# The Specification and Estimation of the Labour Demand Relation Survey of the Literature

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## 1. Introduction

Empirical studies on labour demand typically assume the existence of some underlying production function. The Cobb-Douglas or the Constant Elasticity of Substitution production functions[1] are usually specified for this purpose, and the labour demand relation is derived from these functions. Short run deviations of the actual labour input from its desired level are usually incorporated into the model by specifying a partial adjustment scheme.

In order to examine the implications of the different theoretical postulates, we shall classify work on the subject into three broad categories, as follows:

i. works that assume profit maximisation, and obtain an expression for the desired labour input from the marginal productivity condition for labour, derived from a production function. This category of works normally produces a labour demand relation which includes a wage term and an output term amongst the explanatory variables.

ii. works that also assume profit maximisation, but derive the expression for the desired labour input from the expansion path of the production function. This category of works will be explained with reference to the model suggested by Nadiri (1968). As we shall show, models of this type normally require data on capital services and their costs.

iii. works that substitute the production function into a cost function, and obtain and expression for desired employment by minimising the cost function. We shall explain this category of works with reference to the model suggested by Ball and St. Cyr (1966). As we shall show, models of this type tend to produce unacceptable estimates of the parameter measuring short run returns to labour.

There are a number of variants to the basic models presented in each category, and we shall consider some of these variants whenever important theoretical modifications to the basic models are suggested.[2]

## 2. Models derived from the Marginal Productivity Condition[3]

The basic assumption underlying this category of models is that, in a competitive setting, a firm would theoretically maximise profits when the marginal product of labour and the real wage rate are equal. In a time-series context this relation can be expressed as:

$$\frac{\partial Y_t}{\partial (EH)_t} = W_t \qquad (1)$$

where EH stands for the desired labour input, consisting of persons employed (E) multiplied by the number of hours worked (H)[4], W stands for the hourly wage rate, and Y for output, both measured in real terms.

*2.1. Labour Demand and the C.E.S. Production Function*

Models in this category generally postulate an underlying Constant Elasticity of Substitution (C.E.S.) production function, but different authors have worked with different assumptions regarding market imperfections, returns to scale, and technical change. As we shall show, these different assumptions produce different estimating forms of the marginal productivity condition. An expression for desired labour can be derived from the following C.E.S. production function.

$$Y_t = e^{rt}(k(KU)_t^{-p} + (1-k)(EH)_t^{-p})^{-v/p} \qquad (2)$$

where Y, K and E stand for output, capital stock, and men employed respectively; U and H stand for the rate of utilisation of capital and labour respectively; k is the distribution parameter, measuring the capital intensity of the production function; p is the substitution parameter, from which the elasticity of substitution, $s = 1/(1+p)$ can be derived; v is the homogeneity parameter, which measures the degree of returns to scale, and indicates increasing returns with a value of more than unity, constant returns with a value of unity, and decreasing returns with a value of less than unity but greater than zero; and $e^{rt}$ is an exponential time trend, to capture the rate of neutral technical change.[5]

From equation (2) we can obtain a specific form of equation (1) as follows:

$$\frac{\partial Y_t}{\partial (EH)_t} = W_t = ve^{-prt/v}(1-k)(EH)_t^{-(p+1)}Y_t^{(1+p/v)}$$

which when rearranged, and transformed into logarithms, yields the following expression for desired labour:

$$\ln (EH)_t = s \ln v(1-k) - s \ln W_t + \frac{1+s(v-1)}{v} \ln Y_t - \frac{(1-s)}{v} rt \qquad (3)$$

The parameters of the C.E.S. production function, with the exception of the distribution parameter, can therefore be estimated, using data on real wage rates, real output, man hours, and a time variable.[6]

Many variants of equation (2) have been suggested. For example, the returns to scale parameter may be restricted to equal unity, implying constant returns to scale.[7]

Another variant of equation (2), allows for the possibility of market imperfections. Unfortunately, measuring market imperfections or obtaining suitable proxies for them is not an easy matter, and in most empirical work on the subject, this variable is discussed but omitted from the estimating equation.[8] Such omission may be justified on the grounds that market imperfections can be captured by a constant, and can therefore be ignored[9]. Those studies that do include a market imperfections term generally make use of proxies such as measures of industrial concentration and/or degrees of unionisation[10], on the assumption that these are directly related to market imperfections.

Still another variant of equation (2) allows for non-neutral technical change. For example, Black and Kelejian (1970) and Williamson (1971) multiply the labour input by a term standing for labour augmenting technical change, assuming that the effect is captured by a smooth trend. The capital input is multiplied by a similar term. The marginal productivity condition for labour yields an equation similar to equation (3) but the coefficient on 't' is interpreted differently, since it is not derived from the assumption of neutral technical change. The interpretation of the time variable and its coefficient will be discussed further in section 5 below.

Another way of allowing for non-neutral technical change is by adding variables which measure improvements in the quality of inputs. For example, Lucas and Rapping (1970) allow for labour augmenting technical change by multiplying manhours by a "years of schooling completed" index.

The specification suggested by Dhrymes (1969) allows for the possible dependence of employment on investment, due to the effect of embodied technology of new capital vintages, on labour productivity. Dhrymes also allows for the effect of expected output, by introducing a lagged output term in the estimating equation. The final form of the labour demand equation is similar to equation (3), but includes also lagged output and lagged investment.

*2.2 Elasticity of Substitution and Returns to Scale*

Of special interest in the models derived from the marginal productivity condition on labour is the coefficient on the wage rate, which is a measure of the elasticity of substitution and which we denote by 's'. As was noted earlier, the coefficient 's' is related to 'p' in the C.E.S. production function (2). The elasticity of substitution may be defined as the ratio of the percentage change in factor proportions, to the percentage change in the factors' relative prices. Symbolically this can be represented as:

$$s = \frac{d(KU/EH)/(KU/EH)}{d(W/C)/(W/C)}$$

where C stands for the user cost of capital and the other variables have already been defined.

In the C.E.S. production function, the numerical value of the elasticity of substitution is constant, as the name implies, but it is nor restricted to equal unity, as in the case of the Cobb-Douglas production function.[11]

A conclusion that often emerges from the studies based on the C.E.S. production function is that the elasticity of substitution is significantly less than unity, although this result is by no means universal.[12] This would seem to suggest that models which restrict the value of this parameter to equal unity are mis-specified.

The estimate of the elasticity of substitution has important policy implications, since its value could serve as an indicator of the extent to which fiscal and other policy measures can succeed in altering factor proportions. For example, a low value of the elasticity of substitution suggests that a decrease in the wage rate, given the cost of capital, is not likely to be as successful in inducing entrepreneurs to make adjustments in the capital output ratio, as would have been the case if the value of this parameter had been higher.

Another important parameter of the C.E.S. productions function is the returns to scale parameter, 'v', which forms part of the coefficient on $\ln Y_t$ in labour demand equation (3). The possibility that 'v' is greater than unity, implying increasing returns, is compatible with a situation where firms are able to increase output by a larger proportion than the increase in inputs. We note here that the relation between the coefficient on $\ln Y_t$ and 'v' is non-linear, and that small deviations of this coefficient on $\ln Y_t$ from unity correspond with relatively large changes in the value of 'v'.

In studies on aggregate employment, the estimate of the coefficient 'b' has implications regarding the extent of labour absorbtion. Such information is of course of interest also in disaggregated studies, since it permits the ranking of sectors or industries according to the rate of labour absorption, and therefore may be useful for policy decisions as to which sectors or industries should receive prominence for employment promotion.

An assumption often made in such studies, which affects the specification of the model, and therefore the estimates of 's' and 'v', is that the actual and the desired level of employment differ. This may be accounted for by introducing a partial adjustment scheme. We shall discuss the implications of such a scheme in section 6, but at this stage it should be noted that with the introduction of such a scheme, the coefficient on the wage rates and on output would include in them the effect of the speed of adjustment. In this sense, the elasticity of substitution and the degree of returns to scale may be viewed as having a short run dimension, due to partial adjustment, and a long run dimension, compatible with full adjustment.

### *2.3 Empirical Estimates of the Marginal Productivity Condition*

Empirical studies on labour demand based on the marginal productivity condition of the C.E.S. production function are numerous. Here we limit ourselves to briefly describing some results obtained from three studies, which are based on aggregate time-series data.

An influential piece of work in this category is that by Lucas and Rapping (1970). Their manhours demand relation, which utilises U.S. annual data, contains the assumption of constant returns to scale, and incorporates a partial adjustment scheme to allow for short run deviations of the labour input from its desired level.

The numerical value of their estimate of the long run elasticity of substitution for their preferred equation was 1.09, which was higher than other time series estimates. Lucas and Rapping attributed this finding to the increased possibilities for substitution of production between goods of different factor intensities, as well as for capital labour substitution in the production of each good, which aggregation introduces. Since Lucas and Rapping allow for partial adjustment, they also obtain an estimate of the short run elasticity of substitution, with a numerical value of 0.46.

The assumption of constant returns to scale in Lucas and Rapping's study restricted the long run employment output elasticity to equal unity. However the partial adjustment scheme just mentioned enabled them to obtain an estimate of short run labour demand/output elasticity, with a numerical value of 0.79.

Another study in this category of work is that by Black and Kelejian (1970), who utilise U.S. quarterly data from private non-farm production. The functional form that they postulated allows for non-constant returns to scale, and for the possibility that wages and output differ from their expected values. They reported an estimate of the long run elasticity of substitution of 0.36, which is considerably smaller than that reported by Lucas and Rapping. The estimated value of their employment output elasticity was 0.87 for the long run and 0.56 for the short run, implying a returns to scale parameter with a value of 1.25.

Peel and Walker (1978) applied the marginal productivity condition on labour to the U.K. aggregate employment using quarterly data. They obtained very low estimates of the short run elasticity of substitution ranging from 0.02 to 0.08, with their long run counterparts of 0.094 and 0.304. The difference of these estimates depended on whether the total number of sample observations were considered, or only those where excess supply of labour was assumed to exist. The higher values of the elasticity of substitution came from regressions applied to periods of excess labour supply only. The return to scale parameter also differed according to which observations were considered, and was estimated to have a value of 0.799 for observations of excess labour supply, and of 1.329 to the total number of sample observations. Apart from producing function parameter estimates, this study highlights the necessity of distinguishing between periods of excess labour demand and excess labour supply. This question is discussed in some depth in Briguglio (1982).

### 3. *Models derived from the Expansion Path*

The tangency solution between isoquants and isocosts, employed in establishing theoretical least-cost capital and labour combinations, implies that at any given level of output, firms will employ quantities of labour and capital services so as to equate the ratio of their marginal products, with the ratio of their prices. The locus of points where the ratio of marginal products (i.e. the marginal rates of substitution along the isoquants) equals the price ratio is termed the Expansion Path and can be symbolically expressed as:

$$\frac{\partial Y_t}{\partial (EH)_t} \frac{\partial (KU)_t}{\partial Y_t} = \frac{W_t}{C_t} \qquad (4)$$

where C is the user cost of capital, W is the wage rate, and (EH) and (KU) and labour and capital services respectively.

#### *3.1 Empirical Work based on the "Expansion Path" Relation*

If the underlying production function is assumed to be the C.E.S. function, as in equation (2), equation (4) can be rearranged and transformed into logarithms, yielding the following desired labour input relation:

$$\ln (EH)_t = \ln ((1-k)/k)^s + s \ln (C/W)_t + \ln (KU)_t \qquad (5)$$

where $s = 1/(1+p)$ as before and stands for the elasticity of substitution.

From this specification, the effect of capital services and their costs on employment of labour services may be estimated. It is also possible to estimate the elasticity of substitution, and the distribution parameter of the C.E.S. production function.

An important study using an equation similar to (5) is that by Nadiri (1968), applied to the U.S. manufacturing sector. Nadiri suggested two variants of the model. One constrains the value of the elasticity of K (capital stock) and U (rate of utilisation) to be equal, as implied in equation (5), whereas the other does not. Nadiri found that on the basis of statistical criteria, the constrained model performed better.

In both variants, the coefficient on $\ln(C/W)_t$ proved to be significantly different from zero and positive[13] implying that changes in this ratio induce changes in factor combinations. The value of the elasticity of substitution was however found to be very low, suggesting that factor substitution occurs within a very narrow range.

Another study based on the expansion path equation, this time derived from the Cobb-Douglas production function, is that by Coen and Hickman (1970). They derived two equations, one explaining labour services, and the other explaining capital services. Since both equations are derived from the same production function, they inherit common coefficients, and due to this, the authors had to devise methods to restrict the common coefficients to have equal values. Their preferred method was to estimate the labour demand relation first, and then impose the estimated common coefficients in the investment demand function.

The desired labour input function derived by Coen and Hickman, when transformed into logarithms, takes the following form:

$$\ln(EH)_t = \text{constant} + \frac{b}{(a+b)}\ln(C/W)_t + \frac{1}{(a+b)}\ln Y_t + (r/(a+b))t$$

where 'a' and 'b' are elasticities of output with respect to capital and labour services respectively (as given in the Cobb-Douglas production function) and 'r' is the exponential rate of neutral technological change.

This specification is of course different from equation (5) since the elasticity of substitution is constrained to equal unity, being as it is derived from the Cobb-Douglas production function. One result of Coen and Hickman's study is that, like other specifications derived from the Cobb-Douglas production function, the implied value of 'a', measuring returns to labour alone, turned out to be larger than one. This finding will be discussed further in section 4.

Like the previous category of models, those derived from the expansion path usually incorporate a partial adjustment scheme to allow for short run discrepancies between desired and actual labour employment. The equation actually estimated thus contains a lagged dependent variable. The model suggested by Nadiri and Rosen (1969 and 1973) allows for the dependence of the labour input not only on its own lagged values, but also on the lagged values of other inputs. Like Coen and Hickman, Nadiri and Rosen derive their desired labour input function from the Cobb-Douglas production function, but unlike Coen and Hickman, they allow for the possibility that output elasticities with respect to input stocks differ from those with respect to the rates of utilisation of these input stocks[14].

The results obtained by Nadiri and Rosen suggest that demand for any input is indeed interrelated with that of other inputs in the sense that departures from the desired level of any input have a feedback effect on other inputs.

Nadiri and Rosen's study has been criticised on the grounds that the specification is unnecessarily complex[15], and that it produced meaningless coefficients[16]. Thus while Nadiri and Rosen's model is an attempt to offer a more comprehensive explanation of how demand for labour is determined, it is doubtful whether they did succeed in doing this via the results produced by their model.

### *3.2 Data on Stock, Rate of Utilisation, and User Cost of Capital*

One principal disadvantage of this category of works, compared to the previous based on the marginal productivity condition for labour, is that data on capital stock, its rate of utilisation, and its user cost is required to estimate the coefficients. The inclusion of the variables measuring capital services and their cost poses problems, since data on these variables are not usually readily available, and have to be constructed by the researcher, an exercise requiring information which again is difficult to obtain. For example, to derive a series for the user cost of capital, the data required may include the price of capital goods, the rate of depreciation, the discount rate, tax rates on capital and profits, and if applicable, tax deductions as part of some investment incentive scheme[17].

Also, to construct the capital services variable, a series of data on net capital stock may be required, which in turn may necessitate information on lengths of life of different types of capital stock, and on the rate of deterioration. Also, since the available capital stock may not be fully utilised at all times, data on capital utilisation may have to be constructed to allow for this possibility[18].

The amount and quality of data that is required therefore create formidable problems to the researchers. In some cases, arbitrary (though admittedly plausible) values are given to certain factors which cannot be measured, such as the rate of depreciation. The reliability of the estimates derived from specifications requiring capital data may therefore be impaired by measurement error, the extent and direction of which may not be known.

This is not saying that data for other variables, such as labour and output, are free from measurement errors, and do no require the making of certain arbitrary assumptions in their compilation. But data on capital services and their cost are particularly notorious for these shortcomings.

### *4. Cost-Minimisation Models*

We shall use the model suggested by Ball and St. Cyr (1966), a very influential piece of work, as a basic reference for this category of studies.

#### *4.1 The Ball and St. Cyr Model*

Ball and St. Cyr postulate a production function of the form:

$$Y_t = Ae^{rt}(EH)_t^a \qquad (6)$$

where Y and E stand for output and man employed as before, and H stands for productive (as distinguished from nominal or "paid for") hours; $Ae^{rt}$ is a shift parameter intended to capture the influence of capital and technological change[10]; and 'a' is the elasticity of output with respect to labour, indicating a measure of returns to labour alone and expected to have a value of between zero and unity.

The production function is therefore basically of the Cobb-Douglas type, with the effect of capital included in the time trend, t, which takes the value of 1,2,.....T, where T is the number of observations.

Ball and St. Cyr also postulate a cost function of the form:

$$C_t = W_t^p (EH)_t + F_t \qquad (7)$$

where $C_t$ is total costs, net of materials and fuels, F is fixed costs, and $W^p$ is the wage rate per productive manhour.

Assuming that a standard fixed negotiated wage prevails, a worker's take home pay would be $N(W^n)$ where N is normal hours, and $W^n$ is the agreed payment per hour for normal hours. Therefore if the number of hours actually worked productively, to be denoted by H, is smaller than the number of normal hours, the effective wage per productive manhour, for any given period, would be:

$$W^p = N(W^n)/H \qquad H < N$$

Thus the extent to which productive manhours are less than normal manhours affects the costs to the firm, in the sense that wages paid for non-productive hours tend to increase as N/H increases.

If overtime is worked, at a pay rate $W^o$, then the worker's take home pay would be $((N(W^n)+(H-N)(W^o))$. The effective hourly rate of pay per productive manhour in this case would be

$$W^p = ((N(W^n) + (H-N)W^o))/H \qquad H>N$$

This equation implies that the effective wage increases as the number of overtime hours increases (given that $W^o$ exceeds $W^n$) and that the normal hours wage rate ($W^n$) equals the productive hours wage rate ($W^p$) only if H=N, that is if normal and productive hours are equal. It implies also that as the difference between normal and productive hours increases each way (positively or negatively), the difference between $W^p$ and $W^n$ increases also. In other words, the minimum cost per worker occurs when H=N. Ball and St. Cyr suggest that the relation between $W^p$ and H, can be approximated by the following quadratic equation:

$$W^p = b_0 - b_1H + b_2H^2 \qquad (8)$$

Substituting equation (8) into equation (7) gives:

$$C_t = b_0(EH)_t - b_1E_tH_t^2 + b_2E_tH_t^3 + F \qquad (9)$$

Using the production function (6) to solve for $H_t$, and substituting the resulting expression into equation (9) yields:

$$C_t = b_0M_t - \frac{b_1}{E_t}M_t^2 + \frac{b_2}{E_t^2}M_t^3 + F, \text{ where } M_t = \frac{Y_t^{(1/a)}e^{-(rt/a)}}{A^{(1/a)}}$$

Minimising $C_t$ with respect to $E_t$ and solving for $E_t$ we obtain an expression for desired employment, i.e. that level of employment compatible with minimum costs, which we denote by $E_t^*$, as follows:

$$E_t^* = (2b_2/b_1)A^{-(1/a)}e^{-(rt/a)}Y_t^{(1/a)}$$

which when transformed into logarithms, yields:

$$\ln E_t^* = \text{constant} + (1/a)\ln Y_t - (1/a)rt \qquad (10)$$

Ball and St. Cyr's specification implies that the effect of hours would be absorbed in the constant term. In practice, however, this specification does not allow for the effect of changes in normal hours, since the coefficients and which are expected to change if normal hours change, are themselves part of a constant term.

It should be noted that the coefficient 1/a in equation (10) is the reciprocal of the parameter measuring returns to labour alone in the production function (6). Basing on a priori criteria concerning the law of diminishing returns, one should expect that the value of 'a' is positive and does not exceed unity.

### *4.2 Increasing Returns to Labour*

Ball and St. Cyr obtained unsatisfactory results as far as this coefficient is concerned, since OLS regression yielded implied estimates of the parameter 'a' which exceeded unity. This problem is in fact quite common in specifications similar to Ball and St. Cyr's[20].

Several explanations have been proposed to explain this finding. One explanation is that overhead labour (clerical and managerial, etc.) does not change proportionately with output, whereas direct labour (production workers) does, so that when output increases, the labour output mix changes in favour of direct labour[21]. If direct labour is more productive, labour productivity may increase even with capital held constant, seemingly contradicting the law of diminishing returns. Ball and St. Cyr tested this proposition by considering direct labour only in their employment function, but still obtained unsatisfactory results with respect to the parameter 'a'.

Another explanation suggested for the finding of increasing returns to labour is that this is due to the tendency of entrepreneurs to hoard labour. Some reasons why firms may not want to discharge labour when output falls include contractual obligations and costs of hiring and firing. A firm may thus start from a position of excess labour in a cycle upturn, and as production approaches full capacity, output per man actually increases, even if capital is held constant.

Ball and St. Cyr attempted to test this hypothesis by including a variable measuring the degree of labour underutilisation (derived from the unemployment rate). The results they obtained however, did not yield satisfactory indications that the finding of increasing returns to labour was due to changes in the degree of labour utilisation[22].

### *4.3 Ireland and Smyth's Reinterpretation*

An attempt to explain why the estimates of employment-output elasticity obtained from the Ball and St. Cyr model may be acceptable, was proposed by Ireland and Smyth (1970)[23]. They suggest a different interpretation of the coefficients of the employment function. Instead of deriving the function from a Cobb-Douglas type of production function, as Ball and St. Cyr did, Ireland and Smyth start from a C.E.S. production function of the form shown as equation (2) above.

Taking the ration of the marginal product of labour and the marginal product of capital, i.e. the marginal rate of substitution of labour for capital, the following expression is obtained:

$$\frac{\partial Y/\partial (EH)_t}{\partial Y/\partial (KU)_t} = \frac{(1-k)}{k} \cdot \left(\frac{(KU)}{(EH)}\right)_t^{1+p} \qquad (11)$$

Substituting equation (11) into equation (2) and rearranging, we get:

$$Y_t^{-\frac{p}{v}} = e^{-\frac{prt}{v}} (EH)_t^{-p} \left[ (1-k)(1 - \frac{d(EH)_t}{d(KU)_t} \cdot \frac{(KU)_t}{(EH)_t}) \right] \qquad (12)$$

where p, v, k are coefficients of the C.E.S. production function as indicated in section 2.2.1.

If the ratio of the percentage change in manhours and the percentage change in utilised capital is taken to be constant[24] the term in square brackets on the R.H.S. of equation (12) can be replaced by a constant, so that, after rearranging, the following equation is obtained:

$$Y_t = Ce^{-rt}(EH)_t^v$$

where C is a constant replacing the term

$$((1-k)(1 - \frac{d(EH)_t}{d(KU)_t} \cdot \frac{(KU)_t}{(EH)_t}))$$

Applying the same cost minimisation procedure as in Ball and Cyr's model, and transforming the resulting desired employment function into logarithms, we obtain the following equation:

$$\ln ET^* = \text{Constant} + 1/v \ln Y_t - r/vt \qquad (13)$$

Equation (13) contains the same variables as equation (10) of the Ball and St. Cyr model. However the coefficients now have different meanings. In particular, the coefficient on $\ln Y_t$ is (1/v) where 'v' is the returns to scale parameter. Expressed as such, it is nor unreasonable to expect values of 'v' exceeding unity, since now 'v' no longer measures returns to labour alone, but returns to labour and capital[25].

### *5. The Meaning attached to the Time Variable*

The Interpretation normally given to the time variable is that it is intended to act as proxy for some missing variables that vary smoothly over time. Quite often, the variable considered to have this characteristic is technological change, but other factors such as capital services[26] and normal hours[27] have also been considered for this purpose. Though in a few cases, it appears that the time variable is introduced on an ad hoc basis, generally speaking it is derived rigorously from the underlying production function, and its coefficient is therefore related to properties of the production function. For example in models similar to Ball and St. Cyr's, the time variable comes from the Cobb-Douglas production function, and is supposed to stand for variables measuring technological change and capital stock, When the Cobb-Douglas production function is used to obtain an expression for $E_t$ (see equation (10)) the coefficient on the time trend is -(1/a)r, where 'r' is the rate of growth of output due to technology and capital changes, and 'a' is the returns to labour parameter.

The alternative interpretation proposed by Ireland and Smyth, produces a different coefficient on the time variables, even though the estimating form of Ball and st. Cyr's and Ireland and Smyth's specification have the same variables. In the latter specification the coefficient on the time variable is -1(1/v)r, as shown in equation (13) where 'r' is this time only intended to stand for neutral technical change and 'v' stands for returns to scale and not returns to labour alone.

Another interpretation given to the coefficient on the time variable is that it captures the effect of non-neutral technological change. We illustrate this possibility with respect to those models derived from the marginal productivity condition on labour. We start from a C.E.S. production function, similar to that assumed by Black and Kelejian (1970)and Williamson (1971):

$$Y_t = ((k(e^{r_1 t}(KU)_t)^{-p}) + ((1-k)(e^{r_2 t}(EH)_t)^{-p}))^{-v/p} \qquad (14)$$

which is similar to equation (2) except that technological change is assumed to be non-neutral;[28] since '$r_1$' and '$r_2$' stand for the rate of capital and labour augmenting technological change, which are not restricted to equal each other. Equation (2) may in fact be regarded as a special case of equation (14) where '$r_1$' and are '$r_2$' equal, implying a Hicks neutral technological change.

The resulting expression for desired labour from equation (14) is different from that obtained from equation (2), in that the coefficient on the time trend is not the same in both cases.

With the assumption of neutral technological change, the coefficient on the time variable is ((s-1)/v)r as shown in equation (3), where 'r' is the rate of neutral technical change, 'v' is the returns to scale parameter, and 's' is the elasticity of factor substitution. With non-neutral technical change, the resulting coefficient on the time trend would be $(s-1)r_2$ where this time the coefficient does not include 'r' but which stands for the rate of labour augmenting technical progress[29]. The final estimating forms of both specifications are similar, but, as has been shown, the interpretation of the coefficients is different.

Therefore if one applies regression analyses to equation (3) and obtains statistically significant estimates, there is no way in which one can decide, on the basis of these estimates alone, whether the coefficient on the time variable contains 'r' or '$r_2$' In other words, the estimated coefficient on the time variable need not imply neutral, as opposed to labour augmenting, technical change, as assumed in many studies of labour demand.

Likewise, the meaning of the coefficient attached to the time variable in models derived from the expansion path equation, depends on the initial assumptions made about technical change. However, this model , unlike that derived from the marginal productivity condition, permits the researcher to test whether technological change is neutral or not. As was noted earlier, Hicks neutral technical change implies that '$r_1$' and '$r_2$' are equal, where the ratio of marginal products is not affected by technical change. A significant statistical difference between '$r_1$' and '$r_2$' may therefore indicate that technical change is not neutral, but is biased in favour of capital or labour augmentation depending on whether or not '$r_1$' exceeds '$r_2$ .

This can be shown by using equation (14) to derive an expression for desired labour, which when transformed into logarithms yields:

$$\ln(EH)_t = s\ln\frac{1-k}{k} + s\ln(C/W)_t + \ln K_t + ((s-1)(r_2 - r_1))t \qquad (15)$$

Labour augmenting bias in technical change is implied if the coefficient on the time trend has a negative sign and the elasticity of substitution is smaller than unity, in which case $r_2 > r_1$.[30]

The underlying assumptions regarding the production function are therefore of great importance for interpreting the coefficients on the time variable. The Nadiri model lends itself to a very interesting interpretation, but as was pointed out earlier, its estimation required data on capital services and their cost, which may not be good enough to produce reliable estimates.

### *6. Labour Adjustment*

An important feature common to many studies in the three categories of models we have discussed above, is the allowance made for the possibility that the actual change of employment from one period to another is less than that desired. There are various reasons why firms carry out partial adjustment of their labour requirements, amongst which the most important is perhaps that there are costs associated with labour adjustment. These costs include those incurred in hiring, such as training costs, and firing, such as redundancy payments[31]. This implies that the optimal level of employment is truly optimal only in the long run, since in the short run the firm may find it advantageous to employ more or less people than is technologically necessary.

*6.1 The Partial Adjustment Scheme*

Most studies allow for such partial adjustment by postulating an equation of the form:

$$L_t - L_{t-1} = e(L_t^* - L_{t-1})$$

where $L_t - L_{t-1}$ is the actual change in the labour input; $L_t^* - L_{t-1}$ is the desired change (the asterix indicating the desired level of employment), and 'e' is the adjustment coefficient, expected to have positive values not exceeding unity[32].

According to this formulation, full adjustment is implied if 'e' is equal to unity, in which case $L_t = L_t^*$. Partial adjustment is implied if 'e' is smaller than unity[33].

The partial adjustment scheme is usually incorporated into the model by deriving an expression for desired labour first, as explained above with reference to the three categories of models, and the unobservable desired labour variable is then replaced by solving for it from the partial adjustment equation.

As we have shown, the specific form of the desired labour function is usually log-linear. Since problems of estimation would arise if the linear formulation of the adjustment scheme is incorporated in a log-linear function, the adjustment function may be specified in the following log-linear form:[34]

$$\ln L_t - \ln L_{t-1} = e(\ln L_t^* - \ln L_{t-1}), \qquad 0 < e \leqslant 1$$

from which the following is obtained:

$$\ln L_t^* = (1/e)\ln L_t - ((1-e)/e)\ln L_{t-1}$$

If, as an example, we assume that the desired labour function is

$$\ln L_t^* = a + b\ln W_t + c\ln Y_t + dt$$

we can substitute for $L_t^*$ from the partial adjustment scheme to obtain:

$$\ln L_t = ea + eb\ln W_t + ec\ln Y_t + edt + (1-e)\ln L_{t-1} \qquad (16)$$

which can yield estimates of the parameter 'e', and by substituting into the coefficients attached to $\ln W_t$, $\ln Y_t$ and t, estimates of 'b', 'c' and 'd' can also be obtained.

If the estimate of 'b' is taken to be that of the elasticity of substitution as indicated in equation (3), then the estimate of 'eb' may be regarded as that of the "short run" elasticity of substitution. It follows that if the coefficient (1-e) in equation(16) is found to equal zero, and therefore 'e' is equal to unity, implying full-adjustment, the "short run" and the "long run" elasticity of substitution coincide. This of course applies to the estimates of 'c' and 'd' also.

*6.2 Problems of the Partial Adjustment Scheme*

The specification of the partial adjustment scheme discussed in the preceding sub-section may be criticised on the grounds that it postulates a constant speed of adjustment, and that the lag scheme is arbitrary determined. Moreover, there are estimation problems arising from the presence of a lagged dependent variable in the final estimating equation.

To allow for the possibility of variable speeds of adjustment, some researchers have separated the sample of observations into different sub-periods. For example, it may be postulated that adjustments costs may differ for positive employment changes, as compared to negative employment changes. Thus by applying separate regressions to positive and to negative values of $(L_t - L_{t-1})$ one may test whether adjustment speed differs[35].

Alternatively, tests for non-constancy of the adjustment parameter (and of course of other parameters as well) can be based on a search for structural breaks, by applying variants of the Chow test for stability of the model. One would expect for example, that the pattern of labour adjustment changes as a result of government policy effecting costs of hiring and firing employees[36].

The question of the length of the lag cannot be settled a priori, and unfortunately statistical criteria are not of much help either, since the data may yield significant estimates and good fits for different lag lengths of the dependent variable. For example, Nadiri (1968) obtained almost identical good fits with the Koyck type specification and with a second order Pascal lag. The implied average lag from both schemes however, differed considerably, and the choice between the two depended heavily on the discretion of the author.

Another problem associated with the partial adjustment scheme is that the introduction of the lagged dependent variable in the final equation may give rise to biased estimates. To examine this problem we have to discuss the error term of equation (16), because the severity of the problem depends on the properties of this error term. Let us suppose that the expression for desired labour (including the error term) is:

$$\ln L_t^* = a + b \ln W_t + \ln Y_t + ct + u_t$$

where $u_t$ is assumed to be a random real variable with zero mean, constant variance, and is normal distributed. Moreover, it is assumed that there is no serial correlation in the error term.

Now if the partial adjustment is also specified with an error term as follows:

$$\ln L_t - \ln L_{t-1} = e(\ln L_t^* - \ln L_{t-1}) + v_t$$

the final form of the employment function would be:

$$\ln L_t = ea_t + eb \ln W_t + ec \ln Y_t + edt + (1-e) \ln L_{t-1} + (v_t - eu_t) \qquad (17)$$

As it stands, the error term in year t of equation (17) need not be correlated with the previous values, and may be assumed to be non-autocorrelated. This assumption can be tested using for example, the Durbin test[37].

On the assumption that serial correlation is absent, the only additional problem posed by the presence of a lagged dependent variable, is that the OLS assumption that all explanatory variables are non-stochastic no longer holds, since $L_{t-1}$ is related to the past values of the error term of equation (17).

However, if the error term is found to be serially correlated, the OLS estimates will not be consistent and moreover, the power of the Durbin Watson statistic, normally used to detect first order serial correlated, will be impaired, since its value will be biased towards two, suggesting absence of autocorrelation, when in fact it may be present[38].

Various estimation procedures have been suggested for equations with lagged dependent variables and serially correlated errors. These generally involve the use of Generalised Least Squares Procedures, and, if the value of the autocorrelation coefficient is unknown, the application of iterative schemes to estimate it[39].

Finally, a point should be made regarding the interpretation of the coefficient attached to the lagged dependent variable. Equations containing a lagged dependent variable as one of the regressors may be derived from different behavioural hypothesis. for example, an equation similar to equation (17) but with a different error term would have been obtained if the Koyck transformation or the Adaptive Expectations schemes were applied. These introduce autocorrelation by assumption, even if the original demand relation had non-autocorrelation errors[40]. The interpretation of the coefficient on the lagged dependent variable therefore depends on which scheme is considered applicable.

Again, as Griliches (1967) has pointed out, the presence of a lagged dependent variable in the estimating equation need not be due to partial adjustment or adaptive expectations, but simply due to serially correlated errors caused, for example, by some omitted variables. In such cases the coefficient on the lagged dependent variable may, upon estimation, turn out to be significant, and perhaps have a plausible value and could therefore mistakenly be interpreted in relation to partial adjustment[41]

Despite these problems, the partial adjustment scheme has generally produced sensible results when applied in labour demand models; and its application in such models is theoretically justifiable due to the costs of adjustment referred to above. The advantages of this scheme include also that it is relatively easy to apply, that the meaning of 'e' is intuitively appealing and that the scheme economises on degrees of freedom and reduces the problem of multicollinearity when compared to other more complicated schemes involving more than one lag.

### *7. Conclusion*

In this study, we have discussed methods of deriving a labour demand function, and commented on the interpretation that may be given to the coefficient of these derived equations. We have also shown that the coefficients of the labour demand equations may have important policy implications regarding factor substitution and labour absorption.

Of the three categories of models discussed above, those derived from the expansion path of the C.E.S. production function as in the Nadiri (1968) model, would seem to be the most interesting. For example, equation (15) when estimated produces information regarding the elasticity of substitution, via the effect of the factor price ratio, and regarding the presence of labour saving bias of technological change, via the time variable. It also permits the researcher to asses the effect of capital services on labour demand. Models derived from the marginal productivity condition on labour, such as equation (3) do not produce information on the effect of capital services, and do not permit the researcher to test whether or not technological change is neutral. But they produce additional information regarding returns to scale, via the output variable, which has important implications regarding labour absorption.

One important advantage of models derived from the he marginal productivity condition is that, unlike the Nadiri model, they do not require data on capital and its cost. Important as it may be, data on capital and its cost may, as we have noted, introduce serious measurement errors when it comes to estimating the model.

As regards the Ball and St. Cyr type of models, we have noted that these tend to produce unacceptable estimates of the magnitude of short run returns to labour. Moreover, these models do not permit the testing of the effect of the wage rate on labour employment.

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## NOTES:

1. The properties of the Cobb-douglas and the C.E.S. production functions are discussed in Brown (1966) and Ferguson (1969).
2. In this study we do not discuss problems of estimating simultaneous equations, resulting from the interdependence of the labour demand and labour supply equations. This problem is dealt with in Briguglio (1982) and (1984) as part of the discussion on equilibrium (or its absence in the labour market.
3. A large number of works, based on equations derived from the marginal productivity condition on labour, have been produced, utilisng cross-section and time series data, applied to advanced as well as developing countries. Works on developing countries include, Daniels (1969), Katz (1969), Harris and Todaro (1969), Williamson (1971), Oyelabi (1971), Behrman (1972), King (1972) Senga (1973), House (1973), Tyler (1974) and Roemer (1974). Works on advanced countries include Arrow et al. (1966), Lucas and Rapping (1970), Dhrymes (1969) Black and Kelejian (1970), Phipps (1975), Briscoe and Peel (1975), Crandall et al. (1975) and Peel and Walker (1978). For a discussion on works published before 1967 see Nerlove (1967a and 1967b). See also Gaudè (1975), Bruton (1972) and Acharya (1974), who discuss studies applied to developing countries.
4. We assume here the the variable H represents the number of hours worked productively, and therefore stands for the rate of utilisation of the number of persons employed. This question will be discussed further in section 4.
5. The meaning and effect of the time trend will be discussed in section 5. The reason for dedicating a separate section to this variable is that it is often included in the three categories of models discussed in this study.
6. It should be noted, however, that the usefulness of an equation relating labour demand to wage rates, output and a time trend, need not rest exclusively upon the C.E.S. specification, and therefore its coefficients may not be related to the C.E.S. production function as implied in eqaution (3). Apart from this, a labour demand relation containing the wage rate, output and a time trend as regressors has intuitive appeal in its own right.
7. It can be shown that this formulation is more consistent with the assumption of perfect competition. On this question, see Feldstein (1967).
8. The biases that may be introduced as a result of omitting this variable are discussed in Feldstein (1967) and Katz (1969).
9. See for example, Dhrymes (1969) and Black and Kelejian (1970).
10. See for example, Senga (1973) who used an index of industrial concentration based on size of employment, and Metcalf et al. (1974) who based their measurement on size of sales. The latter also used an index of unionisation (the percentage of the labour force that is unionised) to allow for market imperfections.
11. For a theoretical explanation of how the expression f.. the elasticity of substitution is derived from the Cobb-Douglas and the C.E.S. production functions, see Brown (1966), pp. 35-38, 46-50 and Ferguson (1969), pp. 99-110.
12. For a comparison of estimates of the elasticity of substitution from different studies, see Nerlove (1967a and 1967b), Bruton (1972), Acharya (1974) and Gaudè (1975).

13. Similar results were obtained by Williamson (1971) using a slightly different version of the Nadiri model, applied to data from the Phillipines. Briscoe and Peel (1975) applying a similar model to UK data, did not find statistically significant values of the coefficient on the factor price ratio.

14. In their 1969 study, Nadiri and Rosen specify four separate equations, applied to employees, hours, capital stock, and rate of capital utilisation, respectively. The model was further elaborated to include seven input demand equations in their 1973 study, where production and non-production workers were considered spearately, and inventories were included as an additional input.

15. See Deaton, p.9

16. See Hazledine (1978, p.180). Brechling (1976) commenting on the estimated effect of the input price ratio in Nadiri and Rosen study, remarked that the correctly predicted signs appear as an exception rather than the rule.

17. For example, the user cost of capital, as computed by Nadiri (1969) was defined as:

$$C = \frac{PK}{(1-u)}((1-uv)d+(1-um)r-(1-uz)\frac{\Delta PK}{PK})$$

where PK is the price of capital goods; u is the profit tax rate; v is the ratio of tax deductable depreciation to actual replacement; m is the proprotion of the cost of capital exempt from profit tax; d is the rate of depreciation; r is the discount rate; z is the share of capital gains which is taxed; and $\Delta$ PK/PK is introduced to allow for capital gains via inflation.

18. The complications that arise in measuring capital stock are discussed in Redfern (1955), Dean (1964), Griliches (1963), and Jaszi et al. (1962). A common formula used to calculate net capital stock, given the rate of depreciation, and some benchmark value of net capital stock is:

$$K_{t+1} = I_t + (1-d)K_t$$

where I is real gross Investment, K is net capital stock, and d is rate of depreciation. For a discussion on the problems associated with measuring the rate of capital utilisation see Hilton (1970) and Heathfield (1972).

19. See note 5.

20. See for example, Brechling and O'Brien (1967), Miller (1971), Maitha (1972), Roberts (1974) and Briscoe and Peel (1975). Such a finding is usually treated as a matter of concern because it contradicts the law of diminishing returns to labour, Nerlove (1967a, p. 225) describes such a finding as an indication of the unsatisfactory nature of the results.

21. On this point see also Kuh (1965).

22. The treatment of rates of utilisation of the labour input (hours) as a separate factor of production as suggested and applied by Feldstein (1967) and further tested by Craine (1973). Both studies yielded estimates of the elasticity of output with respect to hours with a larger numerical value than that with respect of men. Craine suggested that this finding is a possible explanation for the "paradox" of increasing returns to labour. Bowers and Deaton (1977) have however questioned the validity of this procedure on the grounds that the hours and men are not separate factors of production.

One reason why the rate of labour utilisation may vary cyclically is that management may push employees to work harder in busy periods, and allow a more relaxed pace, during slack periods. Also workers may themselves be motivated to adjust their productivity to safeguard their jobs.

23. The excessively low values of the estimated coefficient on Yt stimulated several attempts to explain this finding by proposing new specifications of the employment function. See for example Fair (1969) and Hazledine (1979).

24. This is a crucial assumption in the Ireland and Smyth derivation of the desired employment function. Ireland and Smyth did not regard this assumption as too restrictive, and considered it more plausible than the Ball and St. Cyr assumption that capital grows at a constrant rate. See Smyth & Ireland (1967, pp. 537-538).

25. Ireland et al. (1973) confirm this finding by estmating a Cobb-Douglas production function which allows for changing capital utilisation rates in the short run. Thus, even if in the short run capital stock is assumed to be constant, the possibility that its rate of utilisation may vary, may explain why there may be increasing returns to scale even in the short run. Nadiri and Rosen (1969, p. 469) also reinterpret the output employment elasticity as short run returns to labour and capital utilisation, and attribute the finding of a large value of the output employment elasticity to the omission of a variable measuring the rate of capital utilisation. Fair (1969, p. 25) however, asserts ;that even if this elasticity is interpreted as returns to scale, as in the Ireland and Smyth's study, one should not expect values much higher than unity.

26. See for example Ball and St. Cyr (1966, p. 180).

27. See Hazledine (1978), p. 186.

28. In the Hickesian sense, technical innovations may be neutral, labour saving or capital saving according to whether they leave the relative marginal productivities of labour and capital unchanged, they lower the marginal productivity of labour relative to the marginal productivity of capital, or raise the marginal productivity of labour relative to that of capital, given a constant capital labour ratio.

29. The marginal productivity condition for labour, derived from equation (14) when rearranged and transformed into logarithms, yields:

$$\ln (EH)_t = [s \ln (1-k) v] - s \ln W_t + (1+s(v-1))/v \ln Y_t - (1-s) r_2 t$$

The estimating form therefore is similar to equation (3), but the coefficient on 't' is interpreted differently. It should be stressed therefore that although many empirical works assume Hicks-neutral technical change, rather than the more general input augmenting technical change, the estimates of coefficients of the labour demand function need not confirm this assumption.

30. As formulated, equation (15) implies labour augmenting bias in technical change if the coefficient on the time variable is negative, and the estimate of the elasticity of substitution is smaller than unity. It can be shown that labour augmenting technincal change is labour saving if, and only if, the elasticity of substitution is smaller than unity (see David and Van Klundert 1965, pp. 362-363).

31. For a discussion on the costs of adjustment of the labour input see Soligo (1966, pp. 173-175). An interesting study on labour, as a quasi fixed factor of production is that by Oi (1962) whose results indicate that a relatively high "degree of fixity" related to non-wage costs, such as firm specific training, is associated with a relatively smaller chang of employment with given output change.

32. The letter L here stands for either men (E) or manhours (EH). In empirical work the partial adjustment scheme has been applied to men, and manhours, although the fixity of the labour input is more likely to occur in the case of men, since hours can be adjusted more easily. Briscoe and Peel (1975, p. 119) suggest that it is more plausible to specify the demand function in terms of men, rather than manhours, and to postulate that adjustment takes place in terms of employment costs.

33. It can be shown that the partial adjustment shceme can be derived form the minimisation of a quadratic cost function, involving costs of departures from equilibrium, and costs of adjusting employment from one period to another (See Griliches 1967).

34. Deaton (1977) has shown that the log-linear specification of the partial adjustment function, unlike the simple linear specification, may be interpreted to imply higher costs of downward adjustment and lower costs of upward adjustment.

35. See Brechling (1965) and Roberts (1974) for examples of such a test.

36. For example, the effect of the Selective Employment Tax in the U.K. may have affected the pattern of employment adjustment. On this possibility, see Knight and Wilson (1974), Taylor (1972) and Briscoe and Peel (1975).

37. Durbin (1970) proposed a test for serial correlation applicable to equation with a lagged dependent variable. This consists of computing what Durbin calls an 'H' statistic, which is than tested as a standard normal deviate.

38. See Johnson (1972) pp. 305-313 and Nerlove and Wallis (1972) for a discussion on these points.

39. Such methods are discussed in Johnston (1972) Chapter 10.

40. See Johnstone (1972) If the Koyck scheme or the adaptive expectation scheme are used, the implied error term would be $(u_t - \lambda u_{t-1})$ which is not serially independent from its previous value. Hence autocorrelation would be present.

41. See Griliches (1967), p. 34.

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