MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2017

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**31st August 2017**
TIME:	**4.00 p.m. to 7.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.
The marks carried by each question are shown at the end of the question.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are *not* allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. Particles A and B have masses 3 kg and 2 kg respectively, and lie at rest on a smooth horizontal table. The particles are connected together by a light inextensible string, which is initially slack. The coefficient of restitution between the particles is 0.5. A is projected towards B with a velocity of 10 ms^{-1}. Find:

   (a) the velocity of the particles immediately after impact; (4)

   (b) the common velocity of the particles after the string becomes taut; (4)

   (c) the kinetic energy lost in the process. (2)

(Total: 10 marks)

2. ABC is a triangular lamina with AB = 4 m, BC = 5 m, AC = 3 m, and angle A = 90°. The points D, E and F are the midpoints of BC, AC and AB respectively. Forces act on the lamina as follows: 4 N along AB, 3 N along AC, 10 N along CB, P N along DE, and Q N along AD, in the directions indicated by the order of the letters.

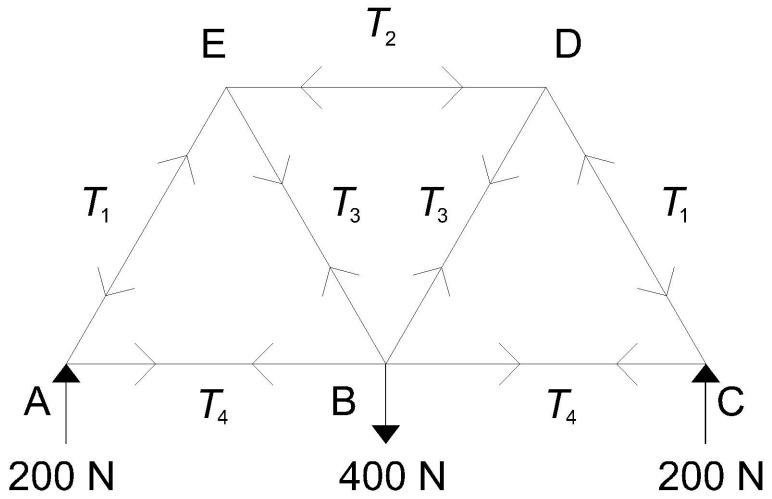
   (a) Show that, unless the system is in equilibrium, it reduces to a single force through D. (2)

   (b) Find the values of P and Q if the resultant is a force of 6 N acting along FD. (4)

   (c) Find the value of P if the resultant acts along AD, and find the magnitude of this resultant in terms of Q. (4)

(Total: 10 marks)

3. The framework shown in the figure below consists of seven identical light rods, each of length $2a$, and smoothly jointed together. The framework rests on supports at A and C, and carries a load of 400 N at B. The system lies in equilibrium with ABC horizontal, and with the framework lying in a vertical plane.



Find the forces in the rods, stating whether they are in tension or in compression.

(Total: 10 marks)

4. A body of mass m can just rest on a rough plane inclined at 30° to the horizontal without slipping down.

   (a) Find μ, the coefficient of friction between the body and the plane. (4)

   (b) Find the least horizontal force needed to prevent the body from slipping downwards if the inclination is increased to 45°. (6)

(Total: 10 marks)

5. A thin uniform lamina ABCD is formed by taking an equilateral triangle ECD, of side 2 cm, and removing the triangle EAB, where A and B are the midpoints of ED and EC respectively.

   (a) Find, in cm, the perpendicular distance of the centroid of the lamina ABCD from CD. You can assume that the centroid of a uniform triangular lamina of height h is at a perpendicular distance of $h/3$ from its base. (7)

   (b) The lamina ABCD is freely suspended from A, and hangs at rest in a vertical plane. Find the angle which AB makes with the downward vertical. (3)

(Total: 10 marks)

6. At time $t = 0$ s, a train passes a signal box B with speed 8 ms^{-1}. It accelerates uniformly until it reaches a speed of 20 ms^{-1} at $t = 40$ s, and maintains this speed until $t = T$ s, at which instant it decelerates uniformly until it comes to rest at the next station S at time $t = 2T$ s.

   (a) Draw a sketch of the velocity time graph for the motion of the train. (4)

   (b) Given that the average speed for the journey from B to S is 13.4 ms^{-1}, find the value of T. (4)

   (c) Draw a sketch of the displacement against time for the first 40 s after the train passes B. (2)

(Total: 10 marks)

7. A coordinate system has origin O, with the x-axis horizontal, the y-axis vertical, and with unit vectors $\mathbf{i}$, $\mathbf{j}$ in the x- and y- directions respectively.

   A basketball is projected from O towards a net with a velocity of 10 ms^{-1} at an angle $\tan^{-1}(2)$ to the horizontal.

   (a) Write down the equation of motion, and hence find the velocity and displacement of the particle at time t seconds after projection in terms of $\mathbf{i}$ and $\mathbf{j}$. (5)

   (b) Find the Cartesian equation of the path of the ball. (3)

   (c) The net is 5 m away from the point of projection. If the ball is to pass through the net, how high above the point of projection should the net be? (1)

   (d) By considering the velocity of the ball, show that it is moving downwards as it passes through the net. (1)

(Total: 10 marks)

8. In an amusement park, there is a ride which is effectively a hollow cylinder which can rotate about a vertical axis.

   The riders stand on the horizontal base of the cylinder and in contact with the curved surface of the cylinder. When the angular speed reaches a certain value, the floor is dropped, but the users remain in contact with the curved surface of the cylinder.

   The radius of the cylinder is 2.5 m, and the speed of rotation is 30 revolutions per minute. The coefficient of friction between the rider and the cylinder is μ.

   Find the smallest possible value of μ so that the ride works effectively.

(Total: 10 marks)

9. A car of mass 900 kg travels up a hill inclined at 10° to the horizontal, against a constant resistance force of 250 N. Its maximum speed is 45 km h^{-1}. Find:

   (a) the power output of the engine; (6)

   (b) the initial acceleration of the engine when it reaches level road at the top of the hill. (4)

(Total: 10 marks)

10. One end of an elastic string, of natural length 1.2 m and modulus of elasticity 150 N, is fixed to a point A. A particle of mass 0.75 kg is attached to the other end. The particle is held at A and then released from rest. At a certain instant, the length of the string is $(x + 1.2)$ m and the velocity of the mass is v ms^{-1}.

   (a) Using conservation of energy, obtain a relation between v and x. (6)

   (b) Find the velocity of the particle when $x = 0.2$ m. (1)

   (c) Find the length of the string when the particle comes to instantaneous rest. (3)

(Total: 10 marks)