discoveries to be made by the learner. Rather than mathematical methods and rules, learners need to acquire abilities to analyze, question, test and find solutions. Thus, developing knowledge and skills relating to the processes which can later be applied in any situation. If such a different approach could be taken in the early stages of Mathematics education, an approach that could be built upon in later years, then great steps could be taken at relieving math anxiety and reducing the common fear. One such approach to this teaching style is examined in *Iso-Taxi,* or *Chinese Checker* geometry.

A New Geometry

There are only three regular polygons that will tessellate a plane - the square, equilateral triangle, and regular pentagon. Of these, only two form a regular grid. Extensive work has been done with the square grid in *Taxi-Cab* geometry, where the placement of points and the movement between points is restricted to the lines of the grid as if the lines represent the city streets and the points represent taxi cabs. If we instead consider the plane of equilateral triangles, and restrict the points and movements similarly on this new isometric grid, a geometry quite different than either Euclidean or *Taxi-Cab* can be developed. In previous work this new geometry has been referred to as *Iso-Taxi* geometry, however, if we examine the physical appearance of our grid, it is noticed to resemble the playing surface of a popular children's game. Hence, for our purposes, we will refer to it as *Chinese Checker* geometry.

Before looking at any geometric properties or geometric sets, we must first understand the isometric *Chinese Checker* plane and the naming of *Chinese Checker* points. The three axes that exist in our plane, *x, y* , and *y'*, are each separated by 60 degrees and divide the plane into hextants which are numbered I-VI in a counter-clockwise manner. (Figure 1) *Chinese Checker* points are then named according to their position relative to the *x*- and *y*-axes (Figure 2), and the slanted *y*-axis creates a situation such that points with the same *x* coordinate do not lie in a vertical line. Instead the geometric solution to an equation such as *x = -2* is a slanted line parallel to the *y*-axis. Also, the *y'*-axis is used only in determining the orientation of points which, in essence, chooses of the appropriate distance equation. (This idea will be further explained.)

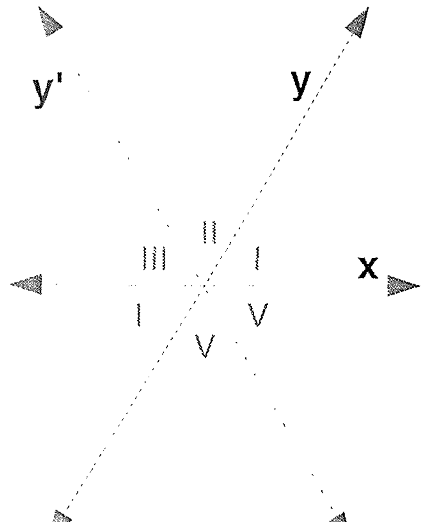


Figure 1

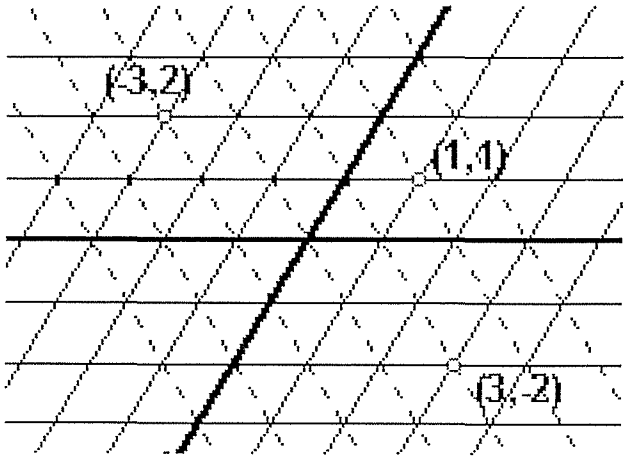


Figure 2

When comparing Euclidean geometry, *Taxi-Cab* geometry, and *Chinese Checker* geometry, points, lines, and angles are the same. The significant difference in the three is the distance equation. The restriction of movements between two arbitrary points to the lines of the playing surface leads to a situation such that the distance equation is dependent upon the relative position of the two points. From this concept, three separate distance equations have been developed:

I. If the points have a *I-IV* orientation, then $d_c = |x_1 - x_2| + |y_1 - y_2|$.

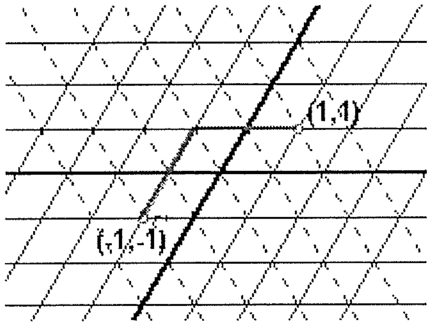
II. If the points have a *II-V* orientation or lie on a line parallel to the *y*- or *y'* axis, then $d_c = |y_1 - y_2|$.

III. If the points have a *III-VI* orientation or lie on a line parallel to the *x*-axis, then $d_c = |x_1 - x_2|$.

In each of the following examples, the orientation of points A and B were determined, and the appropriate distance equation was used.

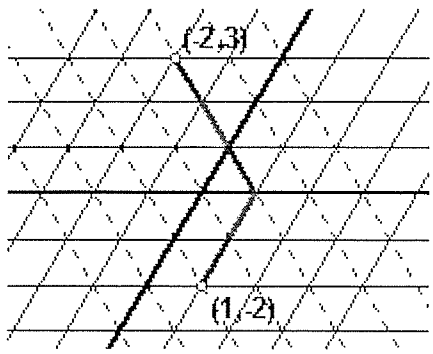
Point $A=(1,1)$ and point $B=(-1,-1)$ have a *I-IV* orientation.

$$\begin{aligned} d_C &= |x_1 - x_2| + |y_1 - y_2| \\ &= |1-(-1)| + |1-(-1)| \\ &= |2| + |2| \\ &= 4 \end{aligned}$$



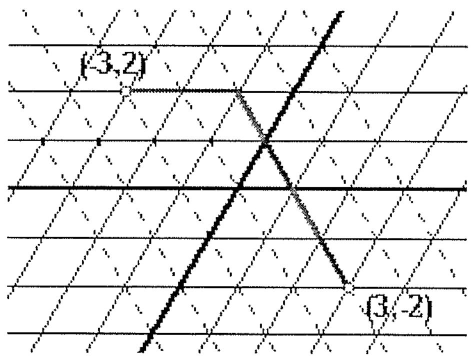
Point $C=(-2,3)$ and point $D=(1,-2)$ have a *II-V* orientation.

$$\begin{aligned} d_C &= |y_1 - y_2| \\ &= |3-(-2)| \\ &= |5| \\ &= 5 \end{aligned}$$



Point $E=(-3,2)$ and point $F=(3,-2)$ have a *II-V* orientation.

$$\begin{aligned} d_C &= |x_1 - x_2| \\ &= |(-3)-3| \\ &= |-6| \\ &= 6 \end{aligned}$$



As previously mentioned, the concepts of points, lines, and angles are unchanged. This is because they are independent of the distance formula. Certain geometric properties and geometric sets, however, are dependent upon the distance formula and are therefore affected by any change in the metric. Consider the circle.

A circle consists of all the points equidistant from a fixed point called the center. When such points are found on the isometric grid, the resulting figure is what we commonly call a hexagon. Furthermore, if we define *pi to* be the ratio of circumference to diameter the value of *pi* can be calculated to be 3. (Figure 3) The area of the circle is also changed when it exists on the isometric grid. Using the unit equilateral triangle that tessellates the plane as a unit of area, we can count the number of such units it takes to tessellate the interior of the figure - 24. Subsequently, we can derive a formula for *Chinese Checker*-area, $A_C = 2\pi r^2$.

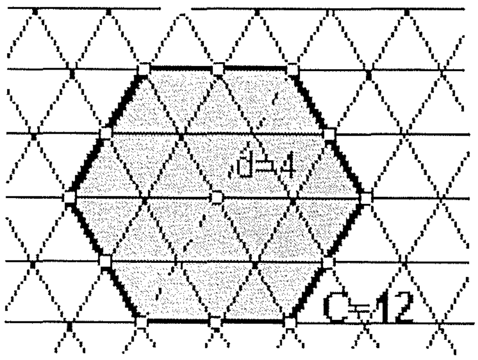


Figure 3

Further Consideration

While there are an endless number of geometric properties and geometric sets that can be examined in this manner, and while this is only one possible educational model, this glance into *Iso-Taxi* geometry, when considered as *Chinese Checker* geometry, allows us to see the impact that teaching style can have on the interest of students. If during instruction on this new geometry students are allowed a hands-on approach with the use of a Chinese Checker board and playing marbles, the subject can be presented in a manner that is much less foreign to the students. Hence, the common fear that is associated with new mathematical topics can be lessened, and the first steps at relieving math anxiety can be taken.