MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2013

SUBJECT:	APPLIED MATHEMATICS
DATE:	15th May 2013
TIME:	9.00 a.m. to 12.00 noon

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. ABCD is a square of side 2 m. L is the midpoint of AB, whilst M is the midpoint of BC. Forces of magnitudes 4, 8 and P N act along $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{DC}$ respectively. The resultant is parallel to $\overrightarrow{LM}$.

   (i) Find the value of P.

   (ii) Find the magnitude of the resultant and the point where its line of action intersects AB.

[5, 5 marks]

2. A solid cylinder and hemisphere both have a radius of 1 cm, and are made up of the same uniform material. The height of the cylinder is 2 cm. The flat base of the hemisphere is then attached to the top of the cylinder, so that the centres of the two faces coincide.

   (i) Find the height of the centre of gravity of the composite solid above the base of the cylinder.

   (ii) The solid is then suspended freely from a point on the circumference of the base of the cylinder. Find the angle which the axis of the solid makes with the vertical.

   Hint: The centre of gravity of a uniform hemisphere of radius r is $3r/8$ from the centre.

   [7, 3 marks]

3. Two identical smooth spheres S and T, each of mass m, can move without friction on a smooth horizontal table. The spheres are moving in opposite directions with speeds u and $3u$ respectively, when they collide directly. After impact, sphere T is reduced to rest.

   (i) Find in terms of u the velocity of the sphere S after impact.

   (ii) Find the coefficient of restitution between the spheres.

   (iii) Find, in terms of m and u, the kinetic energy lost as a result of this impact.

   (iv) Find the impulse between the spheres during the collision.

   [3, 4, 2, 1 marks]

4. A stone is projected at an angle α *above* the horizontal from the top of a vertical cliff 56 m high. The stone falls into the sea 4 s later, at a distance of 32 m from the foot of the cliff. The stone moves in a vertical plane perpendicular to the cliff.

   (i) Find the speed of projection and the angle α.

   (ii) A second stone is projected from the same point, at the same time, in the same vertical plane, at the same speed, and at the same angle α as before, but *below* the horizontal.

   Find how long it will take to reach the sea, and the distance between the points of entry of the stones into the water.

   [5, 5 marks]

5. A light elastic string, of natural length l, has modulus $5mg$. The string is fixed at one end and carries at the other end a particle of mass m which hangs vertically. The mass is pulled to one side by a horizontal force of magnitude $\frac{1}{2}mg$. The system is in equilibrium.

   (i) Find the angle that the string makes with the vertical.

   (ii) Show that the string is extended by $\frac{l\sqrt{5}}{10}$.

   (iii) Find the horizontal distance by which the mass is displaced in terms of l.

[5, 4, 1 marks]

6. A car travelling at 32 ms^{-1} has no tendency to slip on a track of radius 200 m banked at an angle θ.

   When the speed is increased to 40 ms^{-1}, the car is just on the point of slipping up the track.

   (i) Find the angle θ.

   (ii) Find the coefficient of friction between the car and the track.

[4, 6 marks]

7. A lift cage is pulled up a shaft by an inextensible cable. The mass of the lift and its occupants is 3000 kg. The lift starts from rest and accelerates uniformly for 3 s until it reaches a velocity of 2 ms^{-1}, maintains this velocity for 11 s, and then decelerates uniformly to rest in 8 s.

   (i) Draw a velocity time graph for the motion of the lift.

   (ii) Find the tension in the cable when the lift is accelerating upwards with an acceleration of a ms^{-2}, giving your answer in terms of a.

   (iii) Determine the maximum power generated by the engine during the lift's ascent.

   (iv) Find the power generated by the engine during the time when the lift has constant velocity.

   (v) Find the power generated when the lift is decelerating and moving with a velocity of 1 ms^{-1}.

[2, 2, 3, 1, 2 marks]

8. A fixed plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$. A particle of mass m is projected from a point A on the plane, up a line of greatest slope. The coefficient of friction between the particle and the plane is $\frac{1}{3}$. The particle has moved a distance d up the plane when it comes instantaneously to rest at the point B. The particle then moves down the plane along the same line of greatest slope through A.

   (i) Find the total work done against the external applied forces during the motion from A to B.

   (ii) Find the speed of projection from A.

   (iii) Find the kinetic energy of the particle when it has passed through A and moved a further distance $4d$ down the plane from A.

[4, 2, 4 marks]

9. Two identical rods AB and BC, each of length 1 m and weight W, are freely hinged at B. They rest in equilibrium on two smooth pegs at the same horizontal level, with the rods each inclined at 60° to the vertical, and with B below A and C. The system is symmetrical about the vertical through B.

   (i) By considering the equilibrium of one of the rods, find, in terms of W, the reactions at the pegs and at the hinge B.

   (ii) Find the distance between the pegs.

[7, 3 marks]

10. A sphere of weight 30 N rests on a rough plane inclined at 60° to the horizontal. The highest point of the sphere is tied to a point of the plane by a horizontal string. The system is in limiting equilibrium, with the sphere about to slip down the plane.

   (i) Draw a diagram of the system showing the forces acting on the sphere.

   (ii) Find the coefficient of friction between the sphere and the plane.

   (iii) Find the tension in the string.

[3, 4, 3 marks]