IM 02.15m

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2015

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**18th May 2015**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. A parallelogram $ABCD$ has $\angle BAD = 45^{\circ}$ and $AD = DB = BC$. Forces act on the parallelogram as follows:

   $2W$ N acts along $\overrightarrow{AB}$, $2W\sqrt{2}$ N acts along $\overrightarrow{AD}$, $5W\sqrt{2}$ N acts along $\overrightarrow{CB}$,

   P N acts along $\overrightarrow{CD}$, and Q N acts along $\overrightarrow{BD}$.

   Find the values of P and Q if the system reduces to:

   (i) a couple;

   (ii) a single force along BA.

   [7, 3 marks]

2. A car and a van are at rest on a straight horizontal road with the van 25 m in front of the car. At time $t = 0$ seconds, the van moves off with an acceleration of 1.5 ms^{-2}. At time $t = 5$, the car moves off in the same direction with an acceleration of 2 ms^{-2}.

   (i) Sketch on the same diagram the velocity-time graphs which model the motion of the two vehicles.

   (ii) After how many seconds does the car catch up with the van?

[4, 6 marks]

3. ABCD is a uniform rectangular lamina of weight W with sides AB = 6 cm and AD = 5 cm. E and F are points on BC and CD respectively, with CE = CF = 3 cm. The portion ECF is then cut out from the lamina.

   (i) Find the position of the centre of gravity of the remaining portion of lamina, ABEFD.

   (ii) This portion is held in a vertical plane with AB horizontal, by a clamp at A. Find the reaction and couple at A at equilibrium.

[7, 3 marks]

4. A particle of mass 0.1 kg can just rest on a rough plane inclined at 30° to the horizontal without slipping down.

   (i) Find the coefficient of friction between the particle and the plane.

   (ii) The angle of inclination is now increased to 45°. Find the least horizontal force needed to prevent the particle from slipping downwards.

[4, 6 marks]

5. A ball is projected from a point O on level ground with a velocity $30\mathbf{i}+40\mathbf{j}$ ms^{-1}, where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the horizontal and vertical directions respectively. Obtain:

   (i) the velocity and displacement of the ball at time t seconds after projection in terms of $\mathbf{i}$ and $\mathbf{j}$;

   (ii) the time of flight and range of the ball;

   (iii) the greatest height reached by the ball, and

   (iv) the Cartesian equation of the path. **[4, 2, 2, 2 marks]**

6. Two spheres A and B, of equal radius, and having masses m and $2m$ respectively, are initially at rest on a smooth horizontal surface. A is projected towards B with a speed of u ms^{-1}. The coefficient of restitution between the spheres is 0.5. Find:

   (i) the velocity of the spheres after impact;

   (ii) the ratio of the kinetic energies before and after the collision;

   (iii) the impulse of each sphere on the other;

   (iv) the total impulse generated by the collision.

[5, 2, 2, 1 marks]

7. A car of mass 900 kg pulls a trailer of mass 200 kg. The resistance to motion of the car is 200 N and that of the trailer is 80 N.

   (i) Find the power output of the engine if the maximum speed on level ground is 40 ms^{-1}.

   (ii) The car and trailer are travelling at 8 ms^{-1} on a hill, inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{40}$. If the resistance is constant, and the engine is exerting full power, find the acceleration and the tension in the coupling between the car and the trailer.

[3; 4, 3 marks]

8. A particle of mass 0.5 kg is attached to one end of an elastic string of natural length 1 m and modulus of elasticity 50 N. The other end of the string is attached to a fixed point O on a rough horizontal plane. The coefficient of friction between the particle and the plane is 0.4. The particle is projected from O along the plane with a speed of 6 ms^{-1}. Find:

   (i) the greatest distance from O achieved by the particle;

   (ii) the speed of the particle when it returns to O.

[6, 4 marks]

9. A car is travelling round a circular bend of radius 30 m. The coefficient of friction between the car and the road is 0.3. The car has a mass of 600 kg. Find the maximum safe speed for the car if:

   (i) the road is unbanked;

   (ii) the road is banked at an angle of 20° to the horizontal.

[4, 6 marks]

10. Two equal uniform rods AB and BC are smoothly jointed at B. The ends A and C are hinged to supports so that AC is horizontal, B is above AC, and $\angle ABC = 60^\circ$. The rods each have a mass of 20 kg.

    Find the reactions at A, B and C.

[10 marks]