IM 02.14m

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2014

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**16th May 2014**
TIME:	**4.00 p.m. to 7.00 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ $\mathbf{ms^{-2}}$).

1. ABCD is a rectangle with AB = 4 m and BC = 3 m. Forces of 4 N, 5 N and 10 N act along $\overrightarrow{CB}$, $\overrightarrow{DC}$ and $\overrightarrow{DB}$ respectively.

   (i) Find the single force equivalent of this system, and find where its line of action cuts AB.

   (ii) A couple of moment 15 Nm, in the sense ABCD, is now introduced to the system.

   Find the single force that will replace this new system and find where its line of action cuts AB.

[5, 5 marks]

2. A uniform ladder of weight W kg rests in equilibrium with its base on a rough horizontal floor, and its top against a smooth vertical wall. The ladder makes an angle of 60° with the floor.

   (i) Draw a diagram showing the forces in this system.

   (ii) Find the normal reaction and the frictional force at the floor.

   (iii) Find the minimum possible value of the coefficient of friction, μ, between the ladder and the floor.

   [3, 5, 2 marks]

3. A container is made by drilling a conical hole in one end of a uniform right circular cylinder. The axis of symmetry of the cone is the same as that of the cylinder. The cylinder is of radius 2 cm and length 6 cm. The conical hole penetrates 4 cm into the cylinder, and the circular hole at the end is of radius 1.5 cm.

   (i) Find the position of the centre of gravity of the container.

   (ii) The container is then suspended freely from a point on the circumference of the base of the cylinder which is opposite the hole. Find the angle which the axis of the solid makes with the vertical.

   Hint: The centre of gravity of a uniform cone of height h is at a distance of $h/4$ from the centre of the base.

   [7, 3 marks]

4. A golfer hits a golf ball with a speed of V ms^{-1} at an angle of projection α above the horizontal.

   (i) Find from first principles the range of the golf ball as a function of α.

   (ii) The golfer wishes the ball to land on the green which is at the same level as she is. The front edge of the green is 160 m, whilst the back edge is 185 m from the golfer. If the golfer hits the ball with a speed of 49 ms^{-1}, find the possible angles of projection.

   Hint: For any angle θ, $2\sin\theta\cos\theta = \sin 2\theta$ and $\sin(180^{\circ} - \theta) = \sin\theta$.

   [6, 4 marks]

5. In a motor race, two cars A and B are moving on a straight horizontal race track. At $t = 0$, car A is 1 km from the finishing post, and is travelling at 35 ms^{-1} with a uniform acceleration of 0.4 ms^{-2}. At the same instant, car B is 200 m behind A and is travelling at 44 ms^{-1} with a uniform acceleration of 0.5 ms^{-2}. B passes A after a time $t = T$ seconds.

   (i) On the same sketch, draw a plot of the velocity against time for both cars. Interpret the area enclosed by the two velocity-time plots and the lines $t = 0$ and $t = T$.

   (ii) Find in terms of T the velocities of the two cars at time T. Hence, or otherwise, find the value of T and the distance travelled by car A during this interval. How far are the cars from the finishing post at time $t = T$?

   (iii) If the accelerations of the cars are maintained, find the distance of the cars from the finishing post after another 4 seconds.

[2, 4, 4 marks]

6. Two spheres A and B of equal radii are initially at rest on a smooth horizontal surface. A is projected towards B with a speed of 5 ms^{-1}, and the collision between the two spheres reduces A to rest. B continues after the collision to strike a fixed vertical wall at right angles and rebounds to hit A again. The coefficient of restitution between A and B is 4/5, whilst that between B and the wall is 5/8. Sphere A has a mass of 0.2 kg. Find:

   (i) the mass of sphere B and its velocity just after its first collision with A;

   (ii) the velocity of B just after it hits the wall;

   (iii) the speeds of the spheres after the second collision between them.

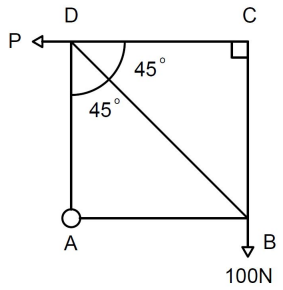
[4, 2, 4 marks]

7. A light inextensible string AB of length $2l$ has a particle attached to its midpoint C. The ends A and B are fastened to two fixed points with A at a distance of l vertically above B. The particle describes a horizontal circle about AB with constant angular speed ω. Both parts of the string are taut during this motion.

   If the tension in CA is three times that in CB, show that $\omega = 2\sqrt{\frac{g}{l}}$.

[10 marks]

8. The square light framework ABCD consists of four identical rods AB, BC, CD and DA arranged in the form of a square together with a fifth rod BD, which is one diagonal of the square. All the rods in the framework are smoothly jointed together.



The framework lies in a vertical plane with DC above AB, both rods being horizontal. The framework is smoothly hinged to a fixed support at A, and carries a vertical load of 100 N at B. The system is held in equilibrium by a horizontal force P at D.

Find the force P, the reaction at A, and the forces in the rods, stating whether they are in tension or in compression.

[10 marks]

9. Two identical elastic strings of length 1 metre and modulus 5 N are each fastened to a particle of mass 0.5 kg. Their other ends are fixed to two points, A and B, 4 metres apart in a vertical line, with A below B.

   (i) If the system is in equilibrium, find the height of the particle above A.

   (ii) The particle is now pulled down to A and released from rest. Find the greatest height above A to which the particle rises.

   Hint: You can assume that both strings are extended at this instant.

[5, 5 marks]

10. A car of mass 1000 kg is working at the rate of 15 kW. The resistance to the motion of the car is kv^2, where v is the speed of the car and k is a constant.

   (i) The maximum speed of the car on a level road is 120 km/h. Find the resistance at this speed, and hence deduce the value of k.

   (ii) Find the acceleration of the car when it is moving at a speed of 40 km/h up a road of inclination θ, where $\sin\theta = 1/25$.

[3, 7 marks]