Cooling of a particle by coupling to its own reflection

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We identify and explore a cooling force resulting from the retarded dipole interaction between an illuminated particle and its reflection. For a one-dimensional example, we find cooling times of milliseconds and limiting temperatures in the millikelvin range. The force, which may be considered the prototype for cavity-mediated cooling, may also be enhanced by plasmon and cavity resonances at the mirror.

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The revolution in atomic physics brought about by the Doppler cooling of atoms [1] and ions [2], the magneto-optical trap [3], and sub-Doppler cooling mechanisms [4, 5, 6], has prompted the proposal of further schemes [7, 8, 9, 10, 11, 12, 13, 14] which might extend the ultracold domain to a broader range of species. In contrast to the initial reliance upon the scattering force of resonant radiation, these more recent schemes have largely been based upon the optical dipole force [15, 16] that results when there is a spatially-varying interaction with off-resonant illumination. Greatly developed as a powerful and flexible means of *tweezing* atoms and microscopic particles [17], the dipole force is at heart conservative and incapable of cooling; when coupled to a dissipative component or otherwise invested with a non-Markovian character, however, it too can form the basis of a cooling mechanism.

An interesting category of forces to have received much recent interest is that in which an atom or ensemble is both trapped by, but can also affect, the field within an optical cavity, resulting in a cavity-mediated cooling force [10, 11, 12, 13, 14]. As with the cooling of a particle as it moves through a polarization gradient [5], intensity variation [6], speckle field [7] or bichromatic standing wave [8], it is a retarded dependence of the atom-field interaction upon the atom's position which gives a dissipative, velocity-dependent component to the otherwise conservative trapping force. Whereas the retardation in the latter cases results from non-adiabatic decay of atomic state populations, however, in cavity-mediated cooling it is caused by decay of the optical field to which the atom is coupled.

Cavity-mediated cooling may be viewed as a combination of two effects: the retarded perturbation by a trapped particle of the optical field in which it is trapped, as described above; and the amplification of this effect by a resonant optical cavity [18]. In this paper, we consider the simplest case of the former, whereby the perturbed field is returned to the atom by a single mirror. This optical binding [19, 20] phenomenon may be interpreted as the classical electrostatic interaction between the dipoles induced in the particle and its reflection; or as tweezing of the particle by light which the particle itself has focused, as shown in Fig. 1. The finite time taken for light from the particle to return via the mirror causes the reflected image to trail behind the moving particle, thus providing a component of the binding force in the direction of the particle velocity. When the sign of this component is such as to oppose the particle motion, cooling ensues.

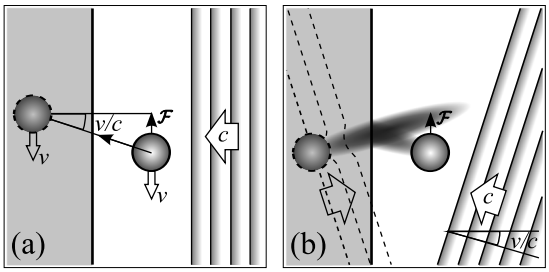

FIG. 1: Retarded binding of a normally-illuminated particle, moving with speed v, to its own reflection, depicted (a) in the laboratory frame, in which the image lags behind; (b) in the rest frame of particle, whereby the 'wake' trails behind.

We analyse the binding between a moving particle and its reflection using a similar approach to that previously applied to the unretarded binding of two point-like particles [21]. We assume an unperturbed illuminating field $\mathcal{E}^0(\mathbf{r}_A)$ of wavelength λ, particle polarizability α, and characterize the round-trip from the particle to its reflection via the mirror by a transit time τ and electric field propagator ζ. The electric field $\mathcal{E}(\mathbf{r}_A)$ experienced by the particle at position $\mathbf{r}_A(t)$ and time t is then, to lowest order in the velocity $\mathbf{v}$ and τ and for $\zeta\alpha, \tau|\mathbf{v}| \ll 1$,

$$\mathcal{E}(\mathbf{r}_A) = \left\{1 - \frac{\alpha\zeta\tau}{1-\alpha\zeta}\mathbf{v}\cdot\nabla\right\}\frac{\mathcal{E}^0(\mathbf{r}_A)}{1-\alpha\zeta} \qquad (1)$$

which corresponds to the result of [21] with the two particles degenerate, modified by the appearance of an additional, velocity-dependent term. With some care over the constraints during differentiation, the dipole force experienced by the particle may be obtained as in [21]. A series expansion in terms of $\alpha\zeta$ reveals the leading terms for a stationary particle to be the dipole force in the un-

perturbed field, then the interaction between two dipoles induced by the unperturbed field and, to the same order, the dipole force upon the polarized particle due to the field propagated from the induced polarization.

For a one-dimensional geometry, the incident illumination combines with its reflection to give the electric field a spatial dependence $\mathcal{E}^0 = \mathcal{E}_0 \sin 2kx$, where $k = 2\pi/\lambda$; $\tau = 2x/c$, and $\zeta = -i \exp 2ikx$ for propagation from a sheet of dipoles a distance x in front of the mirror. With a real susceptibility α, the force $\mathcal{F} = \frac{1}{2}\mathrm{Re}(\alpha\mathcal{E}\nabla\mathcal{E}^*)$ upon the moving particle is therefore

$$\mathcal{F} = \frac{1}{4}\alpha\mathcal{E}_0^2 k \left[\sin 2kx + 2\alpha\left(1-\frac{v}{c}\right)\sin^2 kx(4\cos^2 kx - 1) - 2\alpha k\tau v \sin 4kx\right]. \quad (2)$$

The force thus comprises three terms. The first two are the dipole force exerted upon the particle by the unperturbed field, and a Doppler-shifted optical binding force between the particle and its reflection; the Doppler shift is manifest here not through the change in frequency but through the accompanying change in wavenumber of the components whose superposition yields the field gradient. The third term, which depends upon the particle velocity, the electric field propagator and the round-trip retardation time, is the cooling force, which dominates the velocity-dependent part of the second term when the distance from the mirror is many wavelengths. The second term is supplemented by a further term in (v/c) due to the missing Lorentz component recently noted in [22].

The classical approach is highly instructive, for the arbitrary form of τ and ζ allows the simple geometry of Fig. 1 to be extended to more complex arrangements with curved mirrors and resonant structures, as well as providing a link to the binding and damping of extended [23] or highly polarizable particles and micromechanical structures [24]. In order to examine the dynamics of a particle subjected to the cooling force, however, and to determine the limiting temperature at which the cooling is balanced by heating from off-resonant scattering, we here outline a semi-classical approach [25].

We examine a one-dimensional geometry in which the reflection of a laser-driven atom is imaged back onto itself in a configuration similar to that investigated experimentally in [26], but we consider only a single transverse electromagnetic mode corresponding, for example, to the use of an optical fiber delay line instead of a free-space arrangement. The model thus consists of a single two-level atom coupled to the modes of the electromagnetic field in the right half space, according to the Hamiltonian

$$H = \hbar\omega_a\sigma^+\sigma^- + \frac{p^2}{2m} + \int \hbar\omega a^\dagger(\omega)a(\omega)\mathrm{d}\omega - i\hbar g\int \sin\left(x\frac{\omega}{c}\right)\left[\sigma^+ a(\omega) - a^\dagger(\omega)\sigma^+\right]\mathrm{d}\omega. \quad (3)$$

The first line represents the internal atomic energy given by the transition frequency ω_a, the kinetic energy, and

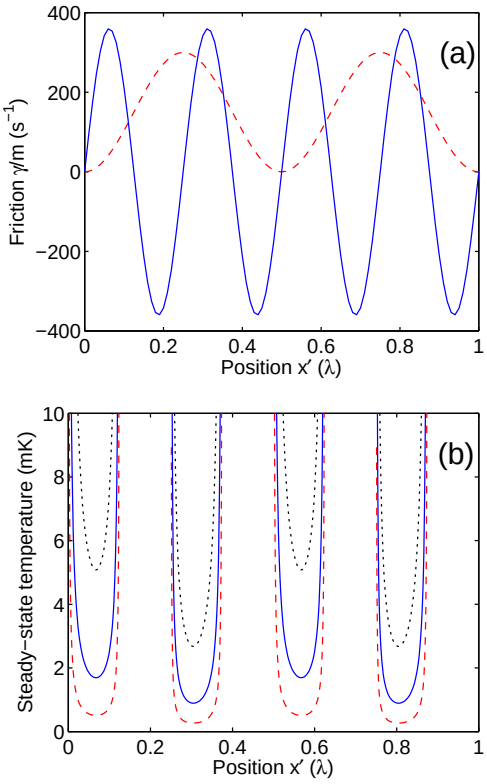

FIG. 2: (a) Spatial dependence of the lowest order friction γ/m (solid line) for $s = 0.1$, $\sigma_a/(\pi w^2) = 0.1$, $x = 3$ m. The dashed line indicates the intensity of the pump wave (arbitrary units). (b) Stationary temperature in a tightly confined harmonic trap versus trap position for $x = 1$ m (dotted), $x = 3$ m (solid), and $x = 10$ m (dashed). Here x' is the position relative to the nearest field node.

the mode energies, respectively, and the second line is the interaction energy between the atomic dipole and the electromagnetic modes with a coupling coefficient g which is assumed constant over the range of relevant frequencies. The density operator ρ of the atom-field system follows the master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}\rho \quad (4)$$

where $\mathcal{L}\rho$ is a standard Liouville term corresponding to spontaneous atomic decay into free space modes with rate 2Γ. For the following discussions we simplify Eqs. (3) and (4) by adiabatic elimination of the atomic excited state.

As a first step, we derive an approximate analytical solution for the friction force resulting from the Hamiltonian (3). To this end we treat the atomic momentum p and position x as classical variables and derive the Heisenberg equations of motion for the mode operators $a(\omega)$. These are then approximated by complex numbers with initial distribution $a(\omega) = A\,\delta(\omega - \omega_0)$ and the stationary solution is found to lowest order in the atom-field

coupling g. We thus obtain the friction force

$$F_v(x) = -\gamma v \tag{5}$$

with the friction coefficient

$$\gamma = \pi\hbar k^2 \tau |A|^2 \frac{g^4}{\Delta^2} \sin(4kx) \tag{6}$$

where $\Delta = \omega_0 - \omega_a$ is the pump detuning, $\tau = 2x/c$ is the propagation time of light from the atom to the mirror and back, and we have assumed $|\Delta| \gg \Gamma$. Using $2\pi g^2 = 4\Gamma\sigma_a/(\pi w^2)$ [27], where $\sigma_a = 3\lambda^2/(2\pi)$ is the atomic scattering cross section and w is the mode waist, and introducing the atomic saturation $s = g^2|A|^2/\Delta^2$, we can rewrite γ as

$$\gamma = 2\hbar k^2 \Gamma \tau s \frac{\sigma_a}{\pi w^2} \sin(4kx). \tag{7}$$

Figure 2(a) shows the variation of γ with atomic position in the standing pump wave: regions of cooling ($\gamma > 0$) alternate with regions of heating ($\gamma < 0$). In order to obtain cooling of a particle by this mirror-induced friction force it will therefore be necessary to trap the particle, e.g. by a far-off resonant dipole trap, to within $\lambda/8$; because the mean kinetic energy of the trapped particle will then be coupled to an equal but uncooled mean potential energy, the cooling rate will be halved. For the chosen parameters, the theory predicts cooling times $\sim (\gamma/m)^{-1}$ of the order of 3 ms at positions of maximum friction.

To estimate the steady-state temperature achievable using this cooling scheme, we need an estimate of the momentum diffusion coefficient. The leading order term in the weak coupling limit assumed above is simply given by the well-known result for diffusion in the standing wave formed by the pump field [28], i.e.,

$$D = \hbar^2 k^2 \Gamma s \left[\cos^2(kx) + \frac{2}{5}\sin^2(kx)\right] \tag{8}$$

where the factor 2/5 accounts for the spatial distribution of circularly polarized spontaneous emission. The stationary temperature is hence given by

$$\begin{aligned} k_B T &= \frac{D}{\gamma} = \frac{\hbar}{\tau}\frac{\pi w^2}{2\sigma_a}\frac{\cos^2(kx) + \frac{2}{5}\sin^2(kx)}{\sin(4kx)} \\ &\approx \hbar\Gamma\frac{\pi w^2}{4\sigma_a \Gamma\tau}. \end{aligned} \tag{9}$$

The final expression of Eq. (9) holds at the position of maximum friction. We see that the expected stationary temperature is given to within a numerical factor by the Doppler limit $\hbar\Gamma$, scaled by the ratio of the cross-section πw^2 of the pump beam to that of the atom and with the atom-mirror round-trip time τ replacing the atomic coherence time $1/\Gamma$, but is independent of detuning, pump intensity and atomic saturation. Figure 2(b) shows the stationary temperature of an atom in a tightly confining harmonic trap versus trap position. For atomic rubidium with the same parameters as in Fig. 2(a), we obtain temperatures of the order of 1 mK. While this is higher than the Doppler temperature of 140 μK, the independence of detuning means that, for sufficiently off-resonant operation ($|\Delta| \gtrsim 10\Gamma$), mirror-mediated cooling will be the dominant cooling effect.

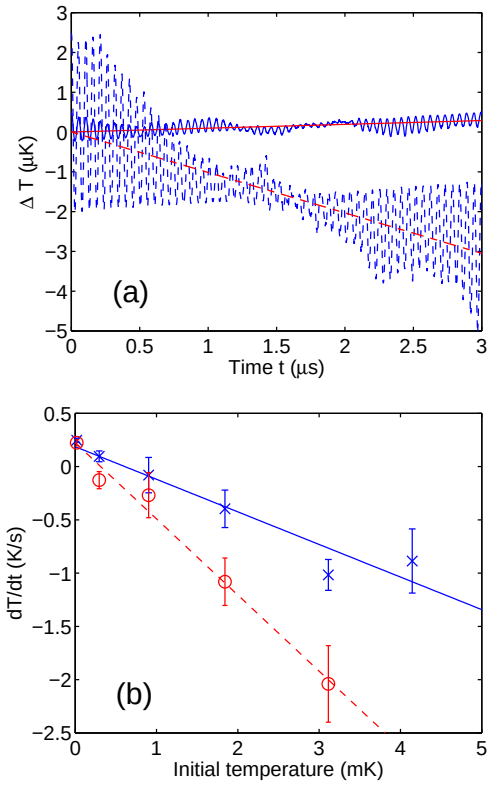

FIG. 3: (a) Change of temperature versus time for initial temperatures of 0.3 mK (solid curves) and 3.1 mK (dashed curves). Simulation results are averaged over 10^4 Monte-Carlo trajectories and fast oscillations with the trap frequency have been filtered out, straight lines are linear fits. Parameters are: mode diameter 1.4 μm, atomic saturation 0.073, trap frequency $\nu_{\rm trap} = 1.5$ MHz, and $\tau = 26.5$ ns. (b) Rate of temperature change versus initial temperature. Data points are simulation results, lines are linear fits. Crosses and solid line are for $\nu_{\rm trap} = 1.5$ MHz, and $\tau = 26.5$ ns, circles and dashed line are for $\nu_{\rm trap} = 750$ kHz, and $\tau = 53$ ns.

We have confirmed the predictions of this simplified analytical model by semiclassical Monte-Carlo simulations of a rubidium atom coupled to a discretized set of modes [29]. This model accurately describes quantum fluctuations of photon numbers as well as of atomic momentum; it is valid for arbitrary coupling strength and arbitrary atomic velocity, and an external harmonic trap is easily incorporated. However, the numerical model does not allow a single trajectory to be followed towards a stationary state because of the relatively long cooling times (of the order of ms as outlined above). Instead, we

performed the simulations for atoms with various initial kinetic energies and monitored the initial change of energy with time. Two examples are shown in Fig. 3(a). For an initially cold sample at 0.3 mK, diffusion dominates over friction and the temperature increases with time. For a larger initial temperature of 3.1 mK, on the other hand, friction dominates and a net cooling effect is observed; and a sample with exactly the stationary temperature remains unchanged over time. Figure 3(b) summarizes the results for $\mathrm{d}T/\mathrm{d}t$ obtained from simulations with varying initial temperatures for two different sets of parameters. A linear fit to the initial temperature, consistent with a simple random walk model, predicts a steady-state temperature of $T = 0.61 \pm 0.17$ mK for trap frequency $\nu_{\mathrm{trap}} = 1.5$ MHz and delay time $\tau = 26.5$ ns, and $T = 0.31 \pm 0.10$ mK for $\nu_{\mathrm{trap}} = 750$ kHz and $\tau = 53$ ns, where the errors result from the uncertainty of 10^4 Monte-Carlo trajectories for each set of parameters. For trapped particles, our analytical expression (9) must be modified to account for the period of order τ after each turning point in the particle's trajectory, during which the newly-reversed motion is accelerated rather than diminished by the retarded cooling force. For the parameters of Fig. 3, this introduces a further factor of 0.64, yielding temperatures of 0.58 mK and 0.30 mK, consistent with our numerical results. There will be a further small correction because the trajectories of the trapped particles extend beyond the region of maximal cooling.

The geometry considered here resembles that in the beautiful experiments of [26], and shares with cavity-mediated cooling schemes the requirement for a tightly focused image and hence achieves cooling only within a small volume. Mirror-mediated cooling, however, permits a number of possible enhancements. Firstly, the mirror may be replaced by an external resonator, such as an etalon or multilayer filter, which allows the same retardation τ to be achieved with a greatly reduced mirror distance. Material or spatial resonances such as the plasmon modes of a micro-structured mirror or antenna might achieve this while also enhancing the strength of the scattered field. As possible examples, we envisage microscopic antennae [30] or hemispherical, plasmon-resonant resonators [31, 32]. These can be produced as arrays so that, like a cobbled street for a bicycle, the dissipative effect is reproduced over an extended area.

Two related phenomena are associated with mirror-mediated cooling geometries. Firstly, the Casimir-Polder force is the effect of retardation upon the van der Waals interaction of a vacuum-polarized particle with its reflection [33]: mirror-mediated cooling results instead from a small component of the much larger laser-induced force. Resistivity in the mirror meanwhile provides an alternative dissipative component that results in friction [34].

Although we have here analyzed the longitudinal cooling of a 1-D geometry, we find that, in three dimensions, transverse cooling is also possible. Higher order terms can be significant, particularly for more macroscopic particles; and we expect that, as with cavity-mediated cooling, collective effects will be significant for ensembles.

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