IM 02.15s

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2015

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**4th September 2015**
TIME:	**4.00 p.m. to 7.00 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. ABCD is a square of side $2a$. Forces of magnitude 2, 3, P and Q act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ of the square. Find the magnitudes of P and Q if the line of action of the resultant of the system

   (i) bisects AB and DC;

   (ii) bisects AB and passes through C.

[6, 4 marks]

2. A light framework ABCD is made up of four identical light rods AB, BC, CD, DA each of length $5a$, smoothly jointed together in the form of a rhombus, together with a fifth rod AC of length $6a$ which acts as a diagonal member in this framework. The system is suspended smoothly from D, and carries a weight W at B.

   Find the reaction at D and the forces in the rods, indicating whether they are in tension or in compression.

   [10 marks]

3. A uniform lamina of weight W is in the form of a triangle OAB, with OA=OB=4 m, and $\angle$ AOB = 1 right angle. A particle of weight $3W$ is attached to the vertex O.

   (i) Find the distance of the centre of gravity of the system from OA and OB.

   (ii) The lamina is then suspended freely from B. Find the angle which BO makes with the downward vertical.

   [7, 3 marks]

4. A bomb is dropped from an aeroplane which is flying steadily with a constant horizontal speed of 200 ms^{-1} at a constant height of 1000 m above level ground. The bomb hits a stationary target lying on the ground.

   (i) Find the time taken by the bomb to reach the target.

   (ii) How far (horizontally) is the plane from the target at the instant that the bomb is released.

   (iii) Taking suitable coordinate axes, find the Cartesian equation of the path taken by the bomb.

   [5, 2, 3 marks]

5. A race track has a circular bend of radius 50 m, and is banked at 45° to the horizontal. The coefficient of friction between the car wheels and the track is 0.6.

   Find the speed limits within which a car can travel round the bend without slipping either inwards or outwards.

   [10 marks]

6. A sphere of mass m moving along a smooth horizontal table with speed u collides directly with a stationary sphere of the same radius and of mass $2m$. The coefficient of restitution for this impact is e.

   (i) Find, in terms of u and e, the velocities of the two spheres after impact.

   (ii) If half the kinetic energy is lost on impact, find the value of e.

[7, 3 marks]

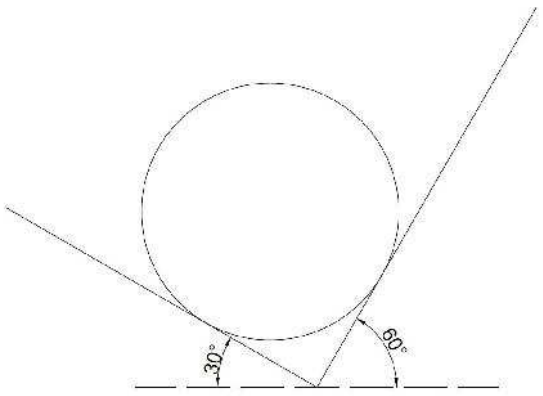
7. A cyclist and his machine have a total mass of 100 kg. When travelling up a hill inclined at $\sin^{-1}\frac{1}{50}$ to the horizontal against a resistance of 20 N, the cyclist can maintain a speed of 12 km/hr.

   (i) Find the rate at which he is working.

   (ii) If the working rate and the resistance to motion are both unchanged, find, in ms^{-2}, the acceleration of the cyclist when travelling on a level road at 10 km/hr.

[6, 4 marks]

8. Two inclined planes intersect in a horizontal line. One plane is inclined at 30° to the horizontal, whilst the other plane is inclined at 60° to the horizontal, as shown in the diagram. A smooth cylinder of weight W rests with its curved surface in contact with the planes, and with its axis horizontal. All contacts can be assumed to be smooth.



   (i) Draw a diagram of the system showing the forces acting on the cylinder.

   (ii) Using Lami's theorem or otherwise, find the forces acting on the cylinder.

[3, 7 marks]

9. A particle of mass m is attached to the midpoint of an elastic spring whose modulus is $2mg$, and whose unstretched length is $2a$. One end P of the spring is attached to the ceiling, whilst the other end Q is attached to the floor of a room of height $4a$. Q lies vertically below P.

   (i) Find the distance from the ceiling of the particle when it is in equilibrium.

   (ii) Find the elastic energy stored in the spring at equilibrium.

[7, 3 marks]

10. Two particles of mass 3 kg and 5 kg respectively are connected by a light inextensible string passing over a smooth pulley which is fixed to the ceiling of a lift. The masses are released from rest with the string taut. Find the tension in the string:

    (i) if the lift is stationary;

    (ii) if the lift is accelerating downwards at 1 ms^{-2}.

[5, 5 marks]