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Predicting the Production of Total Industry in Greece with Chaos Theory and Neural Networks

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Abstract:
This paper explores the use of chaos theory, as well as the neural networks, for predicting the Production of Total Industry in Greece. We have found that our data (from 1961 up to 2011) obey to the chaos theory. More specifically, the results from evaluation show that the minimum emending dimension is 4 suggesting chaos with a high dimensionality. We have also found that it is predictable the behavior of this production in the near future. The same results were evaluated using neural network, confirming our prediction.

Key Words:
Total industry production, chaos false neighbors, neural networks,

***JEL Classification**:*

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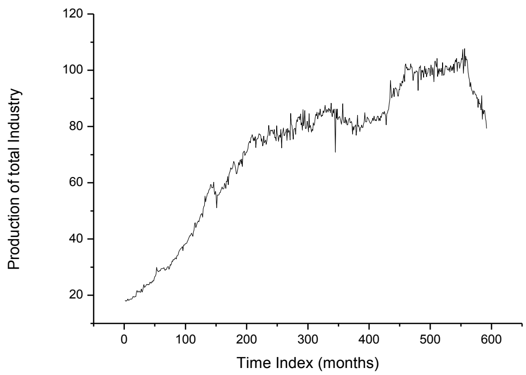
1. Introduction

Non linear dynamics (Medio, 1992) in combination with neural networks had applied in a wide variety of fields, e.g. physics, engineering, ecology and economics. The economist interest is focused on the ability of forecasting an economic time series using time series analysis (Thalassinos and Pociovalisteanu, 2007). In this work we have applied non linear time series analysis in monthly values of Greece Total Industry index. We cover time period from 01.01.1962 until 01.04.2011. We have applied the method of Grassberger and Procaccia (Grassberge and Procaccia, 1983a and 1983b) to evaluate the minimum embedding dimension of each the system. In a second stage using the neural network (Hanias, Curtis, Thalassinos, 2007; Thalassinos *et al.,* 2008 and 2009) we achieved an out of sample multi step time series prediction.

2. Time Series

The data for the Production of Total Industry in Greece are collected from Organization for Economic Co-operation and Development and presented as a signal x=x(t) as it shown at Figure 1 (01.01.1962 – 01.04.2011). The sampling rate is $\Delta t=1$ month and the number of data are N=592.

Figure 1: Time series of Total Industry Production



3. State Space Reconstruction

For a scalar time series, in our case the time series is the Production of Total Industry index, the phase space can be reconstructed using the methods of delays. The basic idea in the method of delays is that the evolution of any single variable of a system is determined by the other variables, with which it interacts. Information about the relevant variables is thus implicitly contained in the history of any single variable. On the basis of this an "equivalent" phase space can be reconstructed.

From our data we construct a vector $\vec{X}_i$, i=1 to N, in the m dimensional phase space given by the following relation (Kantz and Schreiber, 1997; Takens, 1981).

$$\vec{X}_i = \{xi, xi-\tau, xi-2\tau, \ldots xi+(m-1)\tau\} \quad (1)$$

This vector represents a point to the *m* dimensional phase space in which the attractor is embedded each time, where τ is the time delay $\tau = i\Delta t$. The element xi represents a value of the examined scalar time series in time corresponding to the *i-th* component of the time series. Use of this method, reduces phase space reconstruction to the problem of proper determining suitable values of *m* and τ. The choice of these values is not always simple, especially when we do not have any additional information about the original system and the only source of data is a simple sequence of scalar values, acquired from the original system. The dimension, where a time delay reconstruction of the phase space provides a necessary number of coordinates to unfold the dynamics from overlaps on itself caused by projection, is called embedding dimension *m*.

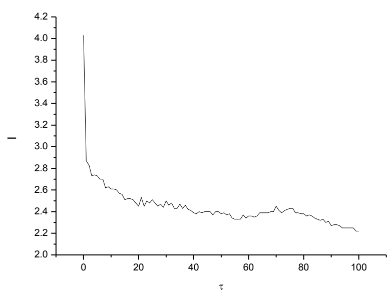
a. Time delay τ

Using the average mutual information we can obtain τ less associated with linear point of view, and thus more suitable for dealing with nonlinear problems. The average mutual information may be expressed by the following formula (Kantz and Schreiber, 1997; Takens, 1981).

$$I(\tau) = \sum_{x_i, x_{i+\tau}} P(x_i, x_{i+\tau}) \log_2 \left(\frac{P(x_i, x_{i+\tau})}{P(x_i) P(x_{i+\tau})} \right) \quad (2)$$

where P(xi) represents probability of value xi and P(xi, xi+τ) is joint probability. In general, I(τ) expresses the amount of information (in bits), which may be extracted from the value in time xi about the value in time xi+τ. As τ, suitable for the phase space reconstruction, is the first minimum of I(t).

Figure 2: Average mutual information I(τ) vs time delay τ



As shown in Figure 2, in our case the mutual information function I(t) exhibits a local minimum at 4 time steps and, thus, we shall consider τ =4 to be the optimum delay time.

b. Embedding dimension m

One method to determine the presence of chaos is to calculate the fractal dimension, which will be non integer for chaotic systems. Even though there exists a number of definitions for the dimension of a fractal object (Box counting dimension, Information Dimension, etc.), the correlation dimension was found to be the most efficient for practical applications. Firstly we calculate the correlation integral [7, 8] for the time series for lim $r \rightarrow 0$ and $N \rightarrow \infty$ by using the equation 3 [2].

$$C(r) = \frac{1}{N_{pairs}} \sum_{\substack{i=1,\\ j=i+1}}^{N} H\left(r - \left\| \vec{X}_i - \vec{X}_J \right\|\right) \tag{3}$$

In this equation, the summation counts the number of pairs for which the distance (Euclidean norm) is less than r, in an m dimensional Euclidean space. H is the Heaviside step function, with H(u)=1 for u>0, and H(u)=0 for $u \leq 0$,where $u : \left(r - \left\| \vec{X}_i - \vec{X}_J \right\|\right)$,

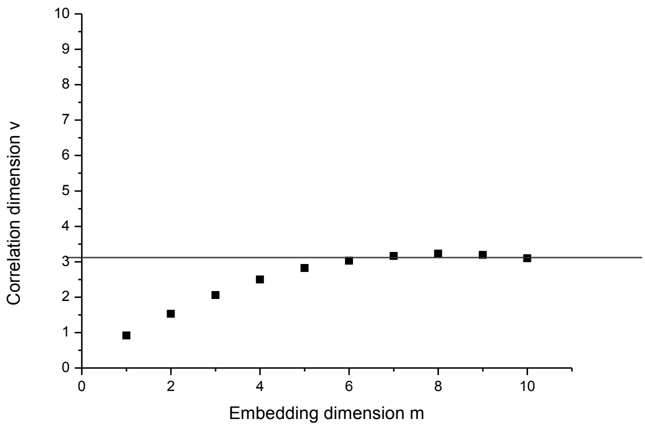
N denotes the number of points and expressed in equation 4.

$$N_{pairs} = \frac{2}{(N-m+1)^2} \tag{4}$$

Where r is the radius of the sphere centered on Xi or Xj.

If the time series is characterized by an attractor, then for positive values of r, the correlation function is related to the radius with a power law C(r)~αrv , where α is a constant and ν is the correlation dimension or the slope of the logC(r) versus logr plot. Since the data set will be continuous, r cannot get to close to zero. To handle this situation, from log C (r) versus log r plot we select the apparently linear portion of the graph. The slope of this portion will approximate ν. Practically one computes the correlation integral for increasing embedding dimension m and calculates the related ν(m) in the scaling region. Using the appropriate delay time τ=4 we reconstruct the phase space. The correlation integral C(r) by definition is the limit of correlation sum of Equation (3) for different embedding dimensions, m=1.10. In Fig.3, the corresponding average slopes v are given as a function of the embedding dimension m, indicating that for high values of m, v tends to saturate at the non integer value of 3.10. For this value of v, the minimum embedding dimension can be mmin = 4 [5], and thus, the minimum embedding dimension of the attractor for one to one embedding will be equal to 4.

Figure 3: Correlation dimension v vs embedding dimension m



4. Time Series Prediction with Chaos Theory

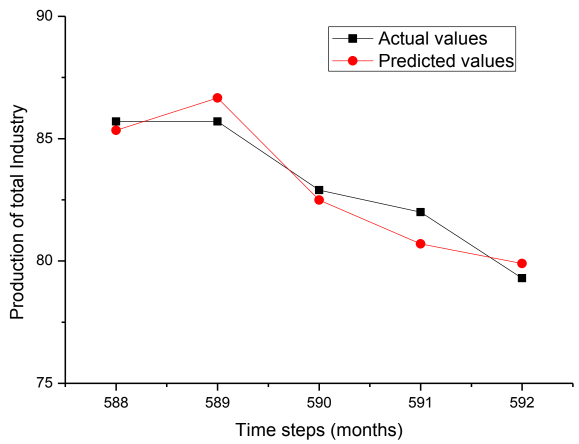
The predictability of a time series using phase space techniques can be considered as a test for the deterministic nature of the system. These prediction techniques have been based on the fact that nearby trajectories, either converge or do not diverge fast enough for small sample steps in the phase space. For this purpose we calculate weighted average of evolution of close neighbours of the predicted state in the reconstructed phase space. The reconstructed m-dimensional signal projected into the state space can exhibit a range of trajectories, some of which have structures or patterns that can be used for system prediction and modelling. To predict k steps into the future from the last m-dimensional vector point, we should find all the nearest neighbours $\{x_{NN}^m\}$ in the ε- neighbourhood of this point.

Let $B_\varepsilon(x_N^m)$ be the set of points within ε of $\{x_N^m\}$ (i.e. the ε-ball). Thus any point in $B_\varepsilon(x_N^m)$ is closer to the $\{x_N^m\}$ than ε [5]. All these points $\{x_{NN}^m\}$ come from the previous trajectories of the system and hence we can follow their evolution k-steps into the future $\{x_{NN+k}^m\}$. This evolution depends on the shape of corresponding strange attractor. The final prediction for the point $\{x_N^m\}$ is obtained by averaging over all neighbours' projections k-steps into the future. The number of k depends on how the stretching and folding is done. The methodology is expressed in equation 5 [9].

$$\{x_{N+k}^{m}\} = \frac{1}{\left|B_{\in}(x_{NN}^{m})\right|} \sum_{X_{NN}^{m} \in B_{\in}(x_{NN}^{m})} x_{NN+k}^{m} \tag{5}$$

where $\left|B_{\in}(x_{NN}^{m})\right|$ is the number of nearest neighbours in the neighbourhood of the point $\{x_{N}^{m}\}$ representing the last known sample from which we want to predict one and two steps into the future. Using the values of τ=4, m=4 and the number of nearest neighbours equal to 27 the actual and predicted time series for k= 5 time steps out of sample ahead are presented at Figure 4.

Figure 4: Actual (squares) and predicted (circles) values for k=5 time steps ahead



The corresponding values with the Normalized mean Square error are presented at Table 1.

Table 1: Actual and predicted values of Production of Total Industry

Time Index	Actual Value	Predicted Value	Error (NMSE)
588	85.7	85.34552	0.08726
589	85.7	86.66168	0.36475
590	82.9	82.48828	0.22426
591	82	80.7035	0.24815
592	79.3	79.89868	0.08415

5. Time Series Prediction with Neural Networks

In order to predict the time series we construct a backpropagation network [4, 10,11] that consists of 1 input layer 1 middle or hidden layers and 1 output layer. The input layer has number of neurons equal to an integer multiple of m as a rule of thumb and the 1st hidden layer has m neurons as a rule of thumb too. We choose the input and 1st hidden layer to have this number of inputs to avoid temporal correlation, and because the attractor is embedded at a m phase space the last hidden layer has m neurons. As an example beginning with the first set of inputs x1,x2,x3…x48 the output is the x49. Then with an iterative process we attempt to predict the next 4 values until x52. We repeat the process for all training sets. We train the network with a training set of 588 exemplars using the 75% of data set. The learning rate was β=0.05 and the momentum α=0.5[4]. Each network was training for 5000 epochs. In Figure 5, in sample actual and predicted values during the learning process are presented, while, in Figure 6, out of sample actual and predicted values are presented using the multistep iterative prediction process.

Figure 5:
In sample, actual (crosses) and predicted values (solid line) during the learning process

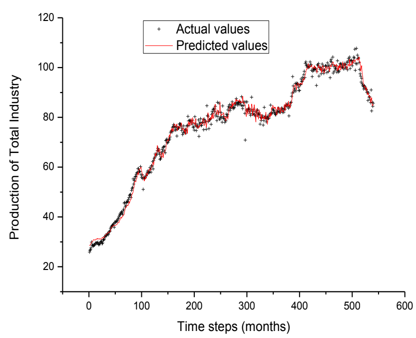
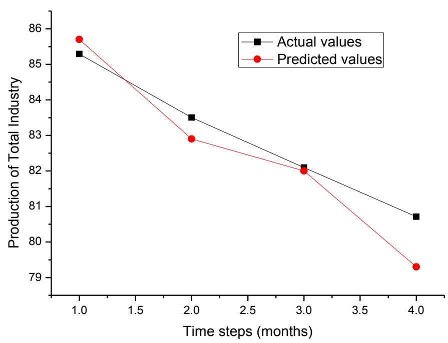


Figure 6:
Actual (squares) and predicted (circled) out of sample values



6. Conclusion

In this paper, we use a non linear analysis in combination with neural networks to predict the Production of Total Industry in Greece. After estimating the minimum embedding we point out that the system is chaotic with high dimensionality. Based on the systems' strange attractor's reconstruction we achieved a 5 time steps out of sample prediction. Also we construct a backpropagation neural network with 1 hidden layer and achieved a reliable 4 time steps out of sample prediction.

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