IM02.19m

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE

EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL

2019 FIRST SESSION

SUBJECT:	**Applied Mathematics**
DATE:	7th May 2019
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, **i**, **j** are unit vectors along the x- and y- axes of a Cartesian system.

(Take $g = 10\ \text{ms}^{-2}$)

1. A cylinder of weight W rests in the angle between a smooth vertical wall and a smooth plane inclined at 60° to the wall.

   (a) Draw a neat and clearly labelled diagram of the system, showing all the forces acting on the cylinder. (3)

   (b) Using Lami's theorem or otherwise, find the forces acting on the cylinder. (7)

(Total: 10 marks)

2. ABCD is a square of side 1 m with its vertices A, B, C, D having Cartesian coordinates $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ respectively. Three forces act on the square as follows: a force of F_1 N acts along $\overrightarrow{AB}$, F_2 N acts along $\overrightarrow{CB}$, and 4 N act along $\overrightarrow{CD}$.

   (a) Given that the line of action of the resultant is given by $3y = 2x + 6$, find the values of F_1 and F_2. (6)

   (b) A couple of moment 3 Nm, in the sense DCBA, is now introduced to the system. Show that the resultant is unchanged, but that its line of action now passes through the origin. (4)

(Total: 10 marks)

3. A rectangular lamina ABCD, of uniform material, has sides AB = 3 m and AD = 1 m. E is a point on DC such that EC = 2 m. The triangular portion BCE is then cut off from the lamina.

   (a) Find the distance of the centroid of the trapezium ABED from AD and AB. (8)

   (b) The trapezium ABED is then suspended freely from the point A. Find the angle which AB makes with the downward vertical. (2)

(Total: 10 marks)

4. A coordinate system has the x-axis horizontal, and the y-axis vertical. A and B are two points on level ground, with A at the origin and B at (60,0). A particle is projected from A towards B with initial velocity 30 ms^{-1} at 30° to AB. At the same instant, a particle is projected from B towards A with the same speed at 30° to BA. Find:

   (a) the velocity and displacement of the particles at time t seconds after projection in terms of **i** and **j**; (6)

   (b) the time t when the particles collide; (2)

   (c) the height of the particles above the ground at the instant of collision. (2)

(Total: 10 marks)

5. A particle moving in a straight line OD with uniform retardation leaves point O at time $t = 0$ s, and comes to instantaneous rest at D. On its way to D, the particle passes points A, B, C at times $t = 1, 2, 4$ seconds respectively after leaving O, with AB = BC = 72 m.

   (a) Draw a velocity-time diagram describing this motion. (2)

   (b) Find the initial velocity of the particle and its retardation. (5)

   (c) Find the length of OA, the time to reach D, and the length of CD. (3)

   (Total: 10 marks)

6. A car of mass 1000 kg has a maximum speed of 15 ms^{-1} up a slope inclined at an angle θ to the horizontal, where $\sin\theta = 0.2$.

   There is a constant frictional resistance equal to $\frac{1}{10}$ of the weight of the car.

   (a) Find the maximum power of the car. (4)

   (b) Find the maximum speed of the car on a level road. (1)

   (c) If the car descends the same slope with the engine working at half its maximum power, find the acceleration of the car at the moment when its speed is 30 ms^{-1}. (5)

   (Total: 10 marks)

7. Two particles P and Q, of mass 2 kg and 3 kg respectively, are connected by a light inelastic string which passes over a smooth fixed pulley. At time $t = 0$ s, the system is released from rest with the string taut and the hanging parts vertical. At time $t = \frac{1}{10}$ s, the particle P picks up a stationary particle of mass 1 kg, which adheres to P.

   You can assume that there is no recoil, that the string always remains taut, and that the particles move with the same velocity after impact.

   (a) Find the speed of P and Q just before impact. (4)

   (b) Show that the common speed of the particles just after impact is $\frac{1}{6}$ ms^{-1}. (4)

   (c) Find the loss of kinetic energy of the system due to the impact. (2)

   (Total: 10 marks)

8. A triangle ABC has AC = BC and $\angle$ C = 1 right angle. D is the midpoint of AB.

   A framework consists of five light rods AD, DB, BC, CA and CD, which are smoothly jointed together, and which have the configuration described above. The system lies in a vertical plane, with ADB horizontal, and with C vertically above D.

   The system rests on two supports at A and B, and carries a load of 300 N at C.

   Find the reactions at A and B, and the forces in the rods, stating whether they are in tension or in compression.

   (Total: 10 marks)

9. A uniform lamina has the form of a semicircle of radius a. The lamina rests in a vertical plane with its curved edge in contact with a smooth vertical wall and rough horizontal ground.

   It can be assumed that the vertical plane containing the lamina is perpendicular to both the wall and the ground, and that the centroid of the lamina is at a distance $\frac{4a}{3\pi}$ from its centre.

   The coefficient of friction between the lamina and the ground is $\frac{1}{5}$.

   Find the inclination of the straight edge of the lamina to the vertical when it is at the point of slipping.

   (Total: 10 marks)

10. An elastic string AB of natural length a and modulus of elasticity $2mg$, has one end, A, fixed. A particle of mass m is attached to the end B and performs horizontal circles with angular velocity $\sqrt{3g/4a}$. Find:

    (a) the extension in the string; (5)

    (b) the cosine of the angle between the string and the vertical; (3)

    (c) the elastic energy stored in the string. (2)

    (Total: 10 marks)