**Remark:** This letter appeared in the Times of the 9th Feb, 1999, in reply to a query by Mr. M. Pace of the 28th Jan, 1999.[6]

Dear Editor,

## Playing with Nines

In his letter which appeared in the issue of the Times of the 28th January, 1999, Mr. M. Pace pointed out an interesting property of the number nine in the set of integers modulo ten. In general, when we work on the scale of n (where n is 3 or more) the number n-1 exhibits the same properties. So, the number 15 shows these properties in the hexadecimal scale. So does seven in the integers modulo 8.

As Mr. Pace pointed out, if a number is divisible by 9 then the sum of its digits is a multiple of 9. He discussed the product 9x7.

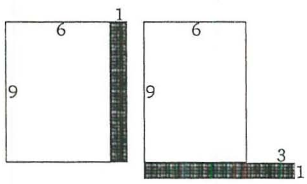


FIGURE 0.2. Areas 9x7= 10 x 6 + 3

One way to see this is to place 9 rows of 7 squares in a rectangular shape. If a column of 9 squares is removed we are left with 9 rows of 6 squares. The 9 squares are now placed as an additional row so that we end up with 10 rows of 6 squares and an extra 3 squares. The product is the area and the sum of the digits of the product is the number of squares in the additional row. It is clear that whatever the number of columns, removing a column and placing the squares removed as an additional row, gives an easy way of working out the product. Besides, the number of squares in the additional row is always nine.

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[6]See p.12.

A more general way to represent the process is to write 9 as 10-1. When a non-zero number z (between 1 and 9) is multiplied by 9 the product may be written as 10z-z or as 10(z-1) + (10-z). Thus the sum of the digits is (z-1) + (10-z) = 9 whatever z is. To take Mr. Paces example namely 9x7, z is 7 and the product has z-1 as the tens digit and 10-z=3 as the units digit. The same holds true for the other example namely 33x9, writing 33 as 3 tens plus 3 units. Now if we were to work 6x7 in the octal scale, we can use the method of fingers mentioned by Mr. Pace. We hold out eight fingers and put down the sixth. The product is 42 modulo 10 which is 5 eights and 2 units or 52 modulo 8 as given by the fingers to the right and left of the sixth.

Playing with numbers can be intriguing as Mr Pace pointed out. The excitement that gripped the mathematical world two years ago when Andrew Wiles solved Fermat's Last Theorem are still vivid in our minds.

Irene Sciriha.

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