IM 02.16m

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2016

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**9th May 2016**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. Two spheres P and Q, of equal radius, and having masses 7 kg and 5 kg respectively, rest on a smooth horizontal table. The spheres are projected towards each other with speeds 8 ms^{-1} and 3 ms^{-1} respectively, and collide with each other directly. After impact, P moves with a speed of 3 ms^{-1} in the same direction as before the impact. Find:

   (i) the velocity of the sphere Q after impact;

   (ii) the coefficient of restitution for this collision;

   (iii) the impulse of each sphere on the other;

   (iv) the total impulse generated by the collision.

[5, 2, 2, 1 marks]

2. A particle of mass 3 kg rests at a point P on the inside surface of a fixed rough hollow sphere, with centre O and coefficient of friction 0.35. OP makes an angle of 50° with the downward vertical. The particle is acted on by a horizontal force which is on the point of moving it up the surface of the sphere.

   Find the magnitude of this force.

   [10 marks]

3. ABCDE is a light framework consisting of a square ABCE of side a, and an equilateral triangle CDE. Angles EAB and ABC are right angles, whilst angles AED and BCD are both 150°. Masses are attached to the framework as follows:

   2 kg at A, 1 kg at B, 4 kg at C, 5 kg at D, and m kg at E.

   (i) Find in terms of m the position of the centroid of this system.

   (ii) When the system is suspended freely from A, the side AB makes an angle of 65° with the downward vertical. Find the value of m in this case.

   [7, 3 marks]

4. Three points A, B, C have coordinates (0,0), (4,0) and (0,3) respectively, where the distances are in metres. D, E and F are the midpoints of BC, AC and AB respectively. Forces act on the system as follows:

   4 N along AB, 3 N along AC, 10 N along CB, P N along DE and Q N along AD,

   in the directions indicated by the order of the letters.

   (i) If the system is in equilibrium, find the values of P and Q.

   (ii) If the system is not in equilibrium, show that the resultant of the system passes through D .

   (iii) If the resultant is a force of 6 N acting in the direction from F to D, find the values of P and Q.

   [6, 2, 2 marks]

5. A golf ball is projected from a point O on horizontal ground with speed 25 ms^{-1} at an angle $\tan^{-1}(\frac{3}{4})$ to the horizontal. A coordinate system is taken, having O as origin, the x-axis horizontal, the y-axis vertical, and unit vectors **i**, **j** in the x- and y- directions respectively. Find:

   (i) the velocity and displacement of the ball at time t seconds after projection in terms of **i** and **j**;

   (ii) the time at which the ball reaches the highest point, and the coordinates of this point;

   (iii) the time when the ball is travelling at 30^{o} above the horizontal;

   (iv) the Cartesian equation of the path.

[4, 2, 2, 2 marks]

6. A train consists of an engine of mass 50,000 kg and a truck of equal mass. The resistance to motion of the engine is 4000 N, whilst that of the truck is 2500 N. The maximum speed of the train on level ground is 120 km/hr. Find:

   (i) the power output of the engine;

   (ii) the maximum speed of the train when it is going up a slope inclined at an angle θ to the horizontal, where $\sin\theta = 0.01$;

   (iii) the acceleration of the train when travelling up the same incline at half the maximum speed found in (ii).

   Hint: It can be assumed that the resistance to motion is constant in all situations, and that the power output of the engine is always equal to that found in (i).

[3, 4, 3 marks]

7. A conical pendulum consists of a particle of mass 5 kg attached to the end B of a light inextensible string AB of length 0.8 m. The end A is attached to a fixed point, whilst the particle at B moves in a horizontal circle, with a constant angular speed of 4 rad/sec. Find:

   (i) the angle that the string makes with the vertical;

   (ii) the reaction at the support at A.

[8, 2 marks]

8. A rectangular block of mass 4 kg rests on a rough horizontal surface. The coefficient of friction between the block and the surface is 0.4. A jet of water, emerging from a circular nozzle of radius 6 cm with speed v ms^{-1}, is aimed directly on one of the vertical faces of the block. It can be assumed that the density of water is 1000 kg/m^3, and that the water is reduced to rest after impact with the block.

   (i) Find, in terms of v, the mass of water discharged per second, and the momentum of the water per second.

   (ii) Find the value of v for which the block will just start to move.

   (iii) Find the initial acceleration of the block if the velocity of the jet is twice that in part (ii).

[3, 3, 4 marks]

9. A catapult is made by fastening an elastic string of natural length 10 cm to points A and B, where AB = 6 cm. The modulus of elasticity of the string is 5 N. A stone of mass 10 grams is placed at the centre of the string, which is then pulled back until the stone is 25 cm from the centre of AB, and equidistant from A and B.

   Using the principle of conservation of energy, find the speed of the stone when it is released.

[10 marks]

10. A smooth cylinder, of radius 1 m, is fixed with its axis horizontal. Two uniform rods AB and BC, each of length 2 m and mass 5 kg, are smoothly jointed at B. They are placed symmetrically on the surface of the cylinder, with B vertically above the axis of the cylinder. The ends A and C are on opposite sides of the vertical plane containing B and the axis of the cylinder. The system is in equilibrium.

   (i) Show that angle ABC is 111.4°.

   (ii) Find the reaction in the joint B.

[7, 3 marks]