European Research Studies Journal
Volume XX, Issue 4A, 2017

pp. 77-101

Economic and Mathematical Model of Business Struggle with a Vertically Integrated Company in the Market

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Abstract:
The paper contains an analysis of the market structure when the inferior process chain company supplies the superior company with input resources and at the same time is vertically integrated with one of the superior companies. The possible anticompetitive effects of such vertically integrated structures are analyzed as well as the influence of information flows between inferior and superior enterprises of the sector on drivers of innovation and national welfare.

These issues are relevant, if important information, in particular, information technology, design and specific characteristics of the product is available to inferior and superior sector enterprises. This situation is typical for high-tech sectors in which the inferior and superior product information exchange needs ensuring compatibility of products and preventing the additional costs of adjustment and increased functionality. The problem of assessing the appropriateness of imposing a ban on various divisions of a vertically integrated company to exchange non-public information received by one of its divisions from external sources is important. In the case of such a restriction, the inferior organizational division of a vertically integrated company may use personal information of the superior competitor only within the powers of its supplier, and cannot make it public for its superior organizational division as an example. The aim of the research is to develop an economic and mathematical model of business struggle with a vertically integrated company in the market. The paper contains the hypotheses and supporting evidence.

Keywords: *economic and mathematic modeling, vertically integrated company, regional economics.*
JEL Classification: *C02, R10*

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1. Indroduction

The basic hypothesis is that information flows between the inferior and superior companies in the sector do not necessarily lead to a reduction of incentives for innovation and national welfare. It shows that the information flows lead to lower incentives for innovation in the superior self-dependent company. However, they lead to increased incentives for the innovation development of a vertically integrated company, at least if the investments in R&D companies are complementary. On the contrary, if the companies are involved in the innovative development competition, the incentives for innovations of a vertically integrated company are decreased with increasing intensity of external effects related to the information flow between the inferior and superior companies in the sector.

However, with both forms of R&D, the "effective" R&D investments of a vertically integrated company are higher with external information flows than if various organizational divisions of a vertically integrated company are prohibited to exchange non-public information obtained by one of its organizational divisions from outside sources. If companies' investments in R&D are complementary, the national welfare is higher with external information flows than if various organizational divisions of a vertically integrated company are prohibited to exchange non-public information received by one of its organizational divisions from external sources, unless the level of externalities is very high, the types of products are not close substitutes, and the costs of innovative development are not too high. Also, if investments in R&D are an alternative, national welfare is higher with external information flows. Thus, this analysis indicates the inappropriate establishment of prohibition to exchange non-public information obtained by one of the various parts of vertically integrated company organizational divisions from third-party sources.

The main purpose of the work is to develop an economic and mathematical model of competition with the presence of vertically integrated companies in the market; to study the impact of information flows externalities on the market equilibrium (the output volume, the wholesale resource price, the competing companies' profit and their investments in R&D); to analyze the impact of externalities and information flows on the companies' incentives to develop innovation and national welfare.

2. Methodology

The following methods are used: economic and mathematical modeling, the theory of optimization, the game theory (Blackwell and Girshick, 1979; Elettreby, 2016; Elsadany, 2017), the utility theory (Von Neumann and Morgenstern 2007), the random functions theory (Pugachev 1960; Pugachev 2013), analysis of differential equations (Ovsiannikov 2014; Frank, Mashevskaya, and Ermolina 2016), comparative static equilibrium (Hale and Lunel 2013), graphics, and constructive research.

The theoretical and methodological basis of the research are the works of scientists in the field of mathematical simulation of economic processes and systems, the enterprise theory, the game theory, methods of optimization, and methods of mathematical statistics.

For the problems analysis, there is a model in which the company is a vertically integrated monopoly in the inferior resource market and at the same time, it competes in the market with the superior self-dependent company. The game is analyzed and includes three stages. At the first stage, integrated and non-integrated companies simultaneously and independently choose their R&D investment. At the second stage, a vertically integrated company offers input resources to a self-dependent company at a wholesale price. Finally, at the third stage, companies produce differentiated products and compete in outputs in the final product market.

The practical significance of the study results is that the represented economic and mathematical models and methods allow solving tactical and strategic objectives when developing effective strategies and forms of horizontal and vertical integration of enterprises with due regard to the market uncertainty. The game theory model of horizontal integration of stochastically conditions allows analyzing the companies' incentives to division and the integrated companies' influence on the national welfare in relation to the type of market uncertainty. The economic and mathematical model of competition with the presence of a vertically integrated company in the market may be used to evaluate the influence of vertically integrated structures to the competition in the superior market and to analyze the intensity of external informational flows on the companies' incentives to develop innovation and national welfare.

3. Results

It is necessary to consider the sector that includes two companies, one of which is a vertically integrated company UD_1, and the second is a self-dependent superior company D_2. The inferior organizational division of a vertically integrated company U is a monopoly supplier of resources used in production. The superior organizational division of a vertically integrated company D_1 receives the monopoly supplier inputs at marginal costs, and a self-dependent superior in technology chain company D_2 receives them at an endogenously determined wholesale price w.

It is essential to consider a "game" that consists of three stages. At the first stage, both companies simultaneously and independently choose their R&D investment. At the second stage, a vertically integrated company UD_1 offers input resources at a wholesale price to a self-dependent company D_2. Finally, at the third stage, companies produce differentiated products and compete in outputs.

Inverse demand functions for UD_1 and D_2 companies are:

$$p_1 = a - q_1 - dq_2\,,\; p_2 = a - q_2 - dq_1\,, \tag{1}$$

where $a > 0$, $0 \le d < 1$ is the degree of product differentiation, and q_1 and q_2 are volumes of UD_1 and D_2 final production. These demand functions are derived with the use of the utility function of a typical consumer that depends on the final product output and has the following form:

$$U = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2dq_1 q_2)\,.$$

Functions that characterize the costs of UD_1 and D_2 are:

$$C_1(x_1, x_2, k, q_1) = (A - x_1 - kx_2)q_1\,,$$
$$C_2(x_2, k, q_1) = (A + w - x_2)q_2\,, \tag{2}$$

where $x_1 \ge 0$ and $x_2 \ge 0$ are volumes of investments by UD_1 and D_2 in R&D,

$$A \ge x_1 + kx_2\,,\; A \ge x_2 - w\,,\; 0 \le k \le 1.$$

If the UD_1 and D_2 companies' investments in R&D are not present, the specific costs of both companies are A. The parameter k determines the degree of external effects associated with innovative developments. The parameter k reflects the extent to which the superior company uses the technological results obtained from competitor's investments in R&D.

Considering the influence of information flows between two organizational divisions of a vertically integrated company it is assumed that the non-integrated superior company does not receive any externalities ($k = 0$), whereas an integrated company receives externalities ($k > 0$), if there is an information flow relative to the research developments undertaken by the company D_2 between different UD_1 company's organizational divisions. In other words, in equation (2) it is set $k = 0$ if various organizational divisions of a vertically integrated company are prohibited to exchange non-public information obtained by one of its divisions from third-party sources, and $k > 0$ with the availability of information flows. Even in the absence of investment in R&D the self-dependent company D_2 faces additional costs associated with buying resources at a wholesale price w from the inferior monopolist U.

The investments in R&D are subject to a law of diminishing returns that is considered in the quadratic function of the costs of R&D

$$U(x_i) = \mu \frac{x_i^2}{2}\,, \tag{3}$$

where $\mu > 0$, $i = 1{,}2$.

According to equation (3), the division cost of R&D increases with the size of the research laboratory. Thus, higher levels of investment in R&D require proportionately higher costs for the functioning of the research laboratory. In addition, a higher cost parameter μ reflects the lower efficiency of investment in R&D. It is possible to assume that the second-order conditions are met, and companies choose strictly positive outputs in R&D.

3.1 Equilibrium output volumes, profits of competing companies and their investments in R&D analysis

The profit of UD_1 and D_2 companies is determined by functions

$$\pi_1 = wq_2 + (a - q_1 - dq_2)q_1 - (A - x_1 - kx_2)q_1 - \mu\frac{x_1^2}{2}, \tag{4}$$

$$\pi_2 = (a - q_2 - dq_1)q_2 - (A + w - x_2)q_2 - \mu\frac{x_2^2}{2}. \tag{5}$$

The profit of an integrated company includes profits from the input resource and the final product sales.

To maximize its profit each company decides on the output volume at the final “game” stage. The first profit maximization order conditions give the following response functions relative to the output volumes

$$R_1(q_2) = \frac{v + (x_1 + kx_2) - dq_2}{2},$$
$$R_2(q_1) = \frac{v + (x_2 - w) - dq_1}{2}, \tag{6}$$

where $v = a - A$ is the market size. The terms in brackets represent an "effective" specific costs reduction for each company. The Cournot-Nash equilibrium final outputs (Nash 1950; Haurie and Marcotte 1985; De Wolf and Smeers 1997; Chen, Lu, Zeng, Dong, Hu, Guo and Chen 2017; Anikina, Gukova, Golodova and Chekalkina, 2016; Bashmakov, Popov, Zhedyaevskii, Chikichev and Voyakin 2015) are determined by the following equations:

$$q_1^C(w, x_1, x_2) = \frac{v(2-d) - d(x_2 - w) + 2(x_1 + kx_2)}{4 - d^2}, \tag{7}$$

$$q_2^C(w,x_1,x_2)=\frac{v(2-d)+2(x_2-w)-d(x_1+kx_2)}{4-d^2}. \tag{8}$$

At the second "game" stage the vertically integrated company UD_1 chooses the wholesale resource price in order to maximize its profit. By substituting (7) and (8) in (4) and calculating the first profit maximization order conditions relative to w, the following Equation (9) can be derived:

$$w(x_1,x_2,k)=\frac{v(8-4d^2+d^3)+d^3x_1+x_2(kd^3+4(2-d^2))}{2(8-3d^2)}, \tag{9}$$

The analysis (9) shows that the wholesale price of the used resource is always positive; therefore, an independent company D_2 always faces an unprofitable cost ratio relative to a vertically integrated company UD_1.

By substituting (9) in (7) and (8), the expression of companies' volumes outputs can be derived as a function of their investments in research and development, given in Eq. (10) and (11):

$$q_1(x_1,x_2,k)=\frac{v(8-2d-d^2)+x_1(8-d^2)-x_2(2d-k(2-d^2))}{2(8-3d^2)}, \tag{10}$$

$$q_2(x_1,x_2,k)=2\frac{v(1-d)-dx_1+x_2(1-dk)}{8-3d^2}. \tag{11}$$

The firms choose their research and development investment volume from the profit maximization, at the first stage. By substituting the wholesale price and volumes of companies' output as a function of their investments in research and development (9) - (11) in the profit function (4) and (5) and differentiating the company UD_1 profit function relative to x_1 and the company D_2 profit function relative to x_2, the companies' reaction curves with regard to their research and development investment volume can be derived as following Eq. (12) and (13):

$$R_1(x_2,k)=\frac{v(8-4d+d^2)-x_2(4d-k(8+d^2))}{2\mu(8-3d^2)-8-d^2}, \tag{12}$$

$$R_2(x_1,k)=\frac{8(1-dk)(v(1-d)-dx_1)}{2\mu(8-3d^2)-8-d^2} \tag{13}$$

On the assumption that the denominators of both response functions are positive, from expression (13) it follows that the response curve $R_2(x_1,k)$ is characterized by a negative slope, i.e., R&D investments are strategic substitutes from the company D_2 perspective. Consequently, company UD_1 increasing R&D investment reduces the D_2 company's incentives to invest in R&D, and if various organizational divisions of a vertically integrated company are prohibited to exchange non-public information received by one of its organizational divisions from outside sources, $k = 0$, and in the case of information flows presence, $k > 0$. On the contrary, R&D investments can be either strategic substitutes or strategic complements from the point of view of the company UD_1. It is reflected in the equation (12), where the response curve slope depends on the numerator factor x_2.

R&D investments are strategic substitutes (strategic complements) only at low (high) degrees of external effects if the parameter k is less (greater) than the critical value

$$k_{cr} = \frac{4d}{8 + d^2}.$$

Figure 1 corresponds to a situation where R&D investments are strategic substitutes for both companies. In this case, when an independent company reduces its R&D investment, an integrated company increases its investment. However, if R&D investments are strategic complement only from the integrated company point of view (Figure 2), an integrated company does not respond to R&D reduction investment by investing less in its own innovative developments. The arrows show the response curves shift with the parameter k growth (see the detailed analysis of the response functions is in the next section). Equations (12) and (13) show the equilibrium levels of R&D investments that are determined by equations

$$x_1^*(k) = v\frac{B + K}{D + L}, \quad (14)$$

$$x_2^*(k) = 8v\frac{(1 - dk)E}{D + L}, \quad (15)$$

with the following parameters

$$B = \mu[16(4 - 2d - d^2) + 3d^2(4 - d)] - 8 > 0,$$

$$K = 8k[1 + d(1 - k)] > 0,$$

$$D = \mu[2\mu(8 - 3d^2)^2 - 16(5 - d^2) + 3d^4) + 8 > 0,$$

$$L = 8dk[2\mu(2 - dk) - 1] > 0,$$

$$E = 2\mu(1 - d) - 1 > 0.$$

Figure 1. *Company' response curves to the volume of their R&D investments (R&D investments are strategic substitutes for both companies)*

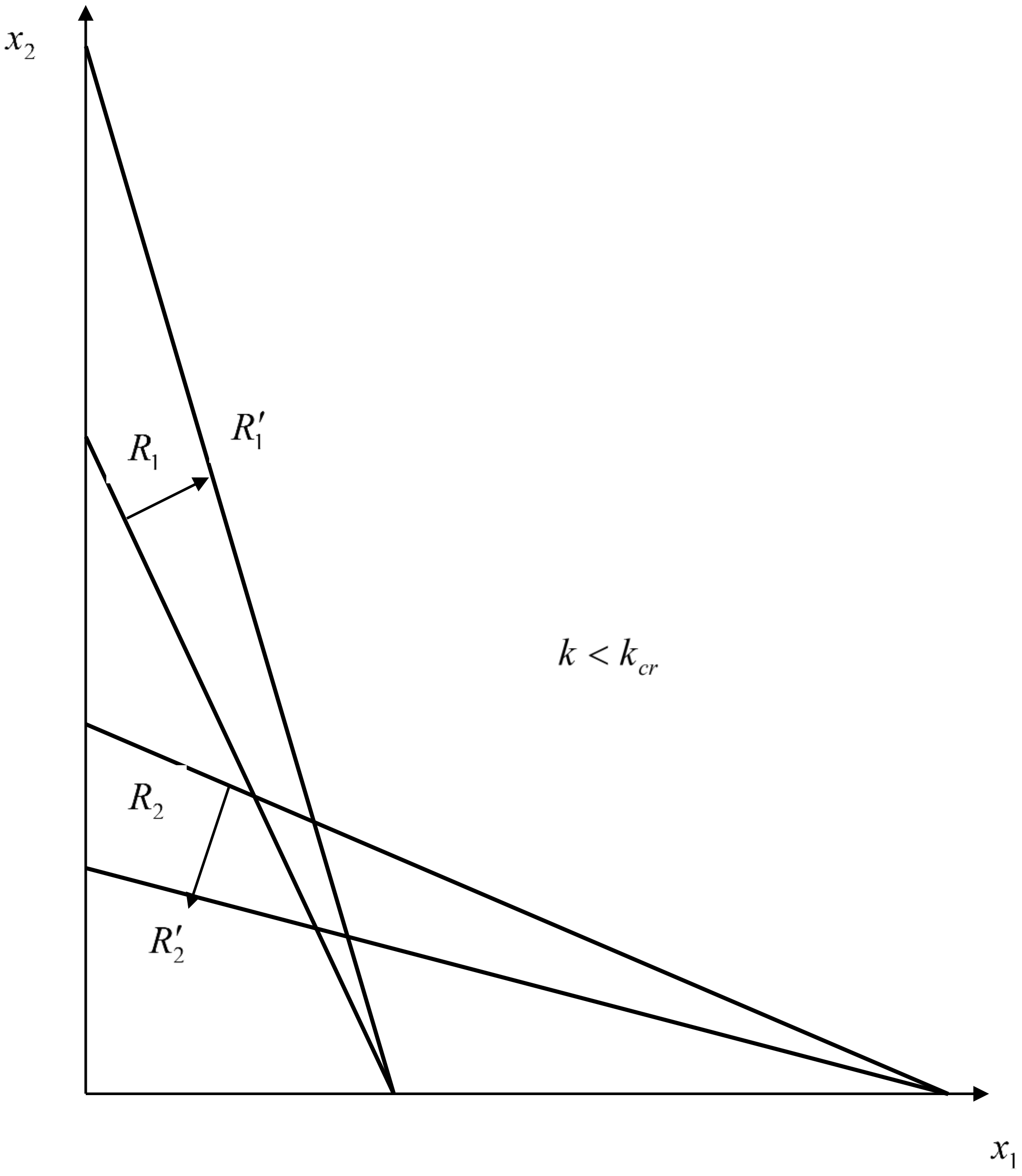
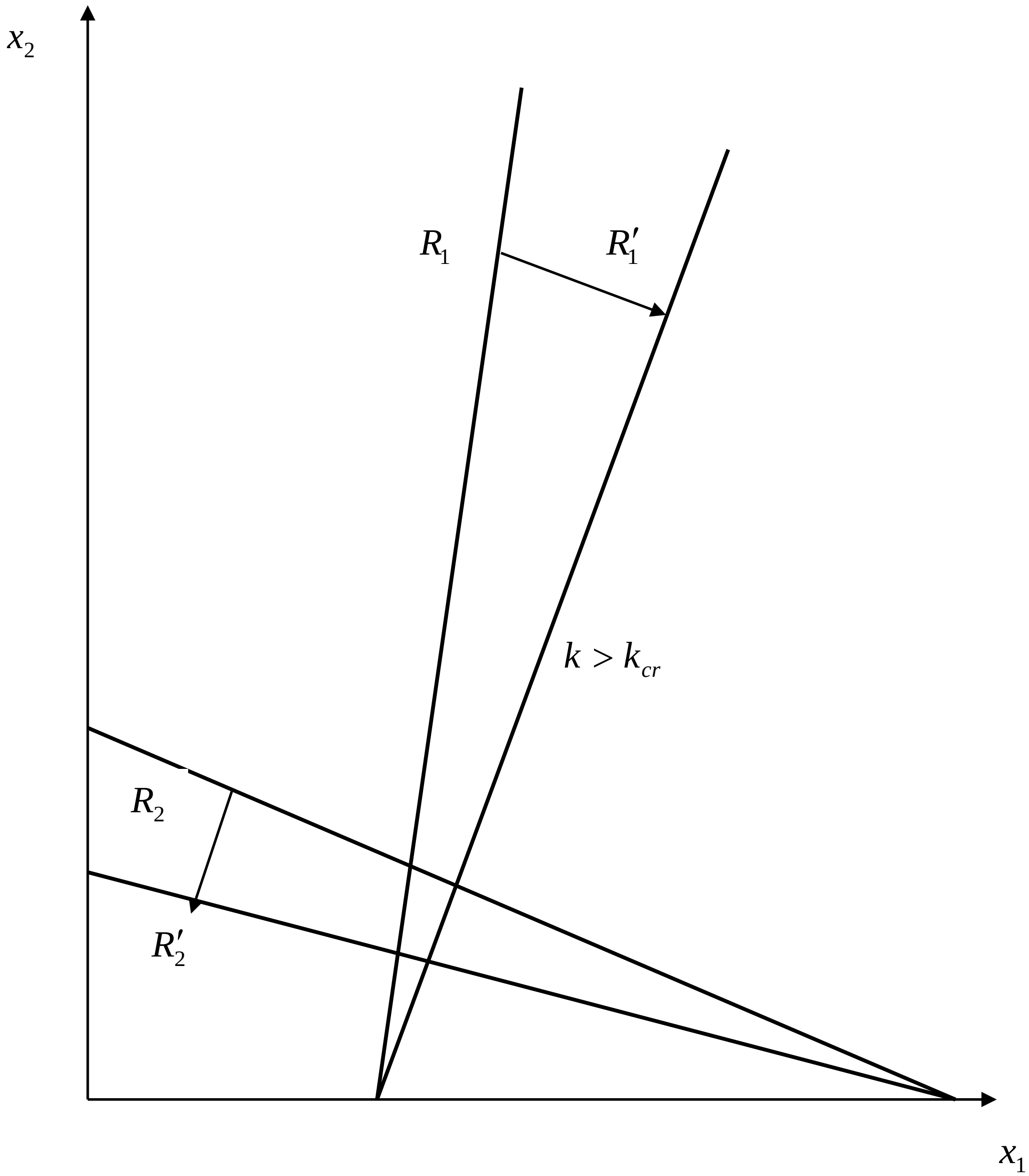


Figure 2. *Companies' response curves to the volume of their R&D investments (R&D investments are strategic complements for both companies)*



Finally, by substituting expressions for the equilibrium levels of the research and development investments in Eq. (14) and (15) into (9), (10) and (11), the expressions for the wholesale price and production volumes equilibrium values can be derived:

$$w^*(k) = \mu v \frac{F + dK}{D + L}, \tag{16}$$

$$q_1^*(k) = \mu v \frac{G + K}{D + L}, \tag{17}$$

$$q_2^*(k) = \mu v \frac{2H}{D + L}, \tag{18}$$

with the following parameters

$$F = \mu[8(8 - 7d^2 + d^3) + 3d^4(4 - d)] - 4(8 + 2d - 3d^2) > 0,$$

$$G = \mu[16(4 - d - 2d^2) + 3d^2(2 + d)] - 8(1 + d) + 3d^2 > 0,$$

$$H = (8 - 3d^2)(2\mu(1 - d) - 1) > 0.$$

Before proceeding to a comparison of situations characterized by the availability of information flows and whether there is a prohibition to exchange non-public information to the various organizational divisions of a vertically integrated company, it should be noted that a vertically integrated company makes higher investments in innovations and produces a higher volume of output than an unintegrated company, and if various divisions of a vertically integrated company are prohibited to exchange non-public information, received by one of its divisions from third-party sources, $k = 0$, and in the case of information flow $k > 0$. This result reflects the competitive advantage of a vertically integrated company with respect to an independent company that pays a higher price for the inputs used.

Statement 1. For all values k, $0 \le k \le 1$:

1. R&D investments of a vertically integrated company outperform R&D investments of a non-integrated company,

$$x_1^*(k) > x_2^*(k);$$

2. the output volume of a vertically integrated company exceeds the output of a non-integrated company,

$$q_1^*(k) > q_2^*(k).$$

Evidence.

1. Calculate the difference

$$x_1^*(k) - x_2^*(k) = v\frac{3d^3\mu(4-d) + 16\mu(3-2d-d^2) + 8k\phi}{D+L}.$$

If $D > 0$, $L > 0$, $0 \le k \le 1$ and $0 \le d < 1$, we have

$$\phi = 1 - d + d(1-k) + 2\mu d(1-d) > 0,$$

then there is inequality

$$x_1^*(k) > x_2^*(k).$$

2. If

$$x_1^*(k) > x_2^*(k)$$

and $w^*(k) > 0$. Then

$$q_1^*(k) > q_2^*(k).$$

4. Discussing the Results

4.1 The influence of external information flows on the market equilibrium characteristics (output volumes, resources' wholesale price, competing companies' profits and their R&D investments)

Situations characterized by the availability of information flows and the prohibition to various organizational divisions of a vertically integrated company to exchange non-public information received by one of its divisions from outside sources are compared in this section. The influence of information flows on innovation, the wholesale resource price, the output volume and the profit of companies are considered. It is reasonable to start with the analysis of R&D investments behavior of each company according to the external information flows intensity, since the requirement to prohibit different organizational divisions of a vertically integrated company to exchange non-public information received by one of its divisions from outside sources corresponds to the case of zero externalities. The results are summarized in the following Statement.
Statement 2.

1. R&D investments of a non-integrated company are reduced with increased intensity of external information flows k, $0 \le k \le 1$;
2. R&D investments of a vertically integrated company grow with an increase in the intensity of external information flows k, if $k \le \max\{1, k\}$, where $k > k_{cr}$, and are always higher in the presence of information flows ($k > 0$), than if various divisions of a vertically integrated company are prohibited to exchange non-public information obtained by one of its divisions from third-party sources. ($k = 0$).
Evidence.

1. If the equilibrium value $x_2^*(k)$ is determined by the intersection point of the two curves of companies' reaction relative to the volume of their R&D investments

$$R_1(x_2^*(k),k) = R_2^{-1}(x_2^*(k),k)$$

and let $k' > k$. The arrows in Fig. 1 and Fig. 2 show the response curves shift if the parameter k grows. From Fig. 1 and Fig. 2 it follows that

$$R_1(x_2^*(k),k') > R_1(x_2^*(k),k)$$

and

$$R_2^{-1}(x_2^*(k),k) > R_2^{-1}(x_2^*(k),k'),$$

then

$$R_1(x_2^*(k)) > R_2^{-1}(x_2^*(k),k').$$

Moreover, the curve $R_2^{-1}(x_2(k),k)$ has a downward slope, while the curve $R_1(x_2(k),k)$ has an upward slope if $k > k_{cr}$ and a downward slope if $k < k_{cr}$, but for all values k there is a chain of inequalities

$$0 > \frac{dR_1}{dx_2} > \frac{dR_2^{-1}}{dx_2}.$$

Therefore, in order that

$$R_1(x_2^*(k'),k') = R_2^{-1}(x_2^*(k'),k'),$$

it is necessary that

$$x_2^*(k') < x_2^*(k),$$

i.e., investments $x_2^*(k)$ should be a decreasing function k.

2. The second part of Statement to be proved, it is known from (12) that

$$R_1(x_2,k) > R_1(x_2,0)$$

for all x_2 and $k > 0$. If $R_1(x_2,0)$ has a downward slope and $x_2(0) > x_2(k)$, the following expression can be derived:

$$R_1(x_2(k),0) > R_1(x_2(0),0).$$

It follows that

$$R_1(x_2(k),k) > R_1(x_2(0),0),$$

i.e., $x_1(k) > x_1(0)$. If $k' > k$, from (12) the following expression can be derived:

$$R_1(x_2(k),k') > R_1(x_2(k),k)$$

for all x_2 and $k' < k_{cr}$. In addition, the curve $R_1(x_2(k),k)$ has a downward slope if $k < k_{cr}$ and $x_2(k) > x_2(k')$.

There is inequality

$$R_1(x_2(k'),k) > R_1(x_2(k),k).$$

It follows from 2 inequalities that

$$R_1(x_2(k'),k) > R_1(x_2(k),k') > R_1(x_2(k),k),$$

i.e., $x_1(k)$ increases by k if $k < k_{cr}$. To show the $x_1^*(k)$ growth by k if $k \le \max\{1,k\}$, differentiate $x_1^*(k)$ relative to k. Assuming the derivative to be zero and solving the resulting equation with respect to k, the following expression can be derived:

$$k = \frac{1}{8d}\{(8-3d^2)\sqrt{[(8+d^2)2\mu-8]\mu} + 8 + (3d^4 + 16d^2 - 64)\mu\}.$$

The check shows that $k > k_{cr}$ for all d and μ, and that $k > 1$ for some parameters values (for example, for sufficiently small d). In this case $x_1^*(k)$ grows by k for all values k.

Thus, external information flows have a negative impact on the independent company's incentives to invest in R&D. Hence, the company D_2 undertakes lower investments in R&D if there are external information flows. On the contrary, the vertically integrated company undertakes higher investments in R&D if there are external information flows. Innovative developments externalities reduce the vertically integrated company's costs; this leads to an output increase that in turn enhances the significance of any cost reduction leading to the company's own investment increase in R&D. Consequently, the company's UD_1 "effective" R&D investments $x_1 + kx_2$ are also higher with external information flows than if various organizational divisions of a vertically integrated company are prohibited to exchange non-public information obtained by one of its divisions from outside sources.

Statement 2 establishes that the intensity increases in external information flows k leads to a decrease of an independent company's D_2 in R&D investments. Fig. 1 and Fig. 2 show this result, where the arrows indicate the response curves' shift direction with increasing intensity of the external information flows k. The intensity increases in the external information flows k rotates the independent company's D_2 response curve counterclockwise around its intersection with the horizontal axes in Figure 1 and Figure 2. It is intuitively clear that the intensity increases in unilateral external information flows from the company D_2 to UD_1 further reduces the independent company's D_2 R&D investments, since in this case a competing company uses its innovations.

On the other hand, the increase of externalities of information flows k intensity rotates the integrated company's UD_1 response curve clockwise around its intersection point with the axis x_1. In particular, if the parameter k is such that the R&D investments are strategic substitutes for the vertically integrated company UD_1, the increase in intensity of externalities k leads to a steeper response curve R_1 with a negative slope. If R&D investments are strategic substitutes, at an intuitive level the increase in intensity of externalities k enhances the impact of externalities on the output volume that leads to a further increase in R&D investments of UD_1.

If R&D investments are a strategic complement to a vertically integrated company UD_1, the Fig. 1 analysis shows that the effect of the intensity increase in external information flows k on R&D investments of a vertically integrated company is positive for intensity values of externalities k, that are higher and close to k_{cr}, whereas for higher intensity values of externalities k this effect may be negative. The latter case has a simple interpretation. If R&D investments are strategic complements for a vertically integrated company UD_1, the intensity increase in external information flows k makes innovation investments a stronger strategic complement for a vertically integrated company UD_1. It is known that the company D_2 will respond to the intensity increase in external information flows k by reducing its R&D investments. If innovative development investments cost enough (i.e., μ is quite high), a vertically integrated company UD_1 also has incentives to reduce its R&D investments to save on the implementing costs. The last positive effect over-compensates the negative impact on profits due to a smaller decrease in division costs.

It is necessary to investigate the influence of external information flows on the company's output volumes. Since R&D investments allow companies to reduce their specific production costs, an increase (decrease) in the company's effective R&D investments should lead to increase (decrease) in the company's output volume. It should be noted, however, that effective reduction of the specific production costs of D_2 and, consequently, of its output, also depends on the price change of the resource used, assigned by UD_1. Comparing the wholesale price of various divisions of a vertically integrated company that are prohibited to exchange non-public information received by one of its divisions from external sources with the wholesale price with the presence of external information flows, a conclusion can be drawn that in the first case the price not only exceeds the price in the second case, but the wholesale price decreases with the intensity increase in the external information flows k (see Statement 3). At the intuitive level, this is due to the external information flows influence on the derivative demand for the resources used. External information flows reduce D_2's incentives to implement innovations and, consequently, reduce its demand for the resources used, forcing UD_1 to assign a lower price for resources to avoid large losses of profit when selling them.

Statement 3. The wholesale price of resources used decreases with the intense growth of external information flows k, if $0 \le k \le 1$.
Evidence.

By differentiating the equation (16) relative to k, the following expression can be derived:

$$\frac{\partial w^*(k)}{\partial k} = 8\mu v[2\mu(1-d)-1]\times$$

$$\times\frac{8dk(2-dk)+12(2-d^2)-\mu(8-3d^2)-\mu(8-3d^2)(2kd^3-5d^2+8)}{(D+L)^2}.$$

The denominator of this expression is always positive. The first factor of the numerator is positive.

$$8\mu v[2\mu(1-d)-1]$$

The second factor decreases with the parameter μ's increase. Consequently, it takes the greatest value with the minimum possible value of the parameter μ that is equal to

$$\frac{1}{2(1-d)}.$$

When substituting

$$\mu = \frac{1}{2(1-d)}$$

to the second denominator factor, the following expression can be derived:

$$\frac{2dk[16(1-d)+d^4-8d(d-dk+k)]-16-5d^4-24(2+d)(1-d)}{2(1-d)}.$$

This expression is always negative if $0 \le k \le 1$ and $0 \le d < 1$, e.g. $w^*(k)$ decreases by k.

A decrease in the wholesale price of the used resource if it is not prohibited for different divisions of the vertically integrated company to exchange non-public information received by one of its divisions from outside sources in case other circumstances are similar reduces the advantage in expenses of superior division of the vertically integrated company UD_1 and improves a competitive position of the

independent superior company. Yet the influence of external information flows on investments in R&D is greater than the influence of external information flows on the wholesale price of the used resource and, consequently, external information flows promote the increase of the output of the vertically integrated company UD_1 and the decrease of the output of the independent superior company.
Statement 4.

1. The output of the independent superior company decreases with the growth of the intensity of the external information flows k, if $0 \le k \le 1$.
2. The output of the vertically integrated company UD_1 increases with the growth of the intensity of the external information flows k, if all

$$k \le \max\{1, \tilde{k}\},$$

where $\tilde{k} > k_{cr}$, and is always higher when there are external information flows ($k > 0$), rather than when it is prohibited for different divisions of the vertically integrated company to exchange non-public information, received by one of its divisions from external sources ($k = 0$).
Evidence.

1. Differentiate the equation (18) relative to k and follow the procedure analogical to the one used in the evidence of Statement 3.
2. Check, that private derivative from $q_1^*(k)$ relative to k is positive if $k = 0$. To show, that $q_1^*(k)$ increases by k, when $k \le \max\{1, \tilde{k}\}$, we differentiate $q_1^*(k)$ on k. Equating the derivative to zero and solving the equation relative to k is obtained, and then

$$\tilde{k} = \frac{1}{8d}\{8 - (64 - 32d^2 + 3d^4)\mu + \sqrt{[(8 - 3d^2)S}\},$$

where

$$S = 8d^2 - 8(8 - 3d^2)\mu + (8 - d^2)^2(8 - 3d^2)\mu^2.$$

It is possible to check that $\tilde{k} > k_{cr}$ at all values of d and μ. Moreover, it is possible to check, that $\tilde{k} > 1$ for some of the parameter values (i.e., if d is small enough), and in this case $q_1^*(k)$ increases by k regardless of the values of k. Calculate if $0 < k \le 1$

$$q_1(k) - q_1(0) = \frac{(8 - d^2)(x_1(k) - x_1(0)) + 2d(x_2(0) - x_2(k)) + k(8 - d^2)x_2(k)}{2(8 - 3d^2)}.$$

As from Statement 2, $x_1^*(k) > x_1^*(0)$ and $x_2^*(0) > x_2^*(k)$, consequently, $q_1^*(k) > q_1^*(0)$.

Before the analysis from the point of view of the national welfare, it is important to analyze whether the vertically integrated company really has incentives to let information flows spread. In other words, it is important to scrutinize whether a vertically integrated company has or not any incentives to introduce prohibition for its different divisions to exchange non-public information received by one of its divisions from external sources. For this purpose, it is important to compare the profit of the vertically integrated company which has prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources, with the profit of a company which allows external information flows.
Statement 5.

1. The income of the independent superior company declines with the growth of the intensity of the external information flows k, if $0 \le k \le 1$.
2. The income of the vertically integrated company UD_1 increases with the growth of the intensity of the external information flows k, at all

$$k \le \max\{1, k\},$$

where $\tilde{k} > k_{cr}$, and always higher if there are external information flows ($k > 0$), in contrast to the situation when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k = 0$).
Evidence.

1. We get equilibria profits D_2, $\pi_2^*(k)$, substituting the equations (14), (15), (16), (17), (18) in (5). Differentiating $\pi_2^*(k)$ on k, we get the resulting Statement.
2. Substituting the equations (14), (15), (16), (17), (18) in (4), we get equilibria profits of the vertically integrated company UD_1, $\pi_{VI}^*(k)$. The aim is to show, that $\pi_{VI}^*(k)$ is a quasiconcave function k if $0 \le k \le 1$. For this purpose, we calculate the derivative from $\pi_{VI}^*(k)$ relative to k. After that, we check that this derivative is positive if $k = 0$, $N(0) > 0$. Then we need to show, that the second derivative from N relative to k (N_{kk}) is negative at all values of k. For that purpose, we check that this derivative is negative for a minimum value of

$$1 - \frac{1}{2\mu},$$

and hence we set its negativity for all the values of μ. This means, that derivative N_{kk} is originally negative and that it continuously decreases on μ. Consequently, its maximum value is at the minimum value of μ. This proves, that $\pi^*_{VI}(k)$ is a quasiconcave function k, if $0 \le k \le 1$. Then it is necessary to check whether $N(1)$ is positive. If $N(1)$ is positive, which is possible at low values of d, then $\pi^*_{VI}(k)$ increases by k if $0 \le k \le 1$; consequently,

$$\pi^*_{VI}(k) > \pi^*_{VI}(0)$$

if $0 \le k \le 1$. On the other hand, if $N(1)$ is negative, we know, that $\pi^*_{VI}(k)$ increases at first, but then decreases on k. Then we set, that $\pi^*_{VI}(1) > \pi^*_{VI}(0)$, which again means $\pi^*_{VI}(k) > \pi^*_{VI}(0)$ if $0 \le k \le 1$.

If it is not prohibited for different divisions of the vertically integrated company to exchange non-public information received by one of its divisions from outside sources, two factors apply. On the one hand, external information flows allow the vertically integrated company to use innovations of a competitor free of charge. As a result, there are effective investments in R&D of the vertically integrated company, which allows it to expand a superior business and to increase its profit from the sales of final products. Nevertheless, external information flows decrease incentives for investments in R&D of the independent company D_2, therefore declining its demand for the resources used. This leads to the decrease of wholesale price of the resources and, consequently, to the decrease of the profit on inferior market of the vertically integrated company.

As follows from Statement 5, the first positive effect surpasses the second negative effect. Pure effect, this way, is in the growth of the profit of the vertically integrated company owing to external information flows. This conclusion means, that in case of the absence of the intervention by a regulator, inferior company has incentives to transmit to superior division the information concerning its competitor on inferior market. As far as the independent company is concerned, the decline in its profit with the growth of the intensity of external information flows is a direct consequence of decrease in its investments in R&D and output (Statements 2 and 4).

4.2 Analysis of the influence of external informational flows on the total national welfare

To answer the question as to whether a regulator of the economic policy should introduce prohibition for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources, it is necessary to analyze it from the national welfare perspective. Namely, it is important to compare collective national welfare when there are external information flows ($k > 0$) and when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k=0$). It is possible to determine collective national welfare as the sum of welfares of consumers and producers

$$W(k)=(a-A)(q_1+q_2)-\frac{1}{2}(q_1^2+q_2^2+2dq_1q_2)+(x_1+kx_2)q_1+ \\ +x_2q_2-\frac{\mu}{2}(x_1^2+x_2^2). \tag{19}$$

It turns out that the pure influence of external information flows on collective national welfare depends on effectiveness of the innovational technology μ, the extent of the differentiation of the product d and the intensity of external information flows k.

Statement 6. At low and moderate values of μ, and at high values of μ if the extent of the differentiation of product d is low enough, collective national welfare is always higher if there are external information flows ($k>0$), rather than when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k=0$). At high values of μ if the extent of the differentiation of product d is high enough, there is such a value of $\bar{k}$ that at all $k>\bar{k}$ national welfare is higher when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k=0$), rather than when there are external information flows ($k>0$), and vice versa if $k<\bar{k}$. Besides, $\bar{k}$ decreases by μ.
Evidence.

To prove that collective national welfare $W(k)$ is a quasi-concave function k if $0\le k\le 1$, we should follow the same procedure as the one we used to prove Statement 5. Thus, we conclude, that if μ and d are high enough, then $W(0)>W(1)$, while the opposite is true at all the other values. Consequently, if $W(0)<W(1)$, then collective national welfare is always higher when there are external information flows ($k>0$), rather than when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k=0$). If $W(0)>W(1)$, quasi-concaveness means that there is the only possible $\bar{k}$, so that all $k<\bar{k}$ collective national welfare is higher when there are external information flows ($k>0$), rather than when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k=0$). And, on the opposite, at all $k>\bar{k}$, where $\bar{k}$ is a solution to the equation $W(0)=W(\bar{k})$. Finally, it is shown that $\bar{k}$ decreases on μ. This can be illustrated by the following. First, it is necessary to show that both the first derivative from $W(k)$ relative to μ, W_{μ}, at the minimum value of μ, and the second derivative from $W(k)$ relative to μ, W_{μ}, are negative. For this purpose, we prove, that the second derivative from $W(k)$ relative to μ, $W_{\mu\mu}$, is negative at the minimum value of μ, and the third derivative from $W(k)$ relative to μ, $W_{\mu\mu\mu}$, is negative. These two results mean, that collective national welfare $W(k)$ is a decreasing function with the increasing speed on μ. Consequently, its maximum value is at the minimum value of μ.

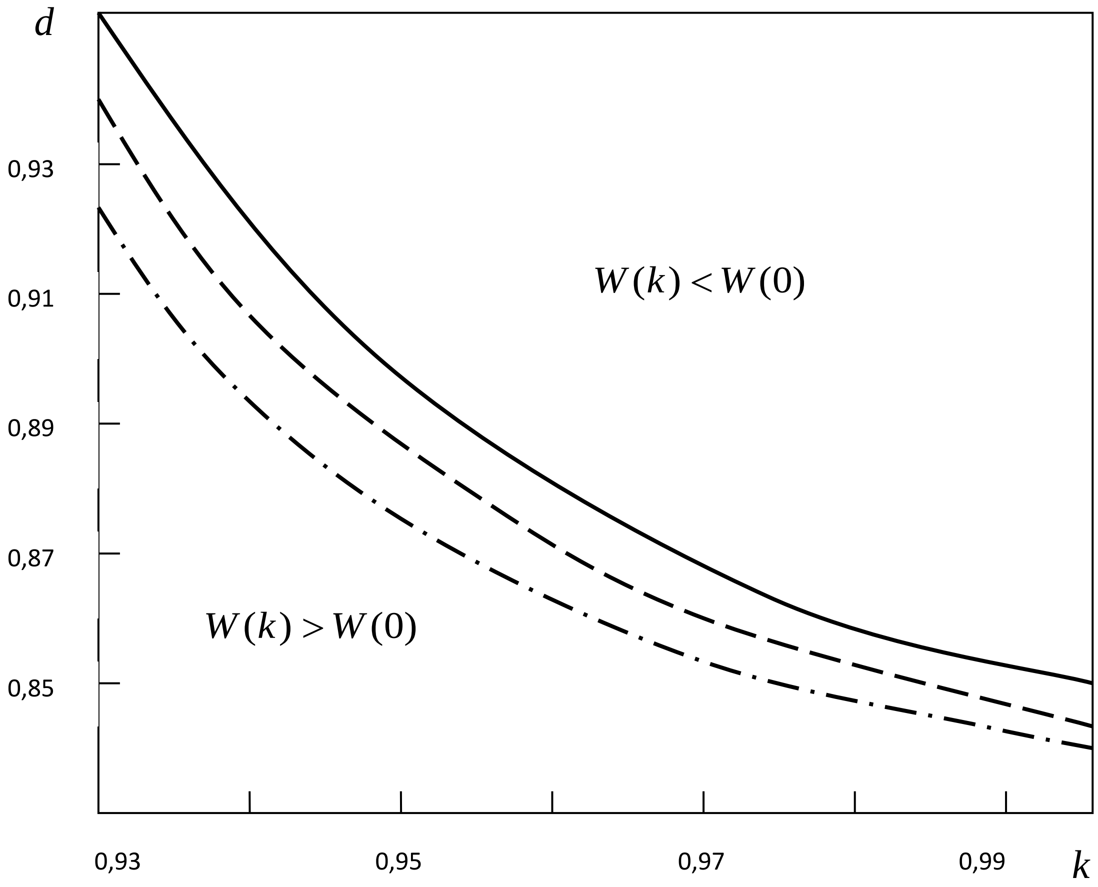
In other words, at low or moderate costs on innovative development, collective national welfare is always higher when there are external information flows ($k > 0$), rather than when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources ($k = 0$). To interpret this income we notice, that according to Statement 1, UD_1 has a bigger part of the market, than D_2, both with and without external information flows. In other words, UD_1 and D_2 are asymmetric even without investments in R&D, and apparently, UD_1 is more effective since it has lower production costs. According to Statements 2 and 4, the presence of external information flows increases the benefit of UD_1 in costs, as it increases its share of the market. Thus, if there are external information flows, output is transferred to the most effective company, which can result into growth of collective national welfare. There will be the same level of national welfare in case of higher values of expenses on innovative developments provided that the production of the companies is differentiated enough.

However, if the expenses on innovative development are high enough and the types of products the companies make are very close substitutes, the result, which was marked above, is changing to the opposite if the intensity external information flows k is high enough, which means $k > \bar{k}$. As it is illustrated on Fig. 3, the national welfare is higher if it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources, rather than when there are external information flows, if, for example,

$$\mu = 10 \text{ and } d = 0{,}84$$

at all $k > 0{,}99$, while if continuous curve - $\mu = 8$, dotted line - $\mu = 9$, dot-dash line - $\mu = 10$ $\mu = 10$ and $d = 0{,}949$ then at all $k > 0{,}926$.

Figure 3. *Comparison of the national welfare if it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources, with the case when there are external information flows.*



Here is an interpretation of this result. If the expenses on innovative development are very high, even a vertically integrated company will make small investments in scientific and research developments and, consequently, the extra savings due to external information flows plays a minor role in comparison with the national welfare perspective. Apart from that, if intensity of external information flows k is high, the problem of the assignment of the results of innovative development for the independent company D_2 is of such a high importance that D_2 makes few investments in R&D and UD_1 wins less from external information flows. It is essential to consider that the problem of the assignment of the results of innovative development for D_2 increases depending on how similar substitutes made by companies are, since the more intense the competition, the higher the negative impact is concerning the inability to apply results of innovative development of D_2. The combination of all the mentioned effects can lead to the changes in the results of the analysis of welfare to the opposite. To make such changes, it is important, that the types of products, which companies make, are very close substitutes, expenses on innovative development are quite high and the intensity of external information flows k is also high.

4.3 Modeling and analysis of the influence of competition between companies in innovation development on market equilibria characteristics

In this section it will be analyzed, how the results above are changing in case of the alternative situation, namely in assumption that companies are involved in competition after coming up with the same innovation. The competition in innovative developments is modeled as a stochastic process. Both companies make investments in R&D, which in case of success leads to one innovation h, and at the same time it will provide savings, when

$$A - h \geq 0 \text{ and } d < \frac{a - A}{a - A + h}.$$

Investments in R&D of a company determine the probability ρ, with which a company will succeed in innovation. Costs on investments in R&D of a company can be determined in the following way.

$$G(\rho_i) = \mu(-\rho_i - \ln(1 - \rho_i)),\ 0 \leq \rho_i \leq 1,\ \mu > 0,\ i = 1,2, \quad (20)$$

so that

$$G(0) = G'(0) = 0,\ G'(\rho_i) > 0,\ G''(\rho_i) > 0,$$

$$\lim_{\rho_i \to 1} G(\rho_i) = \lim_{\rho_i \to 1} G'(\rho_i) = \infty.$$

It is possible to suppose that in terms of the description of one-sided external information flows UD_1 reaches success in R&D, if it either develops innovation on its own or manages to imitate innovation designed by a third company D_2, i.e., with the probability

$$\rho_1 + (1 - \rho_1)k\rho_2,$$

where k - intensity of external information flows, $0 \leq k \leq 1$, and D_2 develops an innovation with the probability ρ_2.

Depending on failure or success in developing an innovation, the profits of integrated and independent companies can be denoted as: π_1^B, π_2^B, if both companies develop innovations; π_1^N, π_2^N, if neither of the companies manages to develop an innovation; π_1^1, π_2^1, if only UD_1 succeeds in R&D; π_1^2, π_2^2, if only D_2 succeeds in R&D. Consequently, the expected profit of UD_1 is as follows:

$$E(\pi_1) = (\rho_1 + (1-\rho_1)k\rho_2)\rho_2\pi_1^B + \rho_1(1-\rho_2)\pi_1^1 + \\ + (1-\rho_1)(1-k\rho_2)\rho_2\pi_1^2 + (1-\rho_1)(1-\rho_2)\pi_1^N - G(\rho_1), \quad (21)$$

And it is possible to write an analogous equation for D_2. The staging of the "game" is as follows. At the final stage of the "game", each company decides on the volume of output to maximize profits. At the second stage of the "game", UD_1 decides on the wholesale price for the resources to maximize profits. At the first stage, companies decide on the volumes of their investments in R&D to maximize profits. Analysis shows that investments in R&D, expected output and profit of the independent company as well as the expected wholesale price are lower if there are external information flows, in comparison with the situation when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources. An opposite conclusion is also correct for the expected volume of production and profit of the vertically integrated company. However, competition in innovative developments changes motivation for innovations in a vertically integrated company, which in this case does not invest more if there are external information flows in comparison with the situation when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources. This result can be explained by the nature of investments in scientific and research developments under considered circumstances in comparison with the economic-mathematical model of competition if there is a vertically integrated manufacturing company. Here, investments in innovative developments of two companies lead to the same amount of savings in case of success. For example, if UD_1 succeeds in developing innovation for savings h, externalities from D_2, in case it also achieves the same success in developing innovation, do not lead to a higher amount of savings, and, consequently, they do not motivate further investments in innovative development. Apart from that, in terms of available externalities UD_1 can achieve the same savings while investing less in innovations, and therefore generate more profit rather than when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources. It is necessary to point out that expected effective investments of UD_1 in innovative developments, i.e., total expected savings

$$x = \rho_1 + (1-\rho_1)k\rho_2$$

are higher if there are external information flows, rather than when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources.

As for the national welfare, calculations show that if companies compete in developing innovations designed for savings, external information flows are more preferable than the situation when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources. This can be explained not only by the above-mentioned asymmetry effect of market shares of two competing companies, but also by the fact that externalities help to avoid excessive investments in innovations. This decreases the amount of total costs, and thus it enhances national welfare.

5. Conclusion

Possible anti-competition effects of vertically integrated structures were analyzed in this paper. Apart from that, the influence of information flows on the motivation for innovation development and the national welfare is considered in terms of inferior and superior enterprises of the sector. For scrutinizing the abovementioned problems, a model was build where vertically integrated company is a monopoly on the inferior resource market and at the same time it participates at the superior market together with a third superior company. It was established that information flows between inferior and superior enterprises in the sector do not necessarily lead to disincentive to development of innovations and the national welfare. It was shown that information flows lead to disincentive to development of innovations with regard to the vertically integrated company, at least if investments in R&D are complementary. However, if enterprises are engaged in competition in innovative development, there are less disincentives to innovation development of the vertically integrated company following the increase in intensity of external effects connected with information flows between inferior and superior companies in the sector.

However, regardless of the abovementioned forms of R&D, "effective" innovations of the vertically integrated company in scientific and research developments are higher if there are external information flows, but lower if it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources. If companies' investments in scientific and research developments are complementary, the national welfare is higher, provided that there are external information flows, than in case when it is prohibited for different departments of the vertically integrated company to exchange non-public information received by one of its departments from external sources, provided that the amount of information received from external sources is not too high, or the types of products are not close substitutes or expenses on development of innovations are not too high. Moreover, if company's investments in R&D are an alternative, the national welfare is higher if there are external information flows.

6. Acknowledgements

The publication has been prepared within the Russian Foundation for Humanities (RFH). Project #17-12-59009 "Features of Development of a Regional Industrial

Complex in the Conditions of the Current Economic Crisis”.

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