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Financial Protection for the Elderly - Contracts Based on Equity Release and Critical Health Insurance

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Abstract:

Purpose: *The purpose of the article is to present a new marriage contract combining the advantages of the reverse annuity contract and the critical health insurance contract. The content of the article focuses on the method of determining premiums and benefits related to this contract.*
Design/Methodology/Approach: *The multistate model is used to describe the survival time of the spouses. The elements of transition probabilities matrix were estimated assuming that the stochastic process describing further life expectancy has Markov property. Empirical examples are presented on the basis of data concerning lung cancer taking into account the incidence, mortality and fatality rates from this disease.*
Findings: *The obtained results indicate that older people can obtain additional financial resources which can improve the quality of life and raise its standard significantly. Moreover, additional financial protection can be ensured in case of a critical illness.*
Practical implications: *The described contract can be used in practice as a form of providing additional financial resources supplementing the home budget of pensioners, especially in the event of a serious illness. The contract can also be used with the option of long-term care insurance. Despite the guaranteed free health care, such contracts can significantly increase the quality of life of patients during a chronic severe illness.*
Originality/Value: *The model presented in the paper is original. It allows flexible modelling of cash flows to provide financial protection in case of low income during retirement and health deterioration of the elderly.*

Keywords: *Financial protection, reverse annuity contract, critical health insurance contract, multistate model, stationary Markov chain.*

JEL classification*: C5, C6, G17, G22, I13, J1.*

Paper Type: *Research study.*

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1. Introduction

The phenomenon of aging societies causes a gradual increase in the number of pensioners in most European countries. Retirees belong to the social group most at risk of material poverty, which applies especially to women at retirement age (Bayar 2013; Bernard and Li 2006; Corden *et al.*, 2010; Fisenko *et al.*, 2018; Vlasov 2017). Economic status of a marriage couple before the husband's death is defined in the literature as the most important factor in determining a widow's financial situation (McGarry and Schoeni 2005). Chronic critical illness can also affect the material situation of pensioners. Many studies confirm that healthcare spending, especially on the long-term care, are a significant out-of-pocket expenses in pensioners' budget (Corden *et al.*, 2010; McGarry and Schoeni, 2005).

In Poland and many post-communist countries, the situation of retirees is untypical. On the one hand, retirees are in a difficult financial situation due to low level of pensions. On the other hand, pensioners are often owners of property. Reverse annuity contract is one of the equity release forms offered to elderly people. This contract is a sales model (home reversion scheme), which is used for individuals in Poland. An owner can receive an additional benefit in exchange for renunciation of his rights to a real estate to a company. The owner is guaranteed, by a notarial act, the right to stay in the property until his death, however he formally is not the owner of this property.

This type of contract could be a beneficial option because it allows obtaining additional funds for current needs, however, in situations of critical diseases, pensioners may not have enough funds for additional treatment or for increasing the quality of life during palliative treatment. The combination of a reverse annuity and insurance against the risk of a critical disease in one contract enables a regular, permanent inflow of funds to the budget of pensioners in the form of annuity benefits and additional health insurance benefits in the event of a critical illness. This type of contract can also be used as a tool of increasing social participation in the co-payment for health services. In all developing countries, growth in demand for medical services and increasing health care expenditures can be observed. It is caused by many factors, including demographic and cultural changes. Aging societies with their bad lifestyle habits require a special, extensive health service system, often focused on long-term care. Due to limited budget resources, the growing needs can be difficult to satisfy. For this reason, a system of co-payment for medical services is proposed in many countries. Additional health insurance is one of the forms of co-payment, enabling the participation of additional funds in financing medical services.

In this paper, we present a marriage contract which combines a reverse annuity contract with insurance against the risk of a critical illness. Premiums and benefits related to this contract are determined based on the multistate model. This model is an extension of the model described in the work (Dębicka *et al.*, 2015) and assumes that spouses are the owners of a property. The paper is organized as follows. In Section 2, the multistate model and the transition probabilities matrix are described. In Section 3, the cash flows connected with the model are presented and the formulas for benefits and premiums are introduced. Empirical examples are analysed in Section 4. These examples illustrate in what way retirees' finances can increase by means of the new contract.

2. Multistate Model

Spouses usually own of properties and due to this reason we consider the marriage contract. The construction of a new contract and the analysis of cash flows require defining a model which describes the further lifetime of the spouses, taking into account the risk of morbidity and the course of a critical disease. Due to the complexity of the studied phenomenon, an introduction of a multistate model is necessary. The multistate model are used in health insurance issue (Haberman and Pitaco, 1999; Pitacco, 2014). We adopt the theory arising from multistate insurance contracts. Let us denote $S = \{1, 2, ..., N\}$ as a state space with T as a set of direct transitions between states of the state space, where (i, j) denotes a direct transition from state i to j $(i \neq j, i, j \in S)$. A multiple state model can be described by (*S*,*T*), which enables us to consider all possible insured risk events up to the end of the contract. The dynamics of the state changes is described by a certain stochastic process associated with the model. We investigate the Markov model based on the stationary Markov chain as discrete-time process $\{X(k), k \geq 0\}$, which describes the state for the contract at time $k = \{0, 1, ..., n\}$

The future life expectancy of spouses depends on many factors related to genetics, work environment, diet, health behaviour, addictions, living conditions and other socioeconomic factors. Some of them are different for spouses, such as working conditions (in case of separate work places) or the genetic predisposition to diseases. In Polish society two sociological phenomena are observed. Firstly, men are more likely to become addicted and, secondly, men have different health behaviour from women, which affects the differentiation of mortality and morbidity rates in male and female populations. In most populations of the world, the excessive mortality of men is observed. Numerous scientific studies confirm that biological sex and gender belong to the most important determinants of inequalities in health status (Laskowska, 2012; Królikowska, 2011). Therefore, the relationship between the life expectancy of husband and wife is not strong. Numerous studies on the interdependence of mutual spouses' survival times confirm this phenomenon (Denuit *et al.,* 2001; Dębicka *et al.*, 2020; Marciniuk, 2016). Thus, the impact of a potential dependence on the amount of insurance premium and benefits is also law.

Furthermore, due to the lack of epidemiological data on the incidence of serious illnesses by individual married couples and information on the course of illnesses in marriages, the future life expectancy of a husband and a wife is modelled separately. Two kinds of cash flows are considered in the model. The first kind of flows is connected with reverse annuity contract. We distinguish a single premium and annuity benefits. A single premium is paid at the beginning of the contract. In accordance with the assumption of the model, the annuity benefits are paid throughout the whole life. Because the Life Expectancy Table has a limit age ω, the benefits are paid for *n*- years, where $n = \max\left\{\omega_x^M, \omega_y^W\right\}$ for Last Surviving Status, where ω_x^M and ω_y^W denote the difference between the age limit ω of the woman's age at entry *x* and the man's age at entry *y*, respectively. Last Surviving Status means that the benefit is still paid even if one of the spouses dies. The value of premium is determined on the basis of the value of real estate *W*. In fact, it is calculated on the basis of the percentage of the value of real estate αW, where $\alpha = [0,1]$ and, in practice, $\alpha = (0; 0.5]$. The annuity benefits are determined on the basis of the pensioner's age, the value of real estate *W* and the further life expectancy.

The second kind of flows is connected with critical (or dread) disease insurance. We consider such an insurance contract which provides benefits paid not only in case of the diagnosis but also in case of deterioration of a patient's condition, and so two kinds of health condition are distinguished, mild and critical. For example, in case of cancer disease, mild state could mean a diagnosis without distant metastases, and critical state – a diagnosis with distant metastases to other organs. The financial resources are obtained at two moments of a disease's course. Firstly, a patient gets benefit at the moments of diagnosis and can spend it on treatment. The second moment is the deterioration of health condition to a critical state. It is important because patients in the terminal state often need palliative care funds as end-of-life costs are often very high in case of a dread illness.

The above cash flows can be considered by the introduction of the multi-state model, which takes into account: the life expectancy before suffering a critical disease, the risk of morbidity of an illness, and the life expectancy at a crucial stage. Due to the fact that further life expectancy in a critical condition depends not only on the age of the insured but also on the duration of the disease, modelling life expectancy requires extension of the Markov model. We introduce the following states:

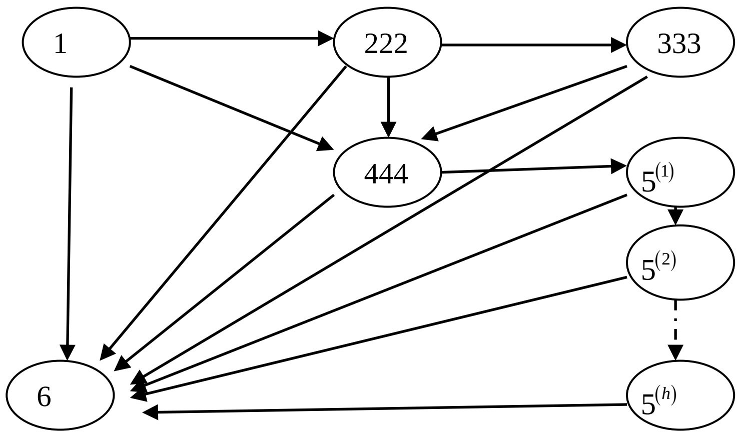
1. the insured is healthy (A – active),
2. the insured got ill in mild stage during last year (I+ – ill during last year),
3. the insured is ill in mild stage for at least one year (I – ill for at least one year),
4. the insured got ill in critical stage during last year (C+ – critically ill during last year),

5. the insured is ill in critical stage for at least one year (C – critically ill for at least one year),
6. the insured is dead (D – dead).

The fifth stage can be extended depending on the duration of a critical illness by the introduction of state $5^{(1)}$, $5^{(2)},\ldots,5^{(h)}$ where *h* denotes the number of years survived in the critical stage. The considered model is extended by one additional state which describes a situation that the insured fell ill in a mild state during a year. This extension is necessary because of the lack of memory of the Markov model and allows to model cash flows.

The multistate model is presented in Figure 1, where circles represent the states and arcs correspond to direct transitions between the states. Marriage contracts include two models estimated separately for a husband and a wife.

Figure 1.*The multistate model for one spouse*



Source: *Own elaboration.*

The probabilistic structure of the analysed model is described by the transition probabilities matrix $\mathbf{Q}(k)$. The elements of matrix - transition probabilities $q_{ij}^{[x]}(k) = P\left(X(x,k+1) = j \middle| X(x,k) = i\right)$ are obtained using methodology of a multi-state life table (increment-decrement table). The values of probabilities depend on some characteristics connected with the whole population, namely the probability of death, morbidity and mortality rates connected with dread diseases in a population. The population parameters are estimated on the basis of life expectancy tables and epidemiological reports related to the morbidity and mortality of critical diseases. The second kind of parameters depends on the characteristics connected with the population of patients who are suffering from critical diseases, namely the percentage of patients diagnosed in critical and mild stages, the probability of health deterioration and the fatality rates. Details connected with the estimation of the model presented in Figure 1 are described in the papers (Dębicka and Zmyślona 2016; 2019).

We obtain the following nonzero transition probabilities matrix (during the time interval [$x+k$, $x+k+1$))

$$\mathbf{Q}^{[x]}(k)=\begin{pmatrix} q_{11} & q_{12} & 0 & q_{14} & 0 & 0 & \cdots & 0 & q_{16} \\ 0 & 0 & q_{23} & q_{24} & 0 & 0 & \cdots & 0 & q_{26} \\ 0 & 0 & q_{33} & q_{34} & 0 & 0 & \cdots & 0 & q_{36} \\ 0 & 0 & 0 & 0 & q_{45^{(1}} & 0 & \cdots & 0 & q_{46} \\ 0 & 0 & 0 & 0 & 0 & q_{5^{(1)}5^{(2)}} & \cdots & 0 & q_{5^{(1)}6} \\ \vdots & \vdots & \ddots & & \ddots & & \cdots & \ddots & \\ \vdots & \vdots & & \ddots & & \ddots & \cdots & & \ddots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

The formulas for the elements of the transition probabilities matrix (1) take the following form presented in Table 1.

Table 1. *The formulas for estimators of the transition probability*

The number of state	Estimators of transition probability
1	$q_{11}=1-\left(q_{x+k}-\varphi_{x+k}\right)-\chi_{x+k}$ $q_{12}=\chi_{x+k}\left(1-\psi_{x+k}\right)$ $q_{16}=q_{x+k}-\varphi_{x+k}$
2 and 3	$q_{ij}=1-q_{x+k}-\xi_{x+k}$, for i=2,3 and j=3 $q_{ij}=\xi_{x+k}$, for i=2,3 and j=4 $q_{i6}=q_{x+k}$, for i=2,3
4	$q_{45^{(1)}}=1-d_{x+k}^{\left(4,5^{(1)}\right)}$ $q_{46}=d_{x+k}^{\left(4,5^{(1)}\right)}$
$5^{(1)},\dots,5^{(h)}$	$q_{5^{(i)}5^{(j)}}=1-d_{x+k}^{\left(5^{(i)},5^{(j)}\right)}$ for i=1,2,...h-1 and j=i+1 $q_{5^{(l)}6}=d_{x+k}^{\left(5^{(l)}6\right)}$ for l=1,2,...h.

Source: *Own elaboration on the basis of (Dębicka, Zmyślona 2016).*

The probabilities depend on the following indicators:

q_{x+k} – the probability of death in the whole population,

φ_{x+k} – the dread disease mortality rate in a population,

χ_{x+k} – the dread disease incidence rate (the morbidity rate) in a population,

ψ_{x+k} – the percentage of patients diagnosed in the critical stage,

ξ_{x+k} – the probability of health deterioration to the critical state,

$d_{x+k}^{(i,j)}$ – the fatality rate in the population of the critically ill.

The probabilistic structure underlying the calculation of benefits and premiums connected with the contract is determined on the basis of the probabilistic structure of the model for husband $\mathbf{Q}^X(k)$ and wife $\mathbf{Q}^Y(k)$ and is given as the Hadamard product:

$$\mathbf{Q}^{(X \circ Y)}(k) = \mathbf{Q}^X(k) \circ \mathbf{Q}^Y(k), \tag{2}$$

where an element of matrix **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** is defined as the following product $q_{ij}^{(X \circ Y)} = q_{ij}^X \cdot q_{ij}^Y$ for each i and j. The course of the process taking into account the survival time of both spouses requires the introduction of the matrix **D** describing the probability structure, given by the formula:

$$\mathbf{D} = \begin{pmatrix} \mathbf{P}^T(0) \\ \mathbf{P}^T(1) \\ \vdots \\ \mathbf{P}^T(n) \end{pmatrix} \in R^{(n+1)\times N \times N}, \tag{3}$$

where $\mathbf{P}(0) = (1, 0, 0, ..., 0)^T \in R^{N\times N}$ denotes the initial distribution vector, $\mathbf{P}^T(t) = \mathbf{P}^T(0)\prod_{k=0}^{t-1}\mathbf{Q}^{(X\circ Y)}(t)$ and $\mathbf{Q}^{(X\circ Y)}(t) = \left[q_{ij}^{(X\circ Y)}\right]_{i,j=1}^{N} \in R^{N\times N}$ is the Hadamard product **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** of transition probabilities matrixes for a wife and a husband described by (1).

3. Modelling Cash Flows

The new contract includes cash flows related to the reverse annuity contract and health insurance. The type of premiums and benefits is related to the stay of the process in a given state and creates an appropriate type of cash flow.

The cash flows related to the reverse annuity contract consist of a single premium and annuity benefits. The single premium is paid at the beginning of the contract:

$$\pi_j(k) = \begin{cases} \alpha W & \text{if } j = 1 \text{ and } k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The level of a single premium (2) depends on some percentage α the real estate W. The annuity benefit $\ddot{b}$ is paid when at least one of the spouses is alive. The amount of paid annuity will depend on the conditions of the health insurance. The periodic premium and a single benefit are cash flows related to the health insurance. The health insurance premium is paid independently by the spouses if they are healthy. Benefits are paid separately to spouses in two situations. First, when the spouse falls ill and, secondly, when his/her health worsens. Funds to cover the health premium pose a part of reverse annuity benefits. The amount of the reverse annuity is decomposed into the part paid to the spouses and the part intended for the payment of the premium.

Therefore, the reverse annuity parameter β is introduced. This parameter specifies the proportion of the annuity which is paid and the remaining part is spent on the health insurance premium. This premium is divided between a husband and a wife in proportion to the R parameter, which is determined on the basis of a population fraction of men, who fall ill with the disease that is the subject of insurance during a year. The decomposition is represented by the formula:

$$\ddot{b} = \beta \ddot{b} + R(1-\beta)\ddot{b} + (1-R)(1-\beta)\ddot{b} \ ,$$

where $\beta \in [0,1]$ is the reverse annuity parameter, $R \in [0,1]$ is a population fraction of men who fall ill with a critical disease.

The annual health insurance premium for a husband and a wife is given by the following formula

$$p_Y = R(1-\beta)\ddot{b} \tag{5}$$

$$p_X = (1-R)(1-\beta)\ddot{b}, \tag{6}$$

respectively.

Thus, the amount of the annuity payable in advance at the beginning of the year is given by:

$\beta \ddot{b}$ in case when both spouses are healthy, (7)

$\beta \ddot{b} - (1-R)(1-\beta)\ddot{b}$ when the wife is healthy and the husband is ill or dead, (8)

$\beta \ddot{b} - R(1-\beta)\ddot{b}$ in case when the husband is healthy and the wife is ill or dead, (9)

$\ddot{b}$ in case when both spouses are ill or one of them is ill and the other is dead or when at least one spouse is alive and they did not take out the health insurance. (10)

Health insurance benefits are payable from below at the end of the year in which a serious illness was diagnosed or when the patient's condition worsened and he is in a critical condition. The benefit c_X is paid only to the wife, when during a year she has fallen ill, or the wife's health deteriorated, and the husband is healthy, died or he is ill without deterioration. Similarly, the conditions for payment of the husband's benefit may be specified. The benefit c_Y is paid only to the husband when during a year he has fallen ill, or his health deteriorates while his wife is healthy, died or is ill. The benefits in the amount $c_X + c_Y$ are paid for both spouses when during a year they both have fallen ill, or their health deteriorates or if one of the spouses has fallen ill while the health state of the other is worsened.

Two matrices are distinguished according to the kind of cash flows. The first matrix $\mathbf{C}_{\text{in}}$ concerns benefits referred to reverse annuity and health insurance. The second matrix $\mathbf{C}_{\text{out}}$ refers to cash flows related to premiums connected with reverse annuity and health insurance. In the analyses, we introduce a discounting factor related to the interest rate:

$$\mathbf{M}^T = \left(1, v, v^2, ..., v^n\right) \in R^{n+1} .$$

The formulas are obtained assuming the principle of equivalence, by the use of the following equation:

$$\mathbf{M}^T \mathbf{Diag}\left(\mathbf{C}_{\text{in}} \mathbf{D}^T\right)\mathbf{S} = \mathbf{M}^T \mathbf{Diag}\left(\mathbf{C}_{\text{out}} \mathbf{D}^T\right)\mathbf{S} , \tag{11}$$

where $\mathbf{D}$ is the matrix given by **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.**, $\mathbf{S} = \left(1,1,....,1\right) \in R^{N \times N}$. Assuming **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.**, premium and benefit formulas take the following form:

$$\ddot{b} = \frac{\alpha W}{\mathbf{M}^T \left(\mathbf{I} - \mathbf{I}_{n+1}\mathbf{I}_{n+1}^T - \mathbf{I}_1\mathbf{I}_1^T\right)\mathbf{D}\left(\mathbf{S} - \mathbf{J}_4\right) + 1} , \tag{12}$$

$$c = \frac{\mathbf{M}^T \mathbf{Diag}\left(\mathbf{C}_{\mathrm{out}} \mathbf{D}^T\right)\mathbf{S}}{\mathbf{M}^T \left(\mathbf{I} - \mathbf{I}_1 \mathbf{I}_1^T\right)\mathbf{D}\left(\mathbf{J}_2 + \mathbf{J}_4\right)}, \tag{3}$$

where $\mathbf{J}_i = \left(0, 0, \ldots, \underset{i}{1}, 0, \ldots, 0\right)^T \in R^{N \times N}$ and $\mathbf{I}_t = \left(0, 0, \ldots, \underset{t+1}{1}, 0, \ldots, 0\right)^T \in R^{n+1}$ are auxiliary vectors and **I** is an identity matrix with n+1 rows and columns. The values of health insurance benefits p_X and p_Y are obtained using formulas **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** and **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.**, on the basis of calculated annuity $\ddot{b}$.
Lastly, we check if the value of a single net premium expressed by the formula:

$$\pi = \mathbf{M}^T \mathbf{Diag}\left(\mathbf{C}_{\mathrm{in}} \mathbf{D}^T\right)\mathbf{S} \tag{4}$$

is equal to an established percentage of the value of the property (αW). The formulas **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** and (3) are obtained using the methodology described in the paper (Dębicka, 2013).

4. Empirical Evidence

Financial needs of retirees are determined by their family, material and health situation. The risk of developing a dread or chronic illness increases with age. The state of health of a pensioner has a huge impact on the level of financial resources, which is noticeable in many countries, including Poland. In the event of a significant deterioration in health, the risk of falling into poverty is significant, especially for women in case of the death of their spouse (Bukowska *at al.*, 2011; McGarry and Schoeni, 2005; Timoszuk, 2017). Older people often do not have enough funds to take out insurance against the risk of serious illness policy. The contract presented in the paper may be an alternative that allows increasing pensioner’s income and obtaining additional funds for treatment and palliative care in case of illness. The level of benefits and pensions payment can be flexibly shaped depending on the needs of a given family of pensioners.

Empirical examples presented in the paper are based on the data concerning the risk of lung cancer mortality and morbidity. Lung cancer belongs to one of the most common cancers in developed countries and is also characterized by high mortality. Transition probabilities matrixes in the analysed model are estimated on the basis of life expectancy tables presented in the paper (Dębicka and Zmyślona, 2016). Empirical data confirms that the maximum survival time in a critical condition (diagnosed with distant metastases or with an inoperable life-threatening tumour) was 4 years (Dębicka and Zmyślona, 2019).

The discounting factor v is closely related to the examined period of the disease and a fixed long-term interest rate i = 5.79 %, because of $v=(1+i)^{-1}$. The interest rate i was estimated on the basis of real Polish market data related to the yield to maturity on fixed interest bonds and Treasury bills from 2008 in the Nelson-Siegel model. A population fraction of men, who fall ill with lung cancer in 2008, R equals 69.1%. The calculations are carried out assuming that the real value of the property is 100,000 euros, which makes it possible to easily rescale the results obtained in case of different property prices.
In the paper, the impact of the beta parameter on the amount of annuity and health insurance benefits is analysed. We consider the different values of the parameter β, which specifies the proportion of the paid annuity. Let β be from 1 to 0 with an accuracy of 0.1, namely $\{1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0\}$. When $\beta=1$, it means that only the marriage reverse annuity is paid. When $\beta=0$, all the benefit is spent on the health insurance premium. When $\beta\in(0,1)$ it involves the combination of the both reverse annuity contract and the dread disease insurance.

Firstly, we analysed the cash flows by an assumption that $\beta=0.9$, which means that 10% of the annuity is intended for paying the premium. Due to the fact that primarily the influence of β on the amount of benefits is being considered, we assumed in this paper that the spouses are at the same age. We take into account the couple aged 65, 70, 75, 80 and 85. The results are presented in Table 2.

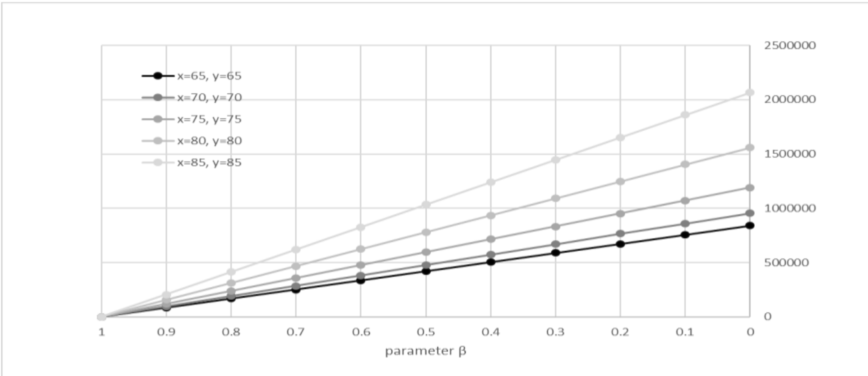
Table 2. *Yearly benefits and premiums for different ages of spouses and* $\beta=0.9$

$x=y$	$\beta\ddot{b}$	c_X	c_Y	p_X	p_Y
65	3677	84178	49458	126	282
70	4166	95584	54131	143	320
75	4912	119034	68035	169	377
80	6046	155928	104101	208	464
85	7751	206669	160762	266	595

Source: *Own elaboration.*

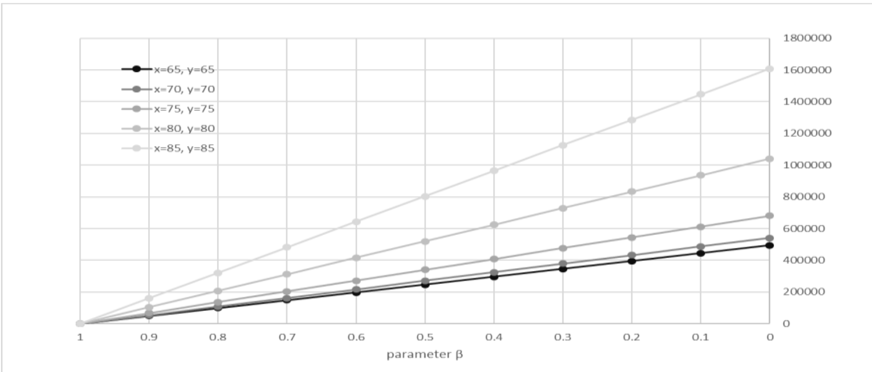
The increase of all benefits with the rise of spouses' age is observed. The dread disease insurance benefit for the wife is much higher than the same benefit for the husband even though the woman pays lower premiums. This phenomenon is caused by the fact that population lung cancer morbidity rates for women are lower than for men. The spouses can receive a yearly marriage reverse annuity, and they can afford to buy the dread disease insurance. On the one hand, annual premium for this insurance is very low, which means that it does not significantly reduce the annuity income. On the other hand, health insurance benefits, in case spouses get sick, are very high, which allows to get significant financial resources, which could be spent on treatment and improving of the quality of life during the disease course. The impact of parameter β on the amount of dread diseases benefits is also noticeable. In Figure 2 and Figure 3, the dread diseases benefits for the wife and the husband, respectively, depending on β are presented. The benefits increase when the parameter β decreases and are clearly higher for women in all age groups. Taking into account the fact that dread and chronic diseases pose the main risk factor of falling into poverty for pensioners, this solution can be very beneficial for pensioners already near $\beta = 0.2$. At this level of the parameter, the obtained health insurance benefits allow improving the quality of life during an illness in all age groups.

Figure 2. *The dread disease benefits for women depending on parameter β*



Source: *Own elaboration.*

Figure 3. *The dread disease benefits for men depending on parameter β*



Source: *Own elaboration.*

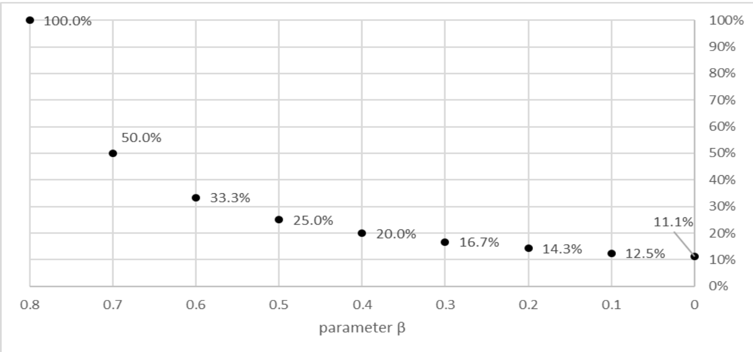
Paying a higher health insurance premium (when β decreases) means receiving higher dread diseases benefits. The growing age of the spouses' causes the increase in the differences between benefits for the same value of parameter β. For example, the benefit for the husband when the wife and the husband are 70-years old (in short written as $x = y = 70$) and $\beta = 0$ is 9.5% higher than the benefit for the husband when $x = y = 65$ and $\beta = 0$. However, when $x = y = 85$ and $\beta = 0$, the benefit for the husband is 54.3% higher than the benefit for the husband, when $x = y = 80, \beta = 0$ and over 224% higher in case of $x = y = 65, \beta = 0$.

The relative increases between benefits (related to annuity and the dread disease insurance) for β and $\beta - 0{,}1$ are calculated as the following rate:

$$relative\ increase = \frac{benefit\ for\ (\beta - 0.1) - benefit\ for\ (\beta)}{benefit\ for\ (\beta)}. \tag{5}$$

The values of (5) are presented in Figure 4. The differences between the consecutive value of the benefit for β are the same for each type of benefit related to the dread disease insurance. If β is smaller, the benefits increase, but the differences between them are getting smaller.

Figure 4. *The relative increase in cash flows connected with annuity and dread disease benefits depending on* β



Source: *Own elaboration.*

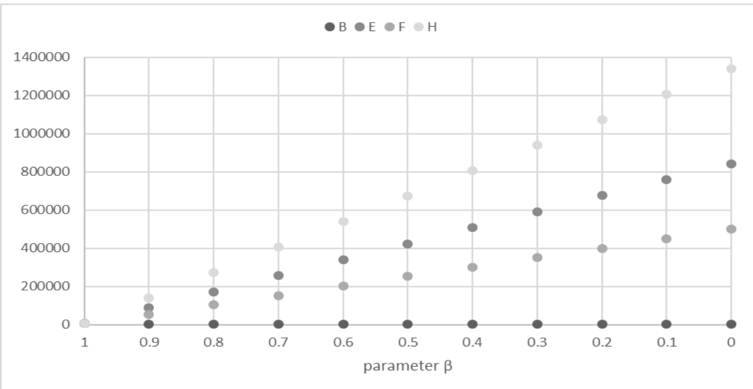
Spouses receive annuity and dread disease insurance benefits. The amount of the benefits depends on life duration and health situation of both spouses. The following benefits can be considered for the duration of the contract:

- $A = \beta \ddot{b}$ (both of spouses are healthy),
- $B = \ddot{b} - R(1-\beta)\ddot{b}$ (husband is healthy, wife is sick or dead),
- $C = \ddot{b} - (1-R)(1-\beta)\ddot{b}$ (husband is sick or dead, wife is healthy),
- $D = \ddot{b} - (1-R)(1-\beta)\ddot{b} + c_Y$ (husband has got sick or his health worsened, wife is healthy),
- $E = \ddot{b} - R(1-\beta)\ddot{b} + c_X$ (husband is healthy, wife has got sick or her health worsened),

- $F = \ddot{b} + c_Y$ (husband has got sick or his health worsened, wife is sick or dead),
- $G = \ddot{b} + c_X$ (husband is sick or dead, wife has got sick or her health worsened),
- $H = \ddot{b} + c_X + c_Y$ (spouses have got sick or his/her health worsened).

These benefits are presented in Figure 5 when spouses are at the same age of 65. The differences between the amounts of certain benefits are not large enough to be visible, therefore some benefits have not been marked off on the chart (benefits B and C - when one of the spouses is healthy and the other is ill or died (only benefit B is visible); benefits D and F- if the husband got sick or his health worsened and the wife is healthy, sick or died (only benefit F is visible); benefits E and G - when the wife got sick or her health worsened and the husband is healthy, sick or died (only benefit E is visible)). The benefits increase alongside the decrease of . The benefits paid to the wife in case of deterioration of her health (diagnosis of a serious illness or critical condition) are definitely higher than for her husband (compare E and F). It is obvious in the considered example because lung cancer mortality rate in men's population is higher and, moreover, women live longer. This difference increases as the value of parameter decreases. This situation is very favourable because women are more likely to fall into poverty, especially when they become widows and their health deteriorates.

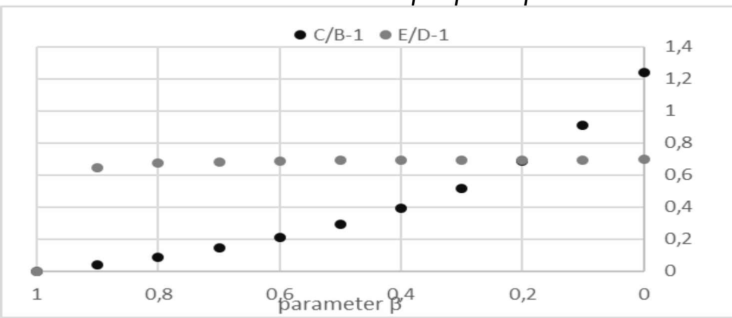
Figure 5. *Benefits for spouses aged 65, depending on parameter β*



Source: *Own elaboration.*

Furthermore, the benefits paid under the contract were compared, depending on the health status of the husband and the wife. The amount of benefits E (in case of health benefits payment for the wife and when the husband is healthy) is about 65-70% higher than in the opposite situation for benefit D (the husband has health benefits paid and the wife is healthy), regardless of parameter beta. The situation is identical when we compare benefits G with benefits F (one of the spouses has health benefits paid and the other is sick or died). Also, benefits are about 65-70% higher when the wife's health benefit is paid. If only an annuity (benefits C and B) is paid, we can observe that the benefits paid when the wife is healthy and the husband is ill or died (benefits C) are higher than the analogous benefits paid when the husband is healthy and the wife is ill or deceased (benefits B). The difference between the amount of benefits increases as the beta parameter decreases (from 4 to 124%). In Figure 6 the relative increases between benefits for wife and husband are presented. The analysis and comparison of the benefits show that the contract protects women's financial situation better when they become widows. Widows have a much worse financial situation than widowers, and therefore the contract provides financial protection particularly for women.

Figure 6. *The relative increases between benefits for wife and husband*



Source: *Own elaboration.*

5. Conclusions

The combination of the two marriage contracts: reverse annuity contract and dread disease insurance poses a new proposition of protection against the effects of longevity. This contract is not offered on the market, but it can be an additional financial resource for pensioners when they are healthy or sick. This contract is very flexible. The spouses have a possibility to establish the level of benefit of a reverse annuity contract which is allocated to the premium of critical illness insurance. Although the benefits obtained from the reverse annuity contract are a little bit lower than when the whole capital αW is used to calculate the reverse annuity benefit, the illness benefits are considerably high. This provides the owner of a real estate with an additional financial protection in case of a dread disease, especially for women. Most importantly, they can stay in the same place where they spent their whole lives and are not forced to move out.

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