MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
ADVANCED LEVEL

MAY 2015

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	I
DATE:	**16th May 2015**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ $\mathbf{ms^{-2}}$).

1. ABCD is a square of side 1 m. Forces F_1, F_2 and 4 N act along $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$ respectively. The equation of the line of action of their resultant, referred to AB and AD as x and y axes respectively, is $3y = 2x + 6$.

   (i) Calculate the values of F_1 and F_2.

   (ii) An anticlockwise coplanar couple of magnitude 3 Nm is now added to the system. Find the equation of the line of action of the resultant of the new system.

[7, 3 marks]

2. A girl throws a cricket ball with a speed of 19 ms^{-1} at an angle of 15° above the horizontal.

   (i) If her brother catches it at the same height, how far apart must they be?

   (ii) Her brother misses the ball completely and it lands 22 metres away from the girl. Find the height above the ground from which it was thrown.

[6, 4 marks]

3. A semi-circular lamina, having diameter AOB and centre O, is made of uniform material and has mass m and radius a.

   (i) Using integration, find the position of the centroid of the lamina.

   (ii) A particle of mass M is fixed to the lamina at the end A of its diameter. The system is then suspended freely from O. At equilibrium, the diameter AOB makes an angle of 45° with the downward vertical. Find the ratio M/m.

[7, 3 marks]

4. A car has an engine capable of developing 15 kW. The maximum speed of the car on a level road is 120 km/hr.

   (i) Calculate the total resistance in newtons at this speed.

   (ii) Given that the total mass of the car is 1000 kg, and that the resistance to motion is proportional to the square of the speed, find the engine's power, in kW, when the car is moving at a constant speed of 40 km/hr up a road of inclination α, where $\sin\alpha = 1/25$.

[3, 7 marks]

5. A bat is used to hit a ball of mass 0.15 kg. Just before impact, the ball is travelling horizontally at speed 20 ms^{-1}. After impact, the ball is travelling horizontally with a speed of 30 ms^{-1} in the opposite direction.

   (i) Find the magnitude of the impulse exerted on the ball.

   (ii) The force on the ball can be modelled as $f(t) = k\sin(100\pi t)$ where t is the time after the ball first makes contact with the bat. Find the value of k if it is assumed that the bat is in contact with the ball for 0.01 sec.

[3, 7 marks]

6. A cyclist rides his bicycle on a track of radius 17 m banked at an angle α to the horizontal. There is no tendency to slip on the track when the cyclist's speed is 10 ms^{-1}.

   (i) Find the angle α.

   (ii) When the speed is increased to 14 ms^{-1}, the bicycle is just on the point of slipping up the track. Find the coefficient of friction between the bicycle and the track.

[4, 6 marks]

7. A particle of mass 3 kg is attached to one end of a light elastic string of natural length 0.8 m. The other end of the string is attached to a fixed point O. When the system hangs in equilibrium, the particle is 1 m below O. The particle is raised to a point 0.8 m above O and then released from rest.

   (i) Show that the modulus of elasticity is 120 N.

   (ii) Find the speed, u ms^{-1}, of the particle as it passes O.

   (iii) Find the speed of the particle just before the string becomes taut.

   (iv) Using conservation of energy from (iii), or otherwise, find the position of the particle when it once again has speed u ms^{-1}.

[2, 2, 1, 5 marks]

8. A uniform cylinder has radius a and weight W. The cylinder rests with its axis horizontal, and with its curved surface in contact with a rough vertical wall and a rough plane inclined at 45° to the wall. The coefficient of friction between the cylinder and the wall, and the cylinder and the plane is μ.

   Assuming that friction is limiting at both lines of contact, find in terms of a, μ and W the greatest couple that will not rotate the cylinder.

[10 marks]

9. A light elastic string of natural length a and modulus λ is stretched between two points A and B distant $2a$ apart on a smooth horizontal table. A particle of mass m is fastened to the midpoint of the string. The mass is then pulled towards A through a small distance and released.

   Show that the mass performs simple harmonic motion about the midpoint of AB. Hence find the periodic time of these oscillations.

   [10 marks]

10. A particle of mass m falls from rest through a resisting medium where the resistance to motion is kv, where v is the velocity of the particle at time t, and k is a positive constant.

    (i) Find the velocity of the particle in terms of m, g, k and t.

    (ii) Show that for large times t, the velocity approaches a limiting value of $\dfrac{mg}{k}$.

    [9, 1 marks]

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MATRICULATION EXAMINATION
ADVANCED LEVEL

MAY 2015

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	II
DATE:	**18th May 2015**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Answer **SEVEN** questions. In all there are 10 questions each carrying 15 marks.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. A *light* straight uniform beam, ABCD, has length $3a$, with AB = BC = CD = a. The beam rests horizontally on supports at A and C, and carries concentrated loads $2W$ and W at B and D respectively. This structural system is in equilibrium.

   (i) Find the reactions at the supports A and C.

   (ii) Find the shearing force and bending moment at any point along the beam.

   (iii) Draw a sketch of the shearing force and bending moment.

   (iv) Find the maximum bending moment and the point on the beam where it occurs.

   [2, 9, 3, 1 marks]

2. The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act at the points whose position vectors are $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. These vector quantities are given as follows:

$$\mathbf{F}_1 = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \mathbf{r}_1 = 4\mathbf{i} - \mathbf{j} + \mathbf{k}$$

$$\mathbf{F}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \mathbf{r}_2 = 3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors along the x- and y-axes of a Cartesian coordinate system.

(i) Find the force through the origin O and the couple which together are equivalent to these forces. Find the magnitude of this couple.

(ii) Show that the lines of action of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are concurrent, and find the position vector of the point of intersection of these two lines.

[10, 5 marks]

3. AB is a light inextensible string of length a. A particle of mass m is attached to the end B of the string, whilst the other end A is attached to a fixed point. When the particle is hanging vertically, it is given a velocity v_0 at right angles to the string, so that the particle starts moving in a vertical circle. The tension in the string is T when AB makes an angle θ with the downward vertical.

(i) Obtain an expression for T in terms of θ, m, a, g and v_0.

(ii) Find, in terms of a and g, the minimum value of v_0 if the particle is to make complete vertical circles.

(iii) Find, in terms of a and g, the value of v_0 if the maximum tension in the string is three times the minimum tension.

[8, 4, 3 marks]

4. A uniform rod AB of weight W and length $2a$ is freely hinged at A to a fixed point on a rough horizontal table. A uniform rough sphere of radius a and weight $W\sqrt{3}$ rests on the table. The rod leans against the sphere so that it makes an angle of 60° with the horizontal, and so that the rod and the centre of the sphere lie in a vertical plane.

(i) Draw a diagram showing clearly the forces on the rod and on the sphere.

(ii) By considering equilibrium of the rod and of the sphere separately, or otherwise, show that the frictional force between the rod and the sphere is $W/6$.

(iii) If the coefficient of friction at each point of contact is μ, find the smallest value of μ which makes equilibrium possible.

[3, 9, 3 marks]

5. A framework consists of five identical light rods forming a regular pentagon ABCDE, together with rods BE and CE which hold it in shape. The framework is suspended from A and carries loads of 50 N at each of C and D. It can be assumed that all joints in this framework are smooth.

   Using the method of sections, or otherwise, calculate the forces in the rods BE and CE.

   [15 marks]

6. Two particles A and B, each of mass m, are connected together by means of a light, inextensible string of length $2a$. They are at rest at a distance a apart on a smooth horizontal table. A is then projected with speed u along the table in a direction perpendicular to AB.

   (i) By considering momenta parallel and perpendicular to the impulse, or otherwise, find the velocities of the two particles immediately after the string becomes taut.

   (ii) Find the impulsive tension in the string and the angular velocity of the system.

   [13, 2 marks]

7. A bead of mass m can slide on a smooth circular ring of wire, which is fixed in a vertical plane, and has radius a. Attached to the bead is one end of a light spring, of natural length a and modulus $3mg$. The other end of the spring is attached to the highest point of the wire. In the subsequent motion, θ is defined to be the angle between the spring and the downward vertical.

   (i) Draw a diagram of this system.

   (ii) Show that the potential energy of the system is given by

   $$\frac{3}{2}mga(2\cos\theta - 1)^2 - 2mga\cos^2\theta + \text{constant}.$$

   (iii) By differentiating the potential energy with respect to θ, and equating the first derivative to zero, show that possible equilibrium positions occur when $\theta = 0$ and $\cos\theta = \frac{3}{4}$.

   (iv) Find the second derivative of the potential energy at these two values of θ, and comment on your result.

   [2, 6, 4, 3 marks]

8. A pendulum consists of a uniform rod AB of mass m and length $2a$ attached at the end B to the rim of a circular ring, also of mass m and having radius a, such that the rod is normal to the ring at B and coplanar with it. The pendulum oscillates in the plane of the ring about the free end A of the rod.

   (i) Find the period of the oscillation.

   (ii) Find the percentage change in the period if the ring were replaced by a disc of the same size and mass.

[10, 5 marks]

9. A uniform disc has centre O, mass m and radius a. It is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through a point P on its rim.

   (i) Using integration and the theorem of parallel axes, find the moment of inertia of the disc about this axis.

   (ii) The disc is held with PO horizontal and released from rest. In the subsequent motion, θ denotes the angle between PO and the horizontal. Find the angular velocity, $\dot{\theta}$, and the angular acceleration, $\ddot{\theta}$, in terms of θ.

   (iii) Find the magnitude of the reaction at P when $\theta = 45^{\circ}$.

[5, 5, 5 marks]

10. A uniform rod, AB, of length $2a$ and mass m is free to rotate about a fixed horizontal axis through A. The rod is held at rest with B vertically above A, and is then slightly displaced. When B reaches the point vertically below A, it collides with a fixed peg and rebounds. The rod is next instantaneously at rest when AB is horizontal.

   (i) Using conservation of energy *before* the impact, find, in terms of a and g, the angular velocity immediately before the rod hits the peg.

   (ii) Using conservation of energy *after* the impact, find the angular velocity immediately after the rod hits the peg.

   (iii) By taking moments about the axis at A, find the impulse at the peg in terms of m, a and g.

   (iv) By considering the motion of the centroid, find the impulsive reaction at A.

[4, 3, 4, 4 marks]