IM02.18m

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE

EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL

2018 FIRST SESSION

SUBJECT:	**Applied Mathematics**
DATE:	8th May 2018
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10\ \text{ms}^{-2}$)

1. ABCD is a rectangle with AB = 4 m and BC = 3 m. Three forces act on the rectangle as follows: a force of 4 N acts along $\overrightarrow{CB}$, 5 N act along $\overrightarrow{DC}$, and 10 N act along $\overrightarrow{DB}$.

   (a) Find the single force equivalent to this system, and find where its line of action cuts AB. (6)

   (b) A couple of moment 15 Nm, in the sense ABCD, is now introduced to the system. Find the force that will replace this new system, and show that its line of action passes through B. (4)

(Total: 10 marks)

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2. A and B are two spheres of equal radii, and have masses of 0.75 kg and 0.60 kg respectively. The spheres are moving directly towards each other along a smooth horizontal surface, with A travelling with a speed of 6 ms^{-1}, and B with a speed of 4 ms^{-1}. When the spheres collide, the direction of motion of sphere B is reversed, and B proceeds to hit a fixed vertical wall at right angles.

   If the coefficient of restitution between the spheres is $\frac{4}{5}$, and that between B and the wall is $\frac{2}{3}$, show that B will collide again with A, and find the speeds of A and B after this second collision.

(Total: 10 marks)

3. A Cartesian set of coordinates is fixed to the surface of a horizontal table, with the x- and y- axes both lying along the surface. A circular lamina, of uniform material and of radius 8 cm, is placed on the table, with its centre C at the origin. A small circle of radius 2 cm and centre P$(4,0)$ is then cut off from the lamina.

   (a) Find the coordinates of the centre of gravity of the remaining shape. (7)

   (b) The remaining shape is then suspended freely from the point A$(0,8)$. Find the angle which AC makes with the downward vertical. (3)

(Total: 10 marks)

4. A uniform ladder of mass 25 kg rests in equilibrium with its base on a rough horizontal floor, and its top against a smooth vertical wall. The ladder makes an angle of 75^o with the horizontal. Find:

   (a) the normal reaction and the frictional force at the floor; (8)

   (b) the minimum possible value of the coefficient of friction μ between the ladder and the floor. (2)

(Total: 10 marks)

5. A vertical tower of height h stands with its base on horizontal ground. A coordinate system is taken, having O as origin at the base of the tower, the x-axis horizontal, the y-axis vertical, and unit vectors **i**, **j** in the x- and y- directions respectively. At time $t = 0$, two particles A and B are projected horizontally from the top of the tower with velocities 14 ms^{-1} and 17.5 ms^{-1} respectively. Find:

   (a) the velocity and displacement of the particles at time t seconds after projection in terms of h, **i** and **j**; (5)

   (b) the height h of the tower, given that the particles hit the ground at two points 10 m apart; (3)

   (c) the Cartesian equation of the path of the slower particle. (2)

(Total: 10 marks)

6. A uniform diving board AB, of length 4 m and mass 40 kg, is fixed at A to a vertical wall and is maintained in a horizontal position by means of a light strut DC. D is a point on the wall 1 m below A, and C is a point on the board where AC = 1 m. An object of mass 60 kg is placed at end B.

   (a) Draw a neat and clearly labelled diagram of the board, showing all the forces acting on it. (3)

   (b) Find the thrust in the strut and the magnitude of the reaction at A. (7)

(Total: 10 marks)

7. A car travels around a bend in a road which is a circular arc of radius 62.5 m. The road is banked at an angle θ to the horizontal, where $\tan\theta = 5/12$. The coefficient of friction between the tyres of the car and the road surface is 0.4.

   Find the least speed at which the car can be driven around the bend before it starts to slip inwards.

(Total: 10 marks)

8. A body of mass 3 kg slides on a smooth horizontal surface, and is attached to a fixed point A by a horizontal spring of natural length 0.5 m and modulus 400 N. The body is pulled out so that the length of the spring becomes 0.6 m, and is given a velocity of 4 ms^{-1} towards A. Find:

   (a) the initial total energy of the system made up of the elastic energy in the string and the initial kinetic energy; (4)

   (b) the maximum compression of the spring in the subsequent motion; (5)

   (c) the maximum extension of the spring in the subsequent motion. (1)

(Total: 10 marks)

9. A car has a mass of 600 kg and a maximum power rating of S watts. The car has a top speed of 40 ms^{-1} on a flat road, and a top speed of 24 ms^{-1} up a hill inclined at an angle θ to the horizontal, where $\sin\theta = 0.2$.

   Given that the resistance to the motion of the car is R N, where R is constant, write down two equations for S and R. Hence find the value of S and the value of R.

(Total: 10 marks)

10. A light framework ACD consists of three rods, AC, CD and AD smoothly jointed together at A, C and D, with $\angle \text{ACD} = 90^\circ$ and $\angle \text{CAD} = \angle\text{CDA} = 45^\circ$. The framework is smoothly hinged at A to a vertical wall, and is held in a vertical plane with AD horizontal and with C above AD, by a fourth rod CB, which is horizontal and which is smoothly hinged to the framework at C, and to the wall at B. B is vertically above A, and BC and AD are both horizontal. The system carries a vertical load of 100 N at D, and is in equilibrium.

    Find the reactions at A and B, and the forces in the rods, stating whether they are in tension or in compression.

(Total: 10 marks)