IM02.19s

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE

EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL

2019 SECOND SESSION

SUBJECT:	**Applied Mathematics**
DATE:	5th September 2019
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

Credit will be given to neat, clearly labelled diagrams.

Unless otherwise stated, **i** and **j** are unit vectors along the x- and y- axes of a Cartesian system of coordinates.

(Take $g = 10\ \text{ms}^{-2}$)

1. ABC is an equilateral triangle of side 2 m. D and E are the midpoints of AB and BC respectively. Rectangular Cartesian axes are taken, with A at the origin, AB along the positive x-axis, and C in the first quadrant. A force of 2 N acts along $\overrightarrow{AC}$, and forces **P** and **Q** act along $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively.

   (a) Express the forces in **i**, **j** notation, and find their resultant. (4)

   (b) Find **P** and **Q** if the system reduces to a couple. (2)

   (c) Find **P** and **Q** if the system reduces to a single force along DE. (4)

(Total: 10 marks)

2. A uniform square lamina ABCD has side 2 cm and mass 0.4 kg. The centre of the lamina is O, and E and F are the midpoints of BC and CD respectively. The portion OECF is removed from the lamina, and a particle of mass 0.3 kg is attached to E. Rectangular Cartesian axes are taken with A at the origin, and with AB and AD along the positive x- and y- axes respectively.

   (a) Find the coordinates of the centroid of this system relative to these axes. (7)

   (b) The composite body is then suspended freely from A. Find the angle which AB makes with the downward vertical. (3)

(Total: 10 marks)

3. Particles A and B have masses 1 kg and 2 kg respectively. They are at rest on a smooth horizontal table, with AB perpendicular to a wall at a distance of 1 m from B. The particle A is then projected towards B with velocity 1 ms^{-1}, which proceeds to hit the wall, bounce back and collide again with particle A.

   It can be assumed that all collisions are direct, occurring along the initial direction of AB, and that the coefficient of restitution for all impacts is 0.5.

   (a) Show that the velocity of A is zero after the first collision. (4)

   (b) Find the velocity of B just before and just after it collides with the wall. (2)

   (c) Find the time elapsed between the first and second collisions between A and B. (4)

(Total: 10 marks)

4. A light framework ABCD consists of four identical light rods, AB, BC, CD and DA arranged in the form of a square, together with a fifth rod BD, which is a diagonal of the square. The framework is freely suspended from A and carries a weight of 10 N at C. The system is in equilbrium, with A above C, and with BD horizontal.

   Find the reaction at A, and the forces in the rods, stating whether they are in tension or in compression.

(Total: 10 marks)

5. A particle is projected from a point on horizontal ground with a speed of 24 ms^{-1} at an angle of 60^o to the horizontal. A wall is situated 20 m away from the point of projection.

   (a) Express in **i** and **j** notation the velocity and displacement of the particle at a time t after projection. (4)

   (b) The particle hits the wall at a height h above the ground. Find h. (4)

   (c) Find the speed of the particle when it hits the wall. (1)

   (d) Find the direction of the motion of the particle when it hits the wall. (1)

(Total: 10 marks)

6. A wooden block A of mass 2 kg rests on a rough horizontal table. It is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a particle B of mass 0.5 kg, which is hanging freely. It can be assumed that the total resistance to the motion of the block is constant and equal to F N.

   (a) The system is released from rest with B at a height of 1 m above the floor. If B hits the floor 2.5 s later, find the acceleration of B while it is falling, and the speed at which it hits the floor. (4)

   (b) By applying Newton's Second Law and using the results obtained in part (a), find the value of F and the tension in the string while B is falling. (6)

(Total: 10 marks)

7. A bend in a road is in the shape of a circular arc of radius r, and is banked at 20^o to the horizontal. A truck, travelling around the bend at 20 ms^{-1}, is about to slip down this slope. The coefficient of friction between the tyres of the truck and the road is 0.2.

   (a) Modelling the truck as a particle, draw a diagram showing **all** the forces acting on it. (3)

   (b) Find the value of r. (7)

(Total: 10 marks)

8. Two smooth planes inclined at angles of 40^o and 50^o to the horizontal intersect in horizontal line. A cylinder of mass 1 kg rests on the planes, with its axis parallel to the line of intersection of the planes.

   (a) Draw a diagram showing the forces acting on the cylinder. (4)

   (b) Using Lami's theorem, or otherwise, find the reactions exerted by the planes on the cylinder. (6)

(Total: 10 marks)

9. A uniform ladder of mass 30 kg rests with its lower end on rough horizontal ground, and its upper end against a smooth vertical wall. Maia, of weight 60 kg, stands on the top of the ladder. The coefficient of friction between the ground and the ladder is 5/12. It can be assumed that the ladder rests in a vertical plane perpendicular to the wall, and that the system is in limiting equilibrium.

   (a) Draw a neat and clearly labelled diagram of the system, showing **all** the forces acting on it. (3)

   (b) Find, to the nearest degree, the angle between the ladder and the horizontal. (7)

(Total: 10 marks)

10. A particle of mass 2 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 100 N. The other end of the string is attached to a fixed point O on a rough horizontal plane. There is a constant frictional resistance of 5 N to the motion of the particle on the plane. The particle is projected from O with velocity 5 ms^{-1}. Find:

   (a) the greatest distance from O achieved by the particle; (6)

   (b) the speed of the particle when it returns to O. (4)

(Total: 10 marks)