MATRICULATION AND SECONDARY EDUCATION CERTIFICATE

EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL

2018 SECOND SESSION

SUBJECT:	**Applied Mathematics**
DATE:	6th September 2018
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

Credit will be given to neat, clearly labelled diagrams.

Unless otherwise stated, **i** and **j** are unit vectors along the x- and y- axes of a Cartesian system of coordinates.

(Take $g = 10\ \text{ms}^{-2}$)

1. ABCDE is a regular pentagon of side 2 m. The vertex A lies at the origin of a Cartesian system of coordinates, with AB lying along the positive x-axis, and with the polygon occupying the region above the x-axis. Forces of 5 N act along $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AD}$. You can assume that the internal angle of a regular pentagon is 108°.

   (a) Express the forces in **i**, **j** notation, and find their resultant. (5)

   (b) Find where the line of action of the resultant cuts AB. (3)

   (c) Find the Cartesian equation of this line of action. (2)

(Total: 10 marks)

2. A child projects a toy railway truck P of mass 1 kg at 1 m s^{-1} along a smooth horizontal straight track so that it collides and couples to an identical truck Q which is stationary and free to move.

   (a) Find the speed of the combined pair of trucks. (3)

   (b) The combined pair of trucks subsequently collides with a third truck, also of mass 1 kg, which is stationary before the impact, and all three trucks then move together. Find the total kinetic energy of the system after this collision, and the ratio of this kinetic energy to the initial kinetic energy of the system. (4)

   (c) Find the impulse generated in the collision in part (b). (3)

(Total: 10 marks)

3. Two uniform square laminae, each of side 3 m, are joined together to form a rectangular lamina, 6 m by 3 m. The common edge of the squares is denoted by OP. The squares are not made of the same material, and the mass per unit area of one of them is twice that of the other.

   (a) Find the coordinates of the centre of gravity of the composite body from OP. (7)

   (b) The composite body is then suspended freely from O. Find the angle which OP makes with the downward vertical. (3)

(Total: 10 marks)

4. A *non-uniform* beam AB, of weight 20 N and length 4 m, has end A freely hinged to a vertical wall. A light string linking B to a point on the wall above A, makes an angle of 60^o with BA, and allows the beam to rest horizontally in equilibrium. The tension in the string is 12 N.

   (a) Draw a diagram showing the forces acting in the system. (2)

   (b) Find the magnitude and direction of the reaction at A. (6)

   (c) Find the distance from A to the centre of gravity of the beam. (2)

(Total: 10 marks)

5. Charles throws a ball from the top of a cliff 50 m high, with a speed of V m s^{-1} at an angle of 30^{o} to the horizontal. A coordinate system is taken, having O as origin at the base of the cliff, and having the x-axis horizontal, and the y-axis vertical. Find:

   (a) the velocity and displacement of the ball at time t seconds after projection in terms of V, **i** and **j**; (4)

   (b) the Cartesian equation of the path of the ball in terms of V, x and y; (3)

   (c) the value of V if the ball is to be intercepted by Philomena, who is in a stationary boat 40 m from the bottom of the cliff. (3)

(Total: 10 marks)

6. A light elastic string of natural length 0.3 m has one end fixed to a point on a ceiling. To the other end of the string is attached a particle of mass 0.2 kg. When the particle is hanging in equilibrium, the length of the string is 0.4 m.

   (a) Find the modulus of elasticity of the string. (2)

   (b) A horizontal force is applied to the particle so that it is held in equilibrium with the string making an angle α with the downward vertical. The length of the string is now 0.45 m. Find α to the nearest degree. (6)

   (c) Find the elastic energy stored in the string in part (b). (2)

(Total: 10 marks)

7. A light framework ABC consists of three identical light rods, AB, BC and CA smoothly jointed together at A, B and C to form an equilateral triangle. The framework rests in a vertical plane with AB resting horizontally on supports at A and B, and with C above AB. The framework carries a load W at C.

   Find the reactions at A and B, and the forces in the rods, stating whether they are in tension or in compression.

(Total: 10 marks)

8. A bend in a railway track forms a horizontal circular arc of radius 1.25 km. A train of mass 40 tonnes travels round the bend at a constant speed of 63 km h^{-1}.

   (a) If the rails are at the same horizontal level, find the force exerted on the sides of the rails. (4)

   (b) If the rails are banked at a certain angle α to the horizontal, the force on the side of the rails is reduced to zero. Find the angle α. (5)

   (c) In part (b), find the height of the outer rail above the inner rail, if the distance between the rails is 2 m. (1)

(Total: 10 marks)

9. A train of mass 100 tonnes travels along a horizontal track with its engines developing a constant power of 60 kW.

   (a) If the greatest speed that the train can attain along the track is 108 km h^{-1}, find the magnitude of the resistance to the motion. (5)

   (b) Find the acceleration of the train when it travels at 3 km h^{-1} down a track inclined at 5^o to the horizontal. You can assume that the engines are working at the same rate as before, and that the resistance is unchanged from part (a). (5)

(Total: 10 marks)

10. A uniform rod AB of weight W and length 2 m rests with its end A in contact with rough horizontal ground at an angle of 60^o to the vertical. It is held in this position by a force T perpendicular to the rod and acting at a point N on AB, where AN = 1.5 m.

    (a) Draw a neat and clearly labelled diagram of the system, showing all the forces acting on it. (3)

    (b) Find the force T, the normal reaction and the frictional force in terms of W. (5)

    (c) If the rod is on the point of slipping, find the coefficient of friction, μ, between the rod and the ground. (2)

(Total: 10 marks)