IM 02.17m

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2017

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**12th May 2017**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. Two spheres A and B, of equal radius, and having masses 0.2 kg and m kg respectively, rest on a smooth horizontal table. The sphere A is projected directly towards B with a speed of 10 ms^{-1} and the collision between the two spheres reduces A to rest.

   The sphere B continues after the collision to strike a fixed vertical wall at right angles, and rebounds to hit A again. The coefficient of restitution between A and B is 4/5, whilst that between B and the wall is 5/8.

   Find m, and the velocities of the spheres after the second collision between them.

   (Total: 10 marks)

2. ABCD is a light framework in the form of a square. It consists of four identical rods AB, BC, CD and DA together with a fifth rod BD acting as a diagonal of the square. The rods are smoothly jointed together at their ends. The system is suspended freely from A and carries weights W at B, W at C, and W at D. The system is in equilibrium.

   Find the reaction at A, and the forces in the rods, stating whether they are in tension or in compression.

(Total: 10 marks)

3. A Cartesian set of coordinates with origin O is fixed to the surface of a horizontal table, with the x- and y- axes both lying along the surface.

   A circular lamina, of uniform material and of radius 6 cm, is placed on the table, with its centre at the origin O.

   Two smaller circles are cut from this circle, one of radius 1 cm and centre P $(-1, -3)$, and the other of radius 3 cm and centre Q $(1, 2)$.

   (i) Find the coordinates of the centre of gravity of the remaining shape. (7)

   (ii) The remaining shape is then suspended freely from the point T $(0, 6)$. Find the angle which TP makes with the downward vertical. (3)

(Total: 10 marks)

4. ABC is a triangle with AB = AC = 1 m, and with $\angle$A = 1 right angle. A is at the origin of a Cartesian system of coordinates, with AB along the positive x-axis, and AC along the positive y-axis. Forces act along the sides of the triangle as follows:

   8 N along BA, 4 N along BC, and 6 N along CA,

   in the direction implied by the order of the letters. Find:

   (i) the magnitude and direction of the resultant; (5)

   (ii) the coordinates of the point where the line of action of the resultant intersects the x-axis; (3)

   (iii) the Cartesian equation of the line of action. (2)

(Total: 10 marks)

5. A particle is projected from a point O on horizontal ground with speed 60 ms^{-1} at an angle $\cos^{-1}(\frac{4}{5})$ to the horizontal. A coordinate system is taken, having O as origin, the x-axis horizontal, the y-axis vertical, and unit vectors $\mathbf{i}$, $\mathbf{j}$ in the x- and y- directions respectively. Find:

   (i) the velocity and displacement of the particle at time t seconds after projection in terms of $\mathbf{i}$ and $\mathbf{j}$; (6)

   (ii) the time the particle takes to reach the point P whose horizontal displacement from O is 96 m; (1)

   (iii) the speed of the particle at this instant; (1)

   (iv) the Cartesian equation of the path. (2)

(Total: 10 marks)

6. A body of mass 3 kg lies on a rough plane which is inclined at 20° to the horizontal. A force of 28 N, applied to the body parallel to and up the slope, causes the body to accelerate up the slope at 1.5 ms^{-2}.

   (i) Find μ, the coefficient of friction between the body and the plane. (7)

   (ii) If the applied force were suddenly removed, the body would travel on up the slope, eventually coming to rest. Explain whether it would then slip down the slope, or whether it would remain at rest. (3)

(Total: 10 marks)

7. A body of mass 2 kg is freely suspended from a spring of natural length 0.75 m and modulus 60 N, the other end of which is fixed to a point A. The body initially hangs freely in equilibrium at a point B. It is then pulled down a further 0.25 m to a point C and released from rest. Find:

   (i) the distance AB; (2)

   (ii) the energy stored in the spring when the body rests at B; (2)

   (iii) the energy stored in the spring when the body is held at C; (1)

   (iv) the kinetic energy of the body when it passes through B after release from C. (5)

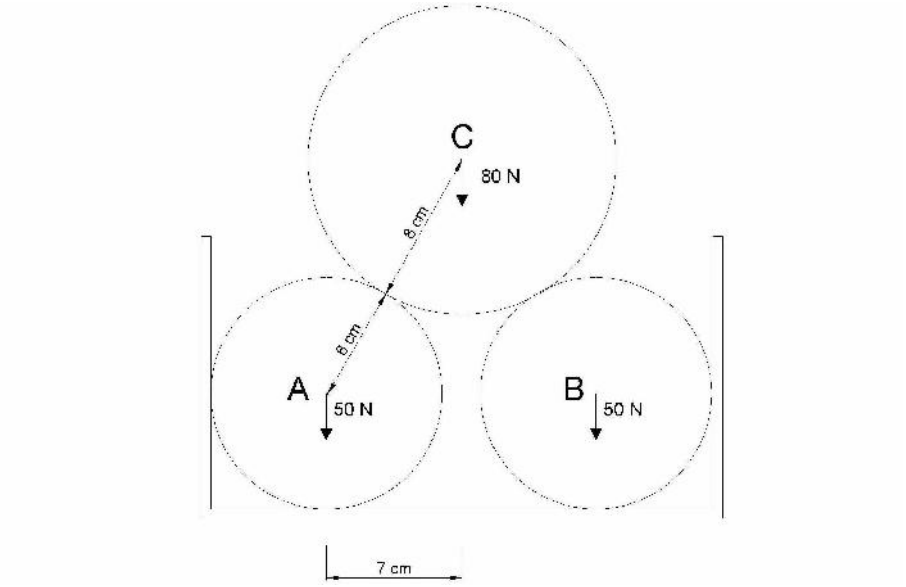
(Total: 10 marks)

8. A film stuntman has to drive a car in a horizontal circular path of radius 105 m around a bend that is banked at 45° to the horizontal. The stuntman finds that he must drive with a speed of at least 21 ms^{-1} if he is to avoid slipping **down** the slope.

   Find the coefficient of friction between the tyres of the car and the road surface.

   (Total: 10 marks)

9. The diagram shows the cross-section of three smooth cylinders, A, B and C, which rest inside a smooth gutter.



   Cylinders A and B, which are identical, each having weight 50 N and radius 6 cm, lie against the sides and on the base of the gutter. Cylinder C, which rests on top of A and B, has a weight of 80 N and radius 8 cm. The distance between the centres of A and B is 14 cm. Find:

   (i) the normal reaction between A and the base of the gutter; (3)

   (ii) the normal reaction between A and C; (4)

   (iii) the normal reaction between A and the side of the gutter. (3)

   (Total: 10 marks)

10. A particle moving in a straight line OD with uniform retardation, leaves the point O at time $t = 0$ and comes to instantaneous rest at D. On its way to D, the particle passes points A, B, C at times $t = 1$, 2 and 4 s respectively after leaving O, where AB = BC = 48 m. Find the length of OA and the length of CD.

    (Total: 10 marks)