Extinction Revisited: "Allee Effect" and Irreversibility in "Schooling" Fisheries

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Abstract - **Important results of Colin Clark's research in the 70s are used again in the discussion of the limits to the privatization of the fisheries. Those results highlighted the possibility of species extinction motivated by special forms of the natural growth function. This paper revisits the situation in which the growth function exhibits a non-feedback, or depensation, curve. The existence of non-shrinkage curves poses problems in determining the sustainable yield and has important implications for resource management. The so-called "Allee Effect" may explain the difficulties of recovery of certain stocks, even when there are a set of limitations to the fishing effort. Ultimately, it explains the extinction of some species: if we face a situation of non-critical feedback, an effect of irreversibility is introduced. These effects are considered in the schooling species fisheries case.**

Keywords - *Natural Growth Law, "Allee Effect", Irreversibility, "Schooling" Fisheries".*

1. Introduction

After almost four decades, important results of Colin Clark's (1973, 1974) research are used again in the discussion of the limits to the privatization of the fisheries as a means of introducing more efficiency in fisheries operations (Clark, Munro & Sumaila (2008, 2010)). Those results highlighted the possibility of species extinction motivated by special forms of the natural growth function of species.

This paper revisits the situation in which the growth function exhibits a non-feedback, or depensation, curve. The existence of non-shrinkage curves poses problems in determining the sustainable yield and has important implications for resource management.

The structure of the paper is the following:

In the first point we present the basic model of fisheries management, the so-called Gordon /Schaefer model. This bio-economic model introduces an equation that reflects the natural growth of the species and describes their biological dynamics.

The second point introduces different forms of this equation and investigates the impacts of non-feedback characteristics of growth functions on the management and conservation policy. In this context, the so-called "Allee Effect" may explain the difficulties of recovery of certain stocks, even when there are a set of limitations to the fishing effort.

In the third point the effects of irreversibility are considered and the possibility of species extinction is discussed. The schooling species fisheries case is used as an example of this kind of preoccupations.

2. The underlying Biological Dynamics of Gordon/Schaefer Model

To design an acceptable bio-economic model of fishing, we must introduce, in its foundation, a biological model of fishing resources growth. In the Gordon (1954) article, the underlying biological foundation is a variant of Schaefer (1957). The populations' dynamics can be easily described with a "Macro-biological Approach". A fish resource population or biomass will, if not subject to human capture, grow in terms of weight, both as a consequence of recruitment of new individuals and as the result of the growth of individual fish in the population. Natural mortality will act as a check on

growth. If we assume stable environmental conditions (especially, if we do not introduce men as predators), along the time, the biomass will approach a natural equilibrium level at which net growth is zero (Coelho, 1989; Coelho, 1999; Smith, 1968).

We define the Law of the Natural Growth as the specific form by each species or resource is regenerated. In fact, each specie regeneration capacity is affected by biological characteristics (birth rate, mortality rate, age structure, etc.) and environmental characteristics (nutrients abundance, temperature, habitat, existence and efficiency of the predators, etc). It was interesting to evaluate all of the factors but difficult. So, when introducing into the model the biological characteristics, we must consider restrictive hypothesis.

If we do not attempt to distinguish among the factors influencing net growth, the growth of the biomass can be viewed as a function of the biomass itself and the population dynamics can be modelled by a very simple differential equation:

$$F(x) = dx/dt$$

x denotes the biomass and F(x) represents the regeneration capacity associated with every level of the stock.

The relation between the rate of growth and the level of the stock is not monotonic. In the Schaefer model, we'll have a quadratic function:

$$F(x) = r \, x \, (1 - x/K)$$

K denotes the carrying capacity and r, constant, denotes the intrinsic growth rate. When integrated, we are facing the popular Lotka/Volterra logistic equation of population dynamics (Neher, 1974; Wilen, 1985).

When we introduce men action of fishing, the first equation is modified:

$$dx/dt = F(x) - H(t)$$

H (t) denotes the capture rate.

The production function is given by:

$$H(t) = h \, E(t) \, x(t)$$

where E(t) denotes the fishing effort at time t (a kind of "capital-jelly" measure of the flow of labour and capital services devoted to fishing; this could be evaluated, for example, in terms of fishing hours), and h, constant, denotes a capture-ability coefficient measuring the different capture conditions between fishing grounds.

If the resources are being captured in a sustainable basis, then dx/dt = 0 and H (t) = F(x). Hence, F(x) can be viewed as the sustainable yield associated with a given biomass level. Since H(t) is a function of E, as well as x, one can establish the sustainable yield/fishing effort relationship:

$$Y = \alpha E - \beta E^2,$$

where Y denotes sustainable physical yield, with $\alpha = h \, K$ and $\beta = h^2 \, K/r$.

With the biological model complete, we can introduce prices and costs. We assume that both the demand for captured resources and the supply of fishing effort are perfectly elastic. The cost function can be expressed as the simple equation:

$$C = c \, E$$

We assume that the total cost is linear with effort. The constant c denotes unit cost of effort.

Sustainable revenue is represented by pY, where p is the unit price of fishing. It has, also, a quadratic form.

We can now solve the model and analyse the behaviour of the "industry".

The main conclusions can be summarized as follows: If fishing was managed by a "sole owner", it would be stabilised at the point where sustainable resource rent - sustainable revenue less total cost - is maximised. In this situation, fisheries are managed in a socially optimal manner. If fishing effort expands beyond this point, overexploitation of the resources occurs.

But, as fishing activities take place in a regime of open access, there is no landlord to appropriate the resource rents generated by fishing. Thus, if fishing was at the point where resource rents are maximised, the "industry" would be enjoying super-normal returns and new fishermen would be attracted to enter the fishing ground. If fishing is unregulated and competitive, fishing effort will expand, leading to overexploitation of biomass. In this case, fisheries would not be in equilibrium until it had expanded to the point where total costs are equal to total revenues, that is, until resource rent had been fully dissipated. This "bionomic equilibrium" reflects the existence of

externalities in the capture process and it's a case of market failure (Filipe et al, 2007; Coelho, 2011).

Besides the relevance of the conclusions, we have to underline the role of the biological model when constructing such a bio-economic model of fisheries management. In fact, the potency of explanation of the basic model depends on the capacity of the growth equation that is introduced, to catch the fundamental characteristics of biological dynamics of the species considered. At the same time, if we do not want to introduce too much mathematical complexity in the model, we must take care of the efficacy/ feasibility of the model and of the biological information needs to estimate the bio-economic model.

3.   Compensation and Non-feedback Control in Biological Models

Also problematic is the possibility that F(x) does not have the usual form. Several alternative forms for the logistic model have been proposed.

The logistic model itself and, in general, models with a growth function such as the first figure - so that the proportional rate of growth

$$r(x) = \frac{F(x)}{x}$$

is decreasing with x - are called models of pure compensation.
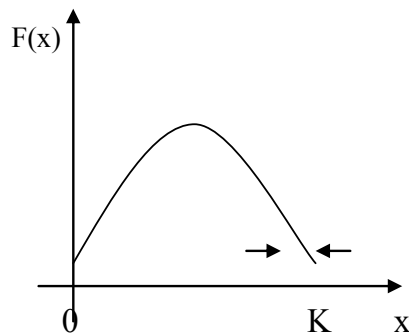
On the other hand, if r (x) is an increasing function of x, for certain values of x, it is said that there is a process of non-feedback or "depensation". For example, there are curves that shows non-feedback for $0 < x < K^*$ and compensation for $x > K^*$. These types of curves are the so-called non-feedback curves.

Also, we can use the expression "curve of non-critical feedback" to refer non-feedback curves with the property F(x) <0 for certain values of x, near x = 0, as in the second figure.
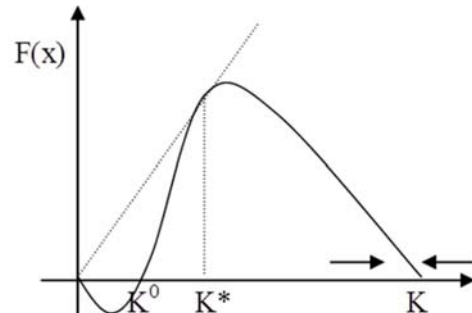
The existence of non-shrinkage curves poses problems in determining the sustainable yield, y, and has important implications for resource management. The first aspect can be seen as follows. Assuming that the stock is subject to a given capture with a constant effort, we have:

$$\frac{dx}{dt} = F(x) - qEx$$

Suppose that we want to build the Yield-Effort curve: y = y (E).

In the case of pure compensation, each level of space (E) produces a unique and stable solution for the population balance X_E and the corresponding yield Y_E is:

$$y_E = f(x_E)$$

Therefore, the curve Yield-Effort rises to a maximum (MSY) and then decreases slowly as the effort is being increased. The sustained yield is zero for values of $E \geq E^*$, where $q E^* = F'(0) = \max r(x) = r^*$. As in the logistic model, the resource stock is driven asymptotically to zero if the catch rate is maintained at a level higher than the intrinsic growth rate r^*.
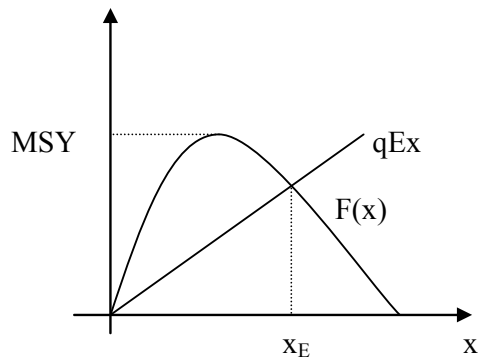
Natural Growth Curve
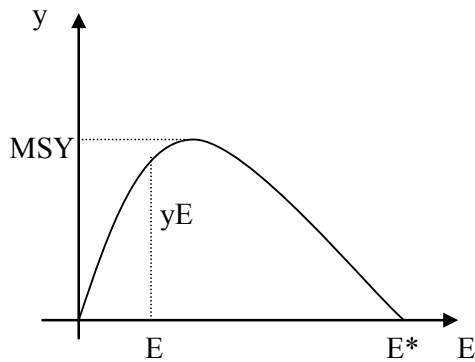
Yield-Effort Curve

In the absence of feedback effect we face the problem of the existence of multiple equilibrium solutions. For each level of effort $E < E^* = \max r(x)/q$ there is a population of stable equilibrium $1xE$ and a yield equilibrium $1yE$, equally stable. But, for values of $E > E+ = F'(0)/q$, there is also a population of unstable equilibrium, $2xE$.

If the initial level of the population $x(0)$ is higher than $2xE$, the equilibrium level is established at $x = 1xE$; however, if $x(0) < 2xE$, equilibrium is established at $x = 0$, assuming that E is constant. Thus, there remains a critical effort E^* such that $y(E) = 0$, but the yield curve-effort is different from pure compensation model, now forming a discontinuity at $E = E^*$, where the yield curve reaches zero if E exceeds the critical level.
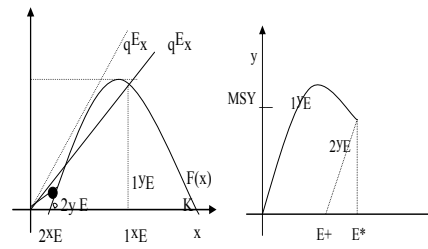

The implications for the management policy are very relevant:

First, the incremental approach of Schaefer model is not appropriate, since a slight increase of E can lead the population to collapse. It reminds us the "butterfly effect": a simple variation in E conducts the population to a possible disaster.

Furthermore, the model introduces a special effect. Suppose that the effort is approaching the level $E > E^*$ and x (t) approaches zero (while x is still positive). If E is reduced to a level below E^*, this does not imply that the system returns to $1yE$. In fact, one can demonstrate that if the reduction is not below $2xE$, the population will continue to decline. That is, to return to $1yE$ may be necessary to reduce the effort until $E+$.

In short: to bring the population to acceptable levels, the reduction in fishing effort may be much higher than desirable. This "Allee Effect", as it is known in Anglo-Saxon literature (see, for example, Southey (1972) and Larkin, Raleigh & Wilimovscky (1964)), may explain the difficulties of recovery of certain stocks, even when there were a set of limitations to the fishing effort. Ultimately, it explains the extinction of some species.

Worse, if we face a situation of non-critical feedback. A new effect is introduced - that of irreversibility.
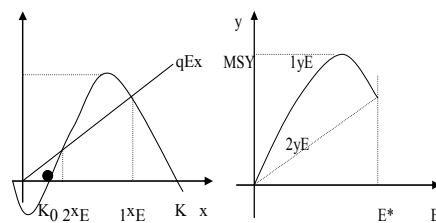
In this case, it can be seen that each level of effort $E \geq 0$ gives rise to two equilibrium solutions

(1xE and 2xE) and that x = 0 is a stable equilibrium solution for any E. If effort goes beyond a supercritical level, the population may find itself reduced to a level lower than K0 (the minimum viable population). So, we find ourselves in a situation of irreversible extinction.

4.   Non-critical Feedback and Species Extinction

In the world's fisheries there are some studied cases of this situation of non-feedback control. These studies also have important practical indications about resource management and conservation measures.

An example is the so-called "Schooling fisheries" case. These species, like sardine, tend to live in large schools. The existence of large schools provides a means of defense against large predators. The mathematical theory that studies the relationships between schools and predators, due to Brock and Riffenburgh (see Clark, 1974) indicates that the detection by predators is an inverse function of the size of the shoal. Since the amount of fish that a predator can consume has an average threshold, when exceeding this limit, the growth of the school implies a reduction of consumption by the predator. Also, other defensive aspects of the school, such as bullying or confusion of predators, are elements of more effective "schools".

However, this type of large schools behavior has allowed the development of highly efficient fishing techniques. With modern fish finding equipment by satellite, with modern nets of fibers, strong and easy to handle, fishing can remain profitable, even for small stocks (Bjorndal (1987), Neher (1990), Mangel and Clark (1983)). Of course, as these stocks are getting scarce they become even less protected.

Furthermore, the existence of these techniques prevents a stock effect on business costs, as opposed to the so-called "search fisheries." For "search" species, the fishing action implies search and detention. The existence of larger populations is essential for fishermen because it reduces the costs of detection (see Neher, 1974). But now, the high capacity of detection of new technology means that costs are no more sensitive to the size of stock, even for schooling fisheries (Bjorndal and Conrad, 1987).This situation is extremely dangerous because of the low biotic potential of some species. The reproductive capacity requires a minimum value below which extinction is inevitable. Since the

efficiency of the school is reduced, the losses due to the effects of predation are relatively large at low stock levels. Clark (1974) states this turns into a situation of non-feedback in the stock-recruitment relationship. And that implies a discontinuity in the curves of yield-effort, so that an infinitesimal increase in stress, below a certain threshold, leads to an unstable state which can lead to extinction.

In the case of non-critical feedback, the path to extinction may be irreversible. According to Clark (1974), a necessary and sufficient condition for the existence of non-critical feedback is that the average fertility is too small to balance the high mortality of some low levels of population, for which the school becomes inefficient. So, the fishing communities can face a possible sudden collapse in the exploitation of small schooling species subject to strong capture. These breakdowns can directly result from overfishing or can be indirectly induced by environmental fluctuations operating on a population of excessive exploitation and decreased resistance.

Bjorndal, Conrad and Salvanes (1993), in a study on the capture of seals in the area of Newfoundland, concluded that, although the stock had not reached the danger of extinction, the existence of a feedback-like effect could be appropriate to describe the dynamic behavior of this specie. The same way, the poor biotic potential of species like whales, especially when subject to severe operating conditions, together with the existence of an intertemporal discount rate higher than the natural growth rate of the species, may be explanatory factors for the near extinction of this species and the need of establishing a "moratorium" on whaling by the International Whaling Commission. On this issue, see, for example, the studies of Conrad (1989) and Clark (1987).

5.   Final Remarks

The widespread implementation of rights based management (RBM) schemes in fisheries management, as ITQs, increased the opportunity for private sector groups to influence fisheries management. This development has given rise to a debate over the extent to which should be encouraged this private influence.

In a provocative paper, Grafton, Kompass & Hilborn (2007) state, on the basis of empirical investigation, that the results of Clark (1973, 1974) are really no more than a theoretical curiosity with no

practical significance. But, other important investigations' results highlight that the conclusions of Clark cannot be safely dismissed. See, for example, the study of Dulvy, Sadovy & Reynolds (2003). The authors point that the possibility of extinction is relevant. In fact, the extinction occurred in about 100 marine fisheries (most of all, it is expected, in situations of positive minimum viable population levels – see Hutchings (2000)). That indicates that there is substantial scientific evidence that we can find several species with positive minimum viable population levels, and that there exist population levels below which the resources cannot replace their original levels of abundance even if we reduce the fishing effort.

According to Clark, Munro & Sumaila (2010) this should imply that there are limits to the privatization of fisheries: there are situations in which the communities should not put under private hands the defense of the common interest. In situations of "depensation" growth curves and of little growth rate of renewal, compared to the existing interest rate, the private management should lead ("efficiently-seeming") to species extinction.

We go further: in situations like this we should also not purely confide in public policy. The management of such situations imposes an important restriction to the managers and underlines the fundamental idea that, in these types of industries, there is more than simply Economics. In fact, Nature and her laws impose a necessary humility to political devisors. A principle of precaution in the definition of total authorized capture levels and in the formulation of other command and control, or economic, tools is simply a question of good sense and ethical posture.

Acknowledgments

References

[1] Anderson, L. (1986), *The Economics of Fisheries Management*, Revised and enlarged edition, The John Hopkins University Press

[2] Anderson, L. (1986), *The Economics of Fisheries Management*, Revised and enlarged edition, The John Hopkins University Press

[3] Arrow, K. and Fisher, A. (1974), Environmental Preservation, Uncertainty, and Irreversibility, *Quartely Journal of Economics*, Vol. 88 (May), pp 312-319

[4] Beddington, J. (1974), Age Structure, Sex Ratio and Population Density in the Harvesting of Natural Animal Populations, *Journal of Applied Ecology*, Vol. 11, pp 915-924

[5] Beddington, J. and May, R. (1977), Harvesting Natural Populations in a Randomly Fluctuating Environment, *Science*, Vol. 197, pp 463-465

[6] Berck, P. (1979), Open Access and Extinction, *Econometrica*, Vol. 47, N° 4, pp 877-882

[7] Bergé, P., Pomeau, Y. and Vidal, C. (1984), *Order within chaos*. New York: John Wiley

[8] Berliner, L. (1992), Statistics, Probability and Chaos, *Statistical Science*, Vol 7, n° 1, pp.69-122

[9] Bishop, R. (1978), Endangered Species and Uncertainty: The Economics of a Safe Minimum Standard, *American Journal of Agricultural Economics*, Vol. 60, N° 1, pp 10-18

[10] Bjorndal, T. (1987), Production Economics and Optimal Stock Size in a North Atlantic Fishery, *Scandinavian Journal of Economics*, Vol. 89, N° 2, pp 145-164

[11] Bjorndal, T. (1988), The Optimal Management of North Sea Herring, *Journal of Environmental Economics and Management*, Vol. 15, N° 1, pp 9-29

[12] Bjorndal, T. (1989), Production in a Schooling Fishery: The Case of the North Sea Herring Fishery, *Land Economics*, Vol. 65, N° 1, pp 49-56

[13] Bjorndal, T. and Conrad, J. (1987), The Dynamics of an Open Access Fishery, *Canadian Journal of Economics*, Vol. 20, N° 1, pp 74-85

[14] Bjorndal, T., Conrad, J. and Salvanes, K. (1993), Stock Size, Harvesting Costs, and the Potential for Extinction: The Case of Sealing, *Land Economics*, Vol. 69, N° 2, pp 156-167

[15] Conrad, J. (1989), Bioeconomics and the Bowhead Whale, *Journal of Political Economy*, Vol. 97, N° 4, pp 974-987

[16] Conrad, J. and Bjorndal, T. (1991), A Bioeconomic Model of the Harp Seal in the Northwest Atlantic, *Land Economics*, Vol. 67, N° 2, pp 158-171

[17] Bromley, D. (1991), Testing for Common Versus Private Property: Comment, *Journal of Environmental Economics and Management*, Vol. 21, N° 1, pp 92-96

[18] Clark, C. (1973), Profit Maximization and the Extinction of Animal Species, *Journal of Political Economy*, Vol. 81, N°4, pp 950-961

[19] Clark, C. (1973), The Economics of Overexploitation, *Science*, Vol. 181, pp 630-634

[20] Clark, C. (1974), Possible Effects of Schooling on the Dynamics of Exploited Fish Populations, *Journal du Conseil International pour L'Exploration de la Mer*, Vol. 36, Nº 1, pp 7-14

[21] Clark, C. (1985), *Bioeconomic Modelling and Fisheries Management*, John Wiley & Sons

[22] Clark, C. (1990), *Mathematical Bioeconomics, The Optimal Management of Renewable Resources*, 2nd edition, Wiley-Interscience Publication, John Wiley & Sons, Inc.

[23] Clark, C., Munro, G. and Sumaila, R. (2010), Limits to the Privatization of Fishery Resources, *Land Economics*, Vol. 86, pp.209-218

[24] Clark, C., Munro, G. and Sumaila, R. (2008), *Limits to the Privatization of Fishery Resources*, Fisheries Centre, University of British Columbia, Working Paper: 07/2008

[25] Clark, C. and Munro, G. (1975), The Economics of Fishing and Modern Capital Theory: A Simplified Approach, *Journal of Environmental Economics and Management*, Vol. 2, nº 2, pp 92-106

[26] Coase, R. (1960), The Problem of Social Cost, *Journal of Law and Economics*, III, pp 1-44

[27] Coelho, M. (2011), Hunting Rights and Conservation: The Portuguese Case, Inter*national Journal of Latest Trends in Finance & Economic Sciences*, Vol.1, nº4, pp.164-170

[28] Coelho, M. (1999), *A Tragédia dos Comuns Revisitada, A Pesca do Bacalhau na Terra Nova: Consequências do Regime das 200 Milhas*; Tese Doutoramento, ISEG/UTL, Lisboa

[29] Coelho, M. (1989), *Gestão e Conservação dos Recursos da Pesca*, Tese de Mestrado, ISE / UTL, Lisboa

[30] Dulvy, N., Sadovy, Y. and Reynolds. J. (2003), Extinction Vulnerability in Marine Populations, *Fish and Fisheries*, Vol.4, pp.24-64.

[31] Feller, W. (1940), On the Logistic Law of Growth and its Empirical verifications in Biology, *Acta Biotheoretica*, Vol. 5, pp 51-66

[32] Ferreira, M., Filipe, J., Coelho, M. and Pedro, M. (2011), Modelling the Dissipative Effect of Fisheries, *China-USA Business Review*, Vol. 10 (11), November 2011, pp. 1110-1114.

[33] Ferreira, M., Filipe, J., Coelho, M. and Pedro, M. (2011), Chaos Effect in Fisheries Management, *Journal of Economics and Engineering*, 2 (1), pp. 4-13. ISSN: 2078-3046

[34] Filipe, J., Coelho, M. and Ferreira, M. (2007), *O Drama dos Recursos Comuns. À procura de soluções para os ecossistemas em perigo*, Edições Sílabo, Lisboa.

[35] Filipe, J., Coelho, M. and Ferreira, M. (2005), Sistemas Dinâmicos, Caos e os Comuns da Pesca, *Revista de Economia Global e Gestão*, N.º 2/2005

[36] Filipe, J., Ferreira, M. and Coelho, M. (2008), *The Relevance of Chaos Theory to Explain Problems of Overexploitation in Fisheries*, Working Paper ISEG/UTL WP/24/2008/DE/SOCIUS

[37] Filipe, J., Ferreira, M., Coelho, M. and Pedro, M. (2009), Complexity, Theory of Chaos and Fishing, In Porath, D. and Bayer, A., *International Suplement special "update", the scientific bi-annual magazine of FH Mainz*, University of Applied Sciences. Mainz, Germany: FH Mainz, University of Applied Sciences

[38] Getz (1988), Harvesting Discrete Nonlinear Age and Stage Structured Population, *Journal of Optimization Theory and Applications*, Vol. 57, Nº 1, pp 69-83

[39] Gordon, H. (1954), The Economic Theory of a Common Property Resource: The Fishery, *Journal of Political Economy*, Vol. 62, pp 124-142

[40] Grabinski, M. (2004), Is There Chaos in Management or Just Chaotic Management?. *Complex Systems, Intelligence and Modern Technology Applications*, Paris

[41] Grabinski, M. (2008), Chaos – Limitation or Even End of Supply Chain Management. *High Speed Flow of Material, Information and Capital*, Istanbul

[42] Grafton, R., Kompass, T. and Hilborn, R. (2007), Economics of Overexpolitation Revisited, *Science*, Vol.318, pp.1601-1606

[43] Hardin, G. (1968), The Tragedy of the Commons, *Science*, Vol 162, pp 1243-1247

[44] Hastings, A., Hom, C. L., Ellner, S., Turchin, P. and Godfray, H. C. J. (1993), Chaos in Ecology: Is Mother Nature a Strange Attractor?, *Annual Review of Ecology and Systematics*, 24: 1-33

[45] Hutchings, J. (2000), Collapse and Recovery of Marine Fisheries, *Nature*, Vol.406, pp.882-885

[46] Lansing, J. S. (2003), Complex adaptive systems, *Annual Review Anthropology* http://www.ic.arizona.edu/lansing/GompAdSys.pdf

[47] Larkin, P. , Raleigh, R. and Wilimovcky, N. (1964), Some alternative premises for constructing theoretical production curves, *Journal of the Fisheries Research Board of Canada*, 21, pp 477-484

[48] Lévêque, G. (2002), *Ecologia: do ecossistema à biosfera*, Lisboa: Instituto Piaget

[49] Levin, S. (2003), Complex adaptive systems: exploring the known, the unknown and the Unknowable, B*ulletin of the American Mathematical Society*, 40

[50] Lopes, R. (1985), L'Économie des Ressources Renouvelables, *Económica*, Paris

[51] Mangel, M. and Clark, C. (1983), Uncertainty, Search, and Information in Fisheries, *Journal du Conseil International pour L'Exploration de la Mer*, Vol. 41, pp 93-103

[52] Maynard Smith, J. (1968), *Mathematical Ideas in Biology*, Cambridge: Cambridge University Press

[53] May, R. (1974), Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles and Chaos, *Science*, Vol. 186, pp 645-647

[54] Myers, N. (1989), A Major Extinction Spasm: Predictable and Inevitable?, in Western, D. e Pearl, M (eds.), *Conservation for the Twenty-first Century*, Oxford University Press, New York, pp 42-49

[55] Neher, P. (1990), *Natural Resource Economics: Conservation and Exploitation*, Cambridge: Cambridge University Press

[56] Neher, P. (1974), Notes on the Volterra - Quadratic Fishery, *Journal of Economic Theory*, Vol. 8, pp 39-49

[57] Olsen, L. & Degan (1985), Chaos in biological systems, *Quarterly Review of Biophysics*, 18(2), 165-225

[58] Schaefer, M. (1957), Some Considerations of Population Dynamics and Economics in Relation to the Management of the Commercial Marine Fisheries, *Journal of the Fisheries Research Board of Canada*, Vol.14, pp 669-681

[59] Scott, A. (1955), The Fishery: The Objectives of Sole Ownership, *Journal of Political Economy*, Vol. 63, pp 116-124

[60] Smith, V. (1968), Economics of Production from Natural Resources, *American Economic Review*, Vol. 58, Jun, pp 409-431

[61] Southey, C. (1972), Policy Prescriptions in Bionomic Models: The Case of the Fishery, *Journal of Political Economy*, Vol. 80, Nº 4, pp 769-775

[62] Wilen, J. (1985), Bioeconomics of Renewable Resource Use, in Kneese, A. and Sweeney, J. (eds), *Handbook of Natural Resource and Energy Economics*, Vol. I, North-Holland, Amsterdam, pp 61-124.