THE APPEAL OF OPERATIVE PROOFS
RECREATION WITH RECTANGLES

IRENE SCIRIHA
DEPT OF MATHEMATICS
FACULTY OF SCIENCE
UNIVERSITY OF MALTA
isci1@um.edu.mt

ABSTRACT. Besides offering an aesthetic attraction, the use of proofs, which rely on simple geometrical figures suggest intuitive ideas. Two such proofs are presented here. One is a recreation with a stair-like decomposition of a rectangle. The second is a simple game that proves Pythagoras Theorem.

1. DECOMPOSE 2000

During the months leading to the first use of the number 2000 in our date, every newspaper carried articles spelling doom either from the Y2K Bug that threatened a worldwide computer meltdown, or from catastrophes foretold by pessimistic sects. To dispel my fears I thought it would be a good idea to decompose the number 2000. Small fragments do not pose any danger!

We look for playful ways to decompose the number 2000 into two arithmetic progressions each containing 40 terms.

The technique we use is often applied in Discrete Mathematics where addition of terms is performed in different ways, often by displaying them as an array and then adding either the rows or the columns or both in turn.

Date: 16th February 2000.

The tools we need are simple. We use the formula for the area of a rectangle which is known to be the product of the length and the breadth. We also use cardboard, a pair of scissors and a pencil.

A 40cm by 50cm rectangle is cut out. A stair-like picture is drawn as shown in Fig.1.

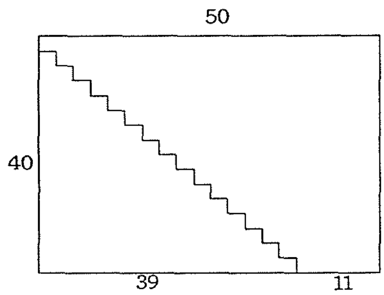

FIGURE 1. *Terms of A.P. are areas of rows.*

The terms of the A.P.s are given by the area of the rows on either side if the stairs. Thus $2000 = (0 + 1 + 2 + ... + 39) + (11 + 12 + ... + 50)$.

More trivial examples are: $2000 = (25 + 25 + ... + 25) + (25 + 25 + ... + 25)$,
$2000 = 2(0 + 1 + 2 + ... + 39) + (11 + 11 + ... + 11)$ and
$2000 = 2(1 + 2 + ... + 40) + (9 + 9 + ... + 9)$. (See Fig.2)

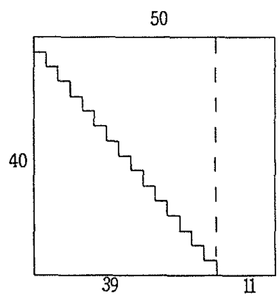

FIGURE 2. *Various divisions of a square.*

2. A Simple Proof of Pythagoras Theorem

We divide two cardboard squares shown in Figs.2 and 3, each of side $a + b$, in two different ways.

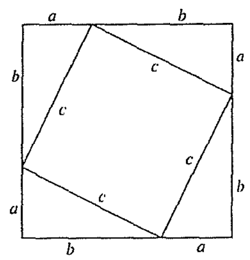

FIGURE 3. *Terms of A.P. are areas of rows.*

The four triangles of Fig.2 are congruent and right angled with hypotenuse length c. If they are cut out, we are left with a square of side length c.

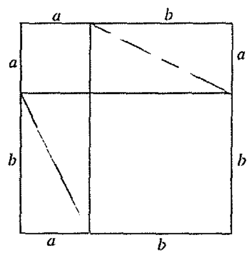

FIGURE 4. *Various divisions of a square.*

These four triangles fit in the rectangles $a \times b$ shown in Fig.3. The rest of the square of Fig.3 is made up of two squares of side a and b respectively.

It follows that $c^2 = a^2 + b^2$.