IM 02.16s

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2016

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**30th August 2016**
TIME:	**4.00 p.m. to 7.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Non-programmable scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ $\mathbf{ms}^{-2}$).

1. The square ABCD has sides of length a. Forces act along the sides of the square as follows: 2 N along $\overrightarrow{AB}$, $5\sqrt{2}$ N along $\overrightarrow{CB}$, $2\sqrt{2}$ N along $\overrightarrow{AD}$, P N along $\overrightarrow{CD}$, and Q N along the diagonal $\overrightarrow{BD}$.

   Find the magnitudes of P and Q if the system reduces to:

   (i) a couple;

   (ii) a single force acting along BA.

   [6, 4 marks]

2. A framework consists of three light rods, AB, BC and CA, smoothly jointed together at their ends to form a triangle ABC. In this triangle, AB = AC, and $\angle$ A = 90°. The framework is suspended smoothly from A, and carries a weight W at B, and another weight W at C.

   Find the reaction at A and the forces in the rods, indicating whether they are in tension or in compression.

   [10 marks]

3. A uniform rod AB of mass 5 kg and length 2 m is attached at its end B to the rim of a uniform disc, which has mass 10 kg, radius 0.6 m and centre C. The rod is tangent to the rim of the disc at B, and both the rod and the disc are in the same plane.

   (i) Taking BC and BA as the x- and y- axes respectively, or otherwise, find the position of the centre of mass of the system.

   (ii) The system is then suspended freely from A. Find the angle which AB makes with the downward vertical.

   [7, 3 marks]

4. A sphere of mass 3 kg moving along a smooth horizontal table with speed 2 ms^{-1} collides directly with a sphere of mass 5 kg moving with velocity 1 ms^{-1} in the same direction as the first sphere. The coefficient of restitution for this impact is 1/4. It can be assumed that both spheres have equal radius. Find:

   (i) the velocities of the two spheres after impact;

   (ii) the impulse acting on each sphere.

   [7, 3 marks]

5. A uniform rod AB of mass 2 kg rests on rough horizontal ground and leans against a smooth vertical wall. The rod is about to slip when its inclination to the horizontal is 60°. It can be assumed that the rod always lies in a plane perpendicular to the wall and the ground.

   Find the coefficient of friction between the rod and the ground.

   [10 marks]

6. A stone is projected horizontally with speed 5 ms^{-1} from the top of a vertical cliff at a height of 20 m above sea level. The stone strikes the sea at a point Q.

   (i) Find the time taken by the stone to reach Q, and find the distance of Q from the base of the cliff.

   (ii) Find the tangent of the angle between the horizontal and the direction of motion of the stone just as it strikes the sea at Q.

   (iii) Taking suitable coordinate axes, find the Cartesian equation of the path taken by the stone.

[7, 1, 2 marks]

7. A ball of mass 1 kg is fastened to one end of a light, inextensible string of length 1 m. The ball is placed on a smooth horizontal table. The string is suspended from a point above the table so that the particle moves as a conical pendulum whilst in contact with the table. The radius of the circle in which the particle moves is 0.5 m.

   (i) If the speed of the particle is 1.5 ms^{-1}, find the normal reaction between the ball and the table.

   (ii) Find the maximum speed of the ball, if it does not lift off the table.

[6, 4 marks]

8. The lid of a chest is hinged about one of its long edges. The lid is uniform and has length 60 cm, width 25 cm and weight 50 N. The lid is in limiting equilibrium when it makes an angle α with the horizontal, where $\alpha = 60°$.

   (i) Find the maximum resistive couple that the hinge can exert.

   (ii) The lid of the chest is kept open when $\alpha = 30°$ by applying a horizontal force P at the midpoint of the longer edge of the lid. Assuming that the resistive couple still acts and is unchanged, find P.

[3, 7 marks]

9. The resistance to motion of a car is kv, where k is a constant and v is the speed. The car, which has a mass of 1000 kg, has a maximum speed of 45 ms^{-1} on the level when its power output is 8 kW.

   (i) Find the value of k.

   (ii) Find the acceleration of the car when it is travelling up an incline of $\sin^{-1}(1/10)$ at a speed of 2 ms^{-1} with the engine working at 6 kW.

[5, 5 marks]

10. A block of mass 2 kg rests on a rough plane which is inclined at 30° to the horizontal. The block is attached to a point on the top of the plane by means of an elastic string of natural length 2 m and modulus 100 N. The coefficient of friction between the block and the plane is 0.25. It can be assumed that the block can only move along a line of greatest slope on the plane.

    Find the distance, along a line of greatest slope, between the lowest and highest positions in which the block will rest in limiting equilibrium.

[10 marks]