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## Fuzzy Technologies Modeling the Level of Welfare of the Population in the System of Effective Management

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### Abstract:

**Purpose:** The article aims to present the use of fuzzy technologies to model the welfare of the population in the system of effective management of the country's economy. In particular, the aim was to develop an approach to assessing the welfare of the country's population based on the use of fuzzy logic and fuzzy set theory.

**Design/Methodology/Approach:** The paper deals with the procedure of fuzzy assessment of the welfare level of the country's population. The expert approach to forming a system of six partial indicators, the value of which is taken into account when calculating the level of welfare of the country's population, is used. After selecting the set of primary indicators, their value is reduced to a fuzzy form. We next find the magnitude of the original integral index of this level in a fuzzy form. Finally, we define the precise value of this integral index, which corresponds to its fuzzy form.

**Findings:** The results obtained indicate the effectiveness of the socio-economic policy of the state during the study period, which resulted in a significant improvement in the welfare of the population of Ukraine despite social and economic instability.

**Practical Implications:** The results obtained indicate the effectiveness of the socio-economic policy of the state during the study period, which resulted in a significant improvement in the welfare of the population of Ukraine despite social and economic instability.

**Originality/Value:** The assessment of the level of welfare of the population, which is based on the use of the theory of fuzzy logic and fuzzy sets and the possibility of taking into account in the calculations of these quantitative and qualitative primary factors that shape this welfare.

**Keywords:** Welfare, fuzzy set theory, linguistic variable, membership function.

**JEL classification:** I31, I32, B23, O12, O15

**Paper Type:** Research paper.

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## **1. Introduction**

Managing any system will be effective if it uses the feedback principle, that is, it considers the reaction of that system to the previous regulatory impact on it. When making management decisions, the decision-maker (DM) needs to know how the management object changes under specific regulatory actions. To do this, you need to determine the status of this object, model it, and evaluate the factors that characterize this state. The above also applies to the economy of any country as a system of governance.

In recent years, the economic development of Ukraine has been accompanied by several negative trends that hinder its growth. The annexation of Crimea and the Russian hybrid war against Ukraine, crises, and other political and economic disturbances have led to a decline in production, unemployment, declining incomes of Ukrainians, an increase in property differentiation of society, and, as a consequence, an increase in inter-territorial displacement, the large-scale spread of poverty and reducing the level of welfare of Ukrainian citizens.

In today's economic environment, the level of welfare of the population is an objective indicator of the assessment of transformation processes in the country's economy, the successful implementation of socio-economic programs for its development, the effectiveness of the public policy, and the progressive development of the country as a whole. The direction and pace of further transformation in the country depend on solving the problem of raising the level of welfare of the population since it ultimately determines the political and economic stability of society as a whole. Therefore, to determine how effective governmental actions are to improve the economic situation in a country, one can use the results of an analysis of the dynamics of the welfare of its population, which indicates the need to develop methods for assessing this level for an arbitrary region or country as a whole.

Given that to assess the welfare of a population in a country, it is necessary to analyze certain groups of factors that characterize it. One can use the classical methods of multidimensional statistics to solve this problem and construct a generalized indicator for these factors. However, these methods have limitations, the primary factors used for calculations should be quantitative. At the same time, the welfare of the population is characterized by many qualitative components, such as working and leisure conditions, the amount and structure of working and leisure time, indicators of the cultural and educational level of the population, health, demographic and environmental situation, etc. Therefore, in order to obtain a more accurate result, it is advisable to use the latest mathematical methods and models for solving well-structured or completely unstructured problems of economic analysis, in particular, the theory of fuzzy logic and fuzzy sets (Zadeh, 1965; Kozlovskiy *et al.*, 2018), which have been successfully tested on managing similar economic problems (Kozlovskiy *et al.*, 2018; 2020). The welfare of the population has been in the view of scientists for a long time. Many scientific works have been devoted to the study of welfare

theories. The welfare of the population has been the subject of research by such authors as R. Adams, R. Barro, A. Bergson, A. Bergsten, K. Arrow, W. Nordhouse, V. Pareto, A. Pigou, M. Ravallion, J. Rawls, P. Samuelson, A. Sen, A. Smith, P. Townsend, J. Tobin, and R. Hicks. Their works reflected methodological aspects of the study of the essence of welfare, research of its components, and measurement of the population's standard of living.

In particular, the subjectivity of the concept of welfare of the population prompted A. Pigou to introduce into the scientific use of economic welfare, which these scientists were offered to measure by the value of Gross Domestic Product (GDP) per capita (Pigou, 1985). The failure to take into account in this approach the degree of inequality in household income, the cost of leisure, as well as products created by the shadow economy, etc., led to William Nordhaus and James Tobin developing the Measure of Economic Welfare (MEW), MEW which Paul Samuelson renamed Net Economic Welfare (Samuelson and Nordhaus, 1995). Some other scientists have proposed modifications of this concept and approaches to their calculation (Stiglitz, Sen, and Fitoussi, 2009; Ignatyuk, 2010; Afssa and Blanchet, 2009). An overview of the literature on this issue is given in (Pryimak and Holubnyk, 2012). However, a unified approach to calculating the level of welfare of the region's population or the country as a whole, which would make it possible to take into account both quantitative and qualitative components of this indicator in scientific publications, was not proposed.

The purpose of this article is to develop an approach to assessing the welfare of the population of the country, based on the use of fuzzy logic and fuzzy set theory and the ability to take into account quantitative and qualitative primary factors that shape this welfare.

## **2. Research Methodology**

The methodological basis of the study is the economic and mathematical apparatus of the theory of fuzzy sets (Rotshtein, 1999; Panoshichen and Kozachko, 2010; Rotshtein and Shtovba, 2009). The development of an effective and rational policy governing the economic system is impossible without information about the current state of this system and data on the dynamics of its development. The state of the country's economy correlates quite well with the level of welfare of its population. Therefore, consider the approaches to modeling and assessing the level of welfare of the country's population. Moreover, the conclusions from the calculations could be used to make management actions to improve the economic situation in this country.

The GDP per capita indicator could be used to assess the level of economic development and welfare of the country's population. Preferably, in a country where this value is higher, the level of income, leisure, health, education, life expectancy, etc., is higher. However, its use as a measure of welfare has significant disadvantages. In particular, it does not consider the differentiation of income of the population, the cost of leisure, the products of the shadow economy, performed in the country non-

market operations, and some other factors. Instead, it includes the magnitudes of some factors that are not related to the population's welfare and thus exceeds the magnitude of the latter. Among such factors are environmental measures taken by the state to improve the environmental situation in its territory.

It is impossible to fully reflect the value of the level of welfare of the population of the country by any other indicator. Therefore, to measure this level, it is necessary to use derivative measurements, that is, to use the values of several primary factors. Moreover, among these factors will be not only quantitative but also qualitative. Suppose we limit ourselves only to primary quantitative factors. In that case, we can use some algorithm of convolution of these factors to solve this problem and construct a complex (integral, generalized) indicator, which would correspond to the level of welfare of the population of the country. According to the chosen algorithm, performing the calculations will not be difficult since several such algorithms have been described and tested in the literature (Pryimak, 2009). It should be noted that many Polish scientists have been involved in the development of the theory of multidimensional statistics in the construction of generalized indicators.

However, using only quantitative factors to assess the welfare of the country's population does not give a complete picture of this level. To solve this problem, we need to consider the qualitative factors, which can be obtained based on expert surveys. This complicates solving this problem and makes it impossible to use traditional statistical methods and models. It is necessary to use theoretical approaches that allow taking into account in the calculations the data obtained from experts, which may be incomplete and inaccurate, that is, operate with the uncertainty that cannot be disclosed accurately and unambiguously. Such theories have been developed recently by scientists. They were called the theories of fuzzy logic and fuzzy sets.

This theory was launched relatively recently in order to be able to formalize quality information. The well-known American mathematician Lotfi A. Zadeh proposed and developed its main provisions in the 1960s. A clear (classical) set theory uses Archimedes' law of absence of a third, according to which a particular element belongs or does not belong to a given set. Instead, in fuzzy set theory, this element may belong to some set not only complete but also some part, such as a quarter or 40%. To indicate the force of belonging of a given element to a particular fuzzy set, use the corresponding number from the interval  $[0, 1]$ , called a function or measure of membership (Siavavko and Rybytska, 2000). Its value can be obtained from expert surveys.

In addition to the above, this theory uses such concepts as linguistic variables and fuzzy sets. Linguistic is called a variable whose value is determined verbally by the verbal characteristics of a property. For example, the unemployment rate may be low, medium, high, critical, etc. These are the so-called values of the term-set linguistic variable that corresponds to this indicator. About the fuzzy set  $B$ , it is defined as the set of pairs of the form (Siavavko and Rybytska, 2000):

$$B = \{(x, \mu_B(x)), x \in X\} \quad (1)$$

Where:  $X$  is the universal set (base scale);

$\mu_B(x)$  is a membership function of the set  $B$  in the universal set  $X$ , which determines the subjective measure of the expert's confidence that a given specific value of the base scale corresponds to a fuzzy set.

The membership function can be either discrete or continuous. Most often, among the continuous membership functions, the membership functions are triangular, trapezoidal and bell-shaped. If the universal set coincides with the set of real numbers, then the corresponding fuzzy set is called a fuzzy number.

To model the level of welfare of the population of the country, we use fuzzy trapezoidal numbers. If the fuzzy number  $\beta$  has a trapezoidal shape, then it can be formally represented by four numbers

$$\beta = (b_1, b_2, b_3, b_4) \quad (2)$$

where  $b_1, b_4, b_2, b_3$  abscissa of lower  $AD$  and upper  $BC$  bases of trapezoid with coordinates  $A(b_1, 0)$ ,  $B(b_2, 1)$ ,  $C(b_3, 1)$ ,  $D(b_4, 0)$  in the Cartesian coordinate system  $(X, \mu)$ .

The procedure for fuzzy assessment of the welfare level of the country's population is as follows. It can be described as follows. After selecting the set of primary indicators, their value is reduced to a fuzzy form; that is, we determine the linguistic estimates of these variables and formalize the function of the variables necessary for their formalization (operate fuzzification of variables). We next find the magnitude of the original integral index of this level in the fuzzy form. Finally, we calculate the precise value of this integral index, which corresponds to its fuzzy form (we perform a defuzzification operation that converts the fuzzy information into an explicit form). To perform these actions, we will use the simplification of this procedure, which A.O. Nedosiekin proposed, to analyze the risk of stock investments (Nedosiekin, 2002).

Let us now dwell on the substantive formulation of the considered problem and the algorithm of fuzzy modeling of the level of welfare of the population of the country. Let this welfare be characterized by a set of  $N$  primary factors  $X_1, X_2, \dots, X_N$ . Among them are both quantitative taken from the statistical yearbooks and qualitative received from experts. Suppose that the metric  $\{X\}$  is sufficient for the accuracy of the analysis. These factors for the study period (year) are respectively  $x_1, x_2, \dots, x_N$ .

Then the desired generalized indicator of the welfare of the population of the DN country depends in some way on these factors  $X_1, X_2, \dots, X_N$ :

$$DN = f(X_1, X_2, \dots, X_N) \quad (3)$$

Our task is to find the type of this function. Moreover, the welfare of the country's population is better for the year for which the value of  $DN$  is greater.

We will assume that the welfare of the population of the country has five states: «bad», «satisfactory», «average», «good», «very good». We assign to each of these states a fuzzy subset  $A_j (j = \overline{1, 5})$  (state: «bad» ( $j = 1$ ), «satisfactory» ( $j = 2$ ), «average» ( $j = 3$ ), «good» ( $j = 4$ ), «very good» ( $j = 5$ )). That is, the term set of the linguistic variable «Welfare of the country's population» will be composed of five components. We construct a normalized generalized DN, that is, a value that can take values from zero to one. Corresponding to the fuzzy subsets  $A_1, A_2, A_3, A_4, A_5$  of the population welfare states of the membership functions  $\mu_1(DN), \mu_2(DN), \mu_3(DN), \mu_4(DN), \mu_5(DN)$ , we define the trapezoidal number of the form (2):

$$\begin{aligned} \mu_1(DN) = \beta_1 = (0,0; 0,0; 0,15; 0,25); \quad \mu_2(DN) = \beta_2 = (0,15; 0,25; 0,35; 0,45); \\ \mu_3(DN) = \beta_3 = (0,35; 0,45; 0,55; 0,65); \quad \mu_4(DN) = \beta_4 = \\ (0,55; 0,65; 0,75; 0,85); \\ \mu_5(DN) = \beta_5 = (0,75; 0,85; 1,0; 1,0). \end{aligned} \quad (4)$$

To reduce the computations, we illustrate the sequence of the following actions with simultaneous calculations of the magnitude of the required generic indicator for specific data.

### 3. Results and Discussion

The first step in the sequence of actions to determine the level of welfare of the country's population is the selection of partial indicators, the value of which is taken into account in its calculation. Using the experts' opinion and the results of the analytical calculations, we selected six indicators ( $N=6$ ) to evaluate this performance: disposable income per person ( $X_1$ ), the share of the population with average per capita total income per month below the statutory subsistence level ( $X_2$ ), average monthly pension allowance ( $X_3$ ), infant mortality rate under one year of age (deaths of children under the age of one in 1,000 live births) ( $X_4$ ), number of people enrolled in institutions secondary education per 10 thousand population ( $X_5$ ), emissions of pollutants and carbon dioxide into the atmosphere by stationary sources of pollution per thousand population ( $X_6$ ).

They are all calculated for the year or at the end of the year. The first, third, and fifth indicators are stimulants, and all others are de stimulants. The disposable income and the average amount of the assigned monthly pension are given in UAH, taking into account inflation. The second indicator describes the poverty level of the population. All indicators considered are relative. The magnitudes of all factors were obtained from official data published by the State Statistics Service of Ukraine, particularly in

the Statistical Yearbook of Ukraine for the relevant years (see, for example, Statistical Yearbook of Ukraine for 2018: Statistical collection. State Statistics Service of Ukraine 2019). All data for the calculations are taken for the years 2006-2018.

**Table 1.** Value of primary indicators for determining the level of welfare of the population of Ukraine for 2006-2018

Year	Indicator					
	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$
2006	7771	21,4	407	9,8	1098	102,82
2007	8253	12,7	390	11	1047	103,29
2008	8693	7,1	492	10	1001	98,15
2009	8061	5,8	524	9,4	978	85,39
2010	9118	8,8	509	9,1	939	90,21
2011	9346	7,8	498	9,0	941	95,93
2012	10100	9,1	502	8,4	927	95,07
2013	10264	8,4	565	8,0	923	94,61
2014	8877	8,6	506	7,8	874	78,09
2015	7408	6,4	377	7,9	885	66,76
2016	7556	3,8	346	7,4	903	72,26
2017	7889	2,4	305	7,6	925	60,97
2018	8375	1,3	359	7,0	959	59,44

**Source:** Own creation.

It should be noted that there were suggestions from the experts to expand the base of primary indicators, for example, at the expense of the indicator «average monthly salary of full-time employees» and others. However, a preliminary analysis of the correlation between the primary factors revealed multicollinearity for some of them. In addition, the calculation of the statistical characteristics of the variation of some of these factors showed the minor importance of this criterion and the inappropriateness of their use in further calculations.

Using these primary indicators in the process of solving this problem creates another problem. These indicators may not be equivocal for assessing welfare. Each of them may have some priority. Therefore, if necessary, each primary indicator can be matched by a specific assessment of its importance (priority), which can be determined by experts or otherwise. However, to simplify the calculations, we assume that the indicators we use are equivalent and do not take their priority.

The next step in the calculations is to determine for each primary indicator  $X_i$  ( $i=(1,6)$ ) the linguistic variable «Level of Performance  $X_i$ » fuzzy subsets of the area of the definition of that indicator  $[D(X)]_i$ , which is an innumerable set of points of the axis of real numbers, and their corresponding membership functions.

We assume that all of these indicators have the same term sets. That is, the linguistic variable «Level of indicator  $X_i$ » is defined equally for each of the primary indicators by five fuzzy subsets  $B_j$  ( $j = \overline{1,5}$ ) of the set  $D(X_i)$ , which in the general case intersect. Let the fuzzy subsets  $B_1, B_2, B_3, B_4, B_5$  of the states of the indicator  $X_i$ ,

respectively, mean «very low», «low», «medium», «high» and «very high». Then for each metric  $X_i$  ( $i = \overline{1,6}$ ), the corresponding membership functions  $\theta_{ij}$  ( $i = \overline{1,6}, j = \overline{1,5}$ ) were constructed. Moreover, with the involvement of experts, it was necessary to describe corresponding to each of these indicators, that is, trapezoidal function functions (Table 2). And at once it was taken into account that the second fourth and sixth of these primary indicators are destimulants.

**Table 2.** Classification of primary indicators

Indicator	Trapezoidal numbers for the values of the linguistic variable «Value of the indicator $X_i$ »				
	«very bad»	«bad»	«medium»	«good»	«very good»
$X_1$	(0; 0; 6500; 7000)	(6500; 7000; 7500; 8000)	(7500; 8000; 8500; 9000)	(8500; 9000; 9500; 10000)	(9500; 10000; $\infty$ ; $\infty$ )
$X_2$	(11; 12; 100; 100)	(8; 10; 11; 12)	(5; 7; 8; 10)	(2; 4; 5; 7)	(0; 0; 2; 4)
$X_3$	(0; 0; 300; 350)	(300; 350; 400; 450)	(400; 450; 500; 550)	(500; 550; 600; 650)	(600; 650; $\infty$ ; $\infty$ )
$X_4$	(11; 12; 1000; 1000)	(9; 10; 11; 12)	(7; 8; 9; 10)	(1; 2; 7; 8)	(0; 0; 1; 2)
$X_5$	(0; 0; 700; 750)	(700; 750; 900; 950)	(900; 950; 1000; 1050)	(1000; 1050; 1100; 1150)	(1100; 1150; 2000; 2000)
$X_6$	(90; 100; 150; 150)	(70; 80; 90; 100)	(50; 60; 70; 80)	(30; 40; 50; 60)	(0; 0; 30; 40)

Source: Own creation.

Now, for each of the primary indices considered  $X_i$  ( $i = \overline{1,6}$ ) and each fuzzy subset  $B_j$  ( $j = \overline{1,5}$ ), let us briefly consider the algorithms for calculating the actual values of their membership functions  $\theta_j(x_i) = \theta_{ij}$  ( $i = \overline{1,6}, j = \overline{1,5}$ ). Moreover, we denote the value of the  $i$ -th primary index  $X_i$  ( $i = \overline{1,6}$ ) in these representations by  $S_i$  ( $i = \overline{1,6}$ ), and we use the table to calculate the classification groups of these quantities and their membership functions.

**Table 3.** Classification of value Disposable income per person

Indicator	Value range	Indicator Value Group	Degree of Confidence (membership function)
Disposable income per person (indicator $X_1$ , magnitude $- S_1$ )	$0 \leq S_1 \leq 6500$	«very low»	$\theta_1 = 1$
	$6500 < S_1 < 7000$	«very low»	$\theta_1 = (7000 - S_1)/500$
	$6500 \leq S_1 < 7000$	«low»	$\theta_2 = 1 - \theta_1$
	$7000 \leq S_1 \leq 7500$	«low»	$\theta_2 = 1$
	$7500 < S_1 < 8000$	«low»	$\theta_2 = (8000 - S_1)/500$
	$7500 \leq S_1 < 8000$	«medium»	$\theta_3 = 1 - \theta_2$
	$8000 \leq S_1 \leq 8500$	«medium»	$\theta_3 = 1$
	$8500 < S_1 < 9000$	«medium»	$\theta_3 = (9000 - S_1)/500$
	$8500 \leq S_1 < 9000$	«high»	$\theta_4 = 1 - \theta_3$
	$9000 \leq S_1 \leq 9500$	«high»	$\theta_4 = 1$
	$9500 < S_1 < 10000$	«high»	$\theta_4 = (10000 - S_1)/500$
	$9500 \leq S_1 < 10000$	«very high»	$\theta_5 = 1 - \theta_4$
	$10000 \leq S_1 \leq \infty$	«very high»	$\theta_5 = 1$

Source: Own creation.



The first of these metrics is disposable income per person  $X_1$ . The scope of this metric  $D(X_1) = (0, \infty)$ . The method of level classification  $X_1$  performed by DM, ie the algorithm for calculating the classification group of each value of this indicator and its membership function, is given in Table 3. The «Interval of values» column of this table shows intervals, the ends of which are abscesses of trapezoidal fuzzy intervals  $\beta_i = (b_{1j}, b_{2j}, b_{3j}, b_{4j})$ . On the upper trapezoid basis, the corresponding  $\theta$ , which corresponds to a given interval of values in which the value  $S_1$ , equals 1, and on the sides of adjacent trapezoids both corresponding  $\theta$  are calculated, with their sum also equal to one.

If the first indicator we considered was a stimulant, then the second – «share of the population with average per capita total income per month below the statutory subsistence level» is a destimulant. Its value is expressed as a percentage, so its area of definition  $[D(X)]_{-2}=(0,100)$ . The algorithm for calculating classification groups and their membership functions for the values of  $S_2$  is given in Table. 4.

**Table 4.** Classification value share of the population with average per capita total income per month below the statutory subsistence level

Indicator	Value range	Indicator Value Group	Degree of Confidence (membership function)
Share of population with average per capita total income per month below the statutory subsistence level (indicator – $X_2$ , magnitude – $S_2$ )	$13 \leq S_2 \leq 100$	«very low»	$\theta_1 = 1$
	$11 < S_2 < 13$	«very low»	$\theta_1 = (S_2 - 11)/2$
	$11 < S_2 < 13$	«low»	$\theta_2 = 1 - \theta_1$
	$10 \leq S_2 \leq 11$	«low»	$\theta_2 = 1$
	$8 < S_2 < 10$	«low»	$\theta_2 = (S_2 - 8)/2$
	$8 < S_2 < 10$	«medium»	$\theta_3 = 1 - \theta_2$
	$7 \leq S_2 \leq 8$	«medium»	$\theta_3 = 1$
	$5 < S_2 < 7$	«medium»	$\theta_3 = (S_2 - 5)/5$
	$5 < S_2 < 7$	«high»	$\theta_4 = 1 - \theta_3$
	$4 \leq S_2 \leq 5$	«high»	$\theta_4 = 1$
	$2 < S_2 < 4$	«high»	$\theta_4 = (S_2 - 2)/2$
	$2 < S_2 < 4$	«very high»	$\theta_5 = 1 - \theta_4$
$0 \leq S_2 \leq 2$	«very high»	$\theta_5 = 1$	

Source: Own creation.

Taking into account the data in Table 2, similar tables (algorithms) were constructed for all other primary indicators. Given that  $X_3$  and  $X_5$  are stimulants and  $X_4$  and  $X_6$  are stimulants, respectively, for the first two of these indicators, these tables are similar to Table 1, and for the other two primary indicators similar Tables 4.

In the next step of the algorithm of estimating the level of welfare of the population of the country on the basis of the values of the corresponding membership functions  $\theta_{ij} (i = \overline{1,6}, j = \overline{1,5})$  defined for each indicator  $X_i (i = \overline{1,6})$  a fuzzy DN must be calculated. These calculations should be performed using known information on the values of all six indicators for each of the years for which the analysis is performed (see Table 1) and the algorithms presented in Tables 3, 4 and similar tables for other

primary indicators.

Coordination of the method of constructing  $DN$  with the chosen number system  $\{\beta\}$  makes it possible to calculate it in the form (Nedosiekin, 2002):

$$DN = (v_1, v_2, v_3, v_4, v_5) = \sum_{j=1}^5 Y_j \otimes \beta_j \quad (5)$$

where the sign « $\otimes$ » expresses the operation of multiplying a real number by a fuzzy number, and the auxiliary coefficients  $Y_j$  ( $j = \overline{1, 5}$ ) are determined by the formulas:

$$Y_j = (\sum_{i=1}^8 p_i \cdot \theta_{ij}) / (\sum_{i=1}^8 p_i) \quad (j = \overline{1, 5}), \quad (6)$$

where  $p_i$  is the priority coefficient of  $X_i$  ( $i = \overline{1, 6}$ ). Recall that for calculations we have taken all  $p_i = 1$  ( $i = \overline{1, 6}$ ).

Since we are used to using real numbers, we finally need to perform the defuzzification operation of the fuzzy  $DN$  number, that is, to move from it to the corresponding real number  $Y$ . The membership of a trapezoidal  $DN$  interval to one of the fuzzy subsets of  $\{A\}$  welfare of a country's population can be determined by using cross-sectional formulas and combining fuzzy sets. The degree of membership  $Z$  of the welfare state of a country to one of the states  $A_j$  is determined using the area  $\Delta$  of some figure by the formula (Nedosiekin, 2002):

$$Z = [\Delta(DN \cap A_j)] / [\Delta(DN \cup A_j)] \quad (7)$$

where  $\Delta$  is defined as the corresponding area bounded by the trapezoidal curves of the membership functions.

However, it is quite difficult to recognize the welfare of the population using formula (7). Therefore, we use the approximate method of solving the problem (Nedosiekin, 2002), which is more convenient in calculations. Its essence is to determine the functions  $\mu_j(DN)$ , ( $j = \overline{1, 5}$ ) by the type of numbers  $\beta$  and taking into account the auxiliary parameters:

$$\bar{b}_j = (b_2^j + b_3^j) / 2, \quad (j = \overline{1, 5}) \quad (8)$$

where  $b_2^j$  and  $b_3^j$ , respectively, the abscissa of the upper base of the  $j$  trapezoid in the notation  $\beta$  (formula (2)). Based on formulas (4) we obtain:  $\bar{b}_1 = (0 + 0,15) / 2 = 0,075$ ;  $\bar{b}_2 = (0,25 + 0,35) / 2 = 0,3$ ;  $\bar{b}_3 = (0,45 + 0,55) / 2 = 0,5$ ;  $\bar{b}_4 = (0,65 + 0,75) / 2 = 0,7$ ;  $\bar{b}_5 = (0,85 + 1) / 2 = 0,925$ .

If the value of  $\mu_j(W) > 0$ , ( $j = \overline{1, 5}$ ) obtained during the analysis, we consider that the welfare state of the population is described by the linguistic value of the subset  $W$

with the level of correspondence  $\mu_j(W)$ . In other cases,  $DN$  does not belong to other subsets of  $A_j$ . That is, the set  $\{\mu\}$  has the singularity that a membership is possible for no more than two intersecting subsets.

It is now possible to write a formula for finding the value of a comprehensive indicator of the welfare of the country's population:

$$Y = \sum_{j=1}^5 \bar{b}_j \cdot Y_j = 0,075 \cdot Y_1 + 0,3 \cdot Y_2 + 0,5 \cdot Y_3 + 0,7 \cdot Y_4 + 0,925 \cdot Y_5 \quad (9)$$

Here we consider the functions of belonging (2), formula (4) and the results of calculations according to formula (8). If one is interested in one of the five considered welfare states of the population of Ukraine in a given year, then it can be determined on the basis of magnitude  $Y$ . To do this, use the rule (4), which is given in Table 5.

**Table 5.** The rule of recognition of the welfare level of the population of Ukraine

Indicator	Value range	Parameter level classification (development level)	Degree of Confidence (membership function)
The level of welfare of the population	$0 \leq Y \leq 0,15$	«bad»	$\mu_1 = 1$
	$0,15 < Y < 0,25$	«bad»	$\mu_1 = 10 \cdot (0,25 - Y)$
	$0,15 < Y < 0,25$	«satisfactory»	$\mu_2 = 1 - \mu_1$
	$0,25 \leq Y \leq 0,35$	«satisfactory»	$\mu_2 = 1$
	$0,35 < Y < 0,45$	«satisfactory»	$\mu_2 = 10 \cdot (0,45 - Y)$
	$0,35 < Y < 0,45$	«average»	$\mu_3 = 1 - \mu_2$
	$0,45 \leq Y \leq 0,55$	«average»	$\mu_3 = 1$
	$0,55 < Y < 0,65$	«average»	$\mu_3 = 10 \cdot (0,65 - Y)$
	$0,55 < Y < 0,65$	«good»	$\mu_4 = 1 - \mu_3$
	$0,65 \leq Y \leq 0,75$	«good»	$\mu_4 = 1$
	$0,75 < Y < 0,85$	«good»	$\mu_4 = 10 \cdot (0,85 - Y)$
$0,75 < Y < 0,85$	«very good»	$\mu_5 = 1 - \mu_4$	
$0,85 \leq Y \leq 1$	«very good»	$\mu_5 = 1$	

**Source:** Own creation.

Using the information on the values of primary indicators (Table 1), in accordance with the described algorithm, we calculated the value of a comprehensive indicator of the level of welfare of the population of Ukraine for 2006-2018. Initially, for each of these years and the corresponding indicator, their membership functions  $\theta_{ij}$  ( $i = \overline{1,6}, j = \overline{1,5}$ ) were calculated, and based on them the auxiliary coefficients  $Y_j$  ( $j = \overline{1,5}$ ). The results of these calculations for 2006 are presented in Table 6.

**Table 6.** The values of  $\{\theta\}$  and  $Y_j$  ( $j = \overline{1,5}$ ) for the level of well-being of the population of Ukraine in 2006

$\{\theta\}$	$\theta_{i1}$	$\theta_{i2}$	$\theta_{i3}$	$\theta_{i4}$	$\theta_{i5}$
$X_1$	0	0,458	0,542	0	0
$X_2$	1	0	0	0	0
$X_3$	0	0,86	0,14	0	0
$X_4$	0	0,8	0,2	0	0

$X_5$	0	0	0	1	0
$X_6$	1	0	0	0	0
$Y_j$	0,333	0,353	0,147	0,167	0

**Source:** Own creation

Now the value of the integral index  $Y$  for this year according to formula (9) will be:

$$Y = 0,075 \cdot 0,333 + 0,3 \cdot 0,353 + 0,5 \cdot 0,147 + 0,7 \cdot 0,167 + 0,925 \cdot 0 = 0,321.$$

Given this value, according to the algorithm of Table 5 we find:  $\mu_2 = 1$ , and  $\mu_1 = \mu_3 = \mu_4 = \mu_5 = 0$ . Hence the following statement: with a high degree of correspondence it can be guaranteed that in 2006 the welfare of the Ukrainian population was «satisfactory».

Similarly, based on the calculated membership functions  $\theta_{ij}$  ( $i = \overline{1,6}, j = \overline{1,5}$ ) for each of the primary indices, as well as the auxiliary coefficients  $Y_j$  ( $j = \overline{1,5}$ ) for 2007-2018 we determine the value of  $Y$  the integral index  $Y$  for each of these years. At the same time, based on the formulas in Table 5, we calculate the membership functions  $\mu_j(DN)$  corresponding to fuzzy subsets  $A_j$  ( $j = \overline{1,5}$ ) of population welfare states. The results of the calculations are presented in table. 7.

**Table 7.** The magnitudes of the complex indicator of the level of welfare of the population of Ukraine  $Y$  and the level of membership  $\mu_j(DN)$  to the set of its states in 2010-2018

Pik	Y	$\mu_j(DN)$				
		$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$\mu_5$
2006	0,321	0	1	0	0	0
2007	0,438	0	0,12	0,88	0	0
2008	0,371	0	0,79	0,21	0	0
2009	0,489	0	0	1	0	0
2010	0,479	0	0	1	0	0
2011	0,472	0	0	1	0	0
2012	0,486	0	0	1	0	0
2013	0,529	0	0	1	0	0
2014	0,465	0	0	1	0	0
2015	0,413	0	0,37	0,63	0	0
2016	0,445	0	0,05	0,95	0	0
2017	0,485	0	0	1	0	0
2018	0,573	0	0	0,77	0,23	0

**Source:** Own creation.

Table 7 shows that the welfare of the population of Ukraine has improved significantly from 2006 to 2018. If in 2006 it was «satisfactory», then in 2018 it can be argued with high degree of correspondence that it was «average» and, to a lesser degree, «good».

#### **4. Conclusions**

Thus, the algorithm of estimating the level of welfare of the country's population, which uses the theory of fuzzy sets and consists of construction based on primary partial factors of a generalized indicator, is expedient to use in practice. The following conclusions can be drawn from the calculations performed using this algorithm:

- Despite various political, economic, financial, and other disturbances in Ukraine, the regulation of its economic processes from 2006 to 2018 has had a positive effect – the level of welfare of the country's population during this period has increased significantly.
- Both the global financial and economic crisis of 2008 and the annexation of Crimea, and the Russian military aggression in the east of the country have significantly affected the welfare of the Ukrainian population. As a result of these events, the level of welfare of the population has decreased significantly.
- If the consequences of this crisis were overcome relatively quickly, the impact of the unannounced war with Russia, which is still going on, has affected the welfare of the Ukrainian population in 2018.
- Fuzzy technologies have proved to be an effective method of modeling the level of welfare of the population in the system of effective management of the country's economy.

#### **References:**

- Afsa, C., Blanchet, D. 2009. A Survey of Existing Approaches to Measuring Socio-Economic Progress. Commission on the Measurement of Economic Performance and Social Progress. Insee-OECD. Retrieved from:  
<http://www.rrojasdatabank.info/dev0040.html>.
- Ignatyuk, A.O. 2010. Methodical approaches to measuring the development of social welfare. Retrieved from:  
[http://www.nbu.gov.ua/portal/Soc\\_Gum/Ekpr/2010\\_34/Zmist/6PDF.pdf](http://www.nbu.gov.ua/portal/Soc_Gum/Ekpr/2010_34/Zmist/6PDF.pdf).
- Kozlovskiy, S., Khadzhynov, I., Lavrov, R., Skydan, O., Ivanyuta, N., Varshavska, N. 2019. Economic-Mathematical Modeling and Forecasting of Competitiveness Level of Agricultural Sector of Ukraine by Means of Theory of Fuzzy Sets under Conditions of Integration into European Market. *International Journal of Recent Technology and Engineering (IJRTE)*, 8(4), 2277-3878.
- Kozlovskiy, S., Mazur, H., Vdovenko, N., Shepel, T., Kozlovskiy, V. 2018. Modeling and Forecasting the Level of State Stimulation of Agricultural Production in Ukraine Based on the Theory of Fuzzy Logic. *Montenegrin Journal of Economics*, 14, 3, 37-53.
- Kozlovskiy, S., Nikolenko, L., Peresada, O., Pokhyliuk, O., Yatchuk, O., Bolgarova, N., Kulhanik, O. 2020. Estimation level of public welfare on the basis of methods of intellectual analysis. *Global Journal of Environmental Science and Management*, 6(3), 355-372.
- Nedosiekin, A.O. 2002. Fuzzy-multiple stock investment risk analysis. *Sezam*, 181.
- Pigou, A. 1985. *Economic Theory of Welfare - T. 1 / Transl. from English*. Progress.

- Pryimak, V.I. 2009. *Mathematical methods of economic analysis: Educational Manual* Center for Educational Literature, 296.
- Pryimak, V.I., Holubnyk, O.R. 2012. Ranking of countries by the level of economic welfare of their population. *Scientific Bulletin of NLTU of Ukraine*, 22(8), 134-141.
- Samuelson, P.A., Nordhaus, W.D. 1995. *Macroeconomics. Fundamentals*, 544;
- Siavavko, M., Rybytska, O. 2000. *Mathematical modeling under uncertainty*. Lviv: Scientific production company Ukrainian Technologies, 320.
- Statistical Yearbook of Ukraine for 2018: Statistical collection*. State Statistics Service of Ukraine. 2019. Zhytomyr LTD «BOOK-DRUK», 482.
- Stiglitz, J.E., Sen, A., Fitoussi, J.P. 2009. *Report by the Commission on the Measurement of Economic Performance and Social Progress*. CMEPSP. Retrieved from: [http://www.stiglitz-sen-fitoussi.fr/documents/rapport\\_anglais.pdf](http://www.stiglitz-sen-fitoussi.fr/documents/rapport_anglais.pdf).
- Zadeh, L.A. 1965. Fuzzy Sets. *Information and Control*, 8, 338-353.