MATRICULATION AND SECONDARY EDUCATION CERTIFICATE

EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL

2020 FIRST SESSION

SUBJECT:	**Applied Mathematics**
DATE:	27^{th} April 2020
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, **i**, **j** are unit vectors along the x- and y- axes of a Cartesian system.

(Take $g = 10 \text{ ms}^{-2}$)

1. A lamina consists of a square ABCD of side 1 m, which is attached along the side BC to an isosceles triangle BCE, with base BC and a height of 1.6 m, with E and AD on opposite sides of BC. The material of the square weighs 4 kg/m^2, whilst that of the triangle weighs 2 kg/m^2.

   (a) Find the distance of the centroid of the lamina from AD. (7)

   (b) The lamina is suspended freely from C, and a weight W is attached to E, so that AB is horizontal at equilbrium. Find the value of W. (3)

(Total: 10 marks)

2. ABCD is a square of side $2a$, with A at the origin, and AB and AD along the x- and y-axes of a Cartesian coordinate system. Forces F, $2F$, $3F$ and $4F$ act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ of the square. These forces are to be replaced by three forces acting along the sides of the triangle ABC.

   (a) By taking moments about A, B and C, or otherwise, find in terms of F the magnitude and sense of each of these forces. (5)

   (b) If the force acting along AC is reversed, find the distance from A of the point where the line of action of the resultant now cuts AB, produced if necessary. (3)

   (c) Find the Cartesian equation of this line of action. (2)

(Total: 10 marks)

3. A tennis ball is projected with a velocity of 25 ms^{-1} from a point O on level ground at an acute angle α to the horizontal. Whilst travelling horizontally, it hits a smooth vertical wall which is 20 m from O.

   (a) By equating the time to cover the horizontal distance to the wall, and the time at which the vertical velocity is zero, or otherwise, verify that the angle of projection $\alpha = 20^{\circ}$, to the nearest degree. (5)

   (b) Using this value of α, show that the ball hits the wall at a height of 3.66 m. (2)

   (c) If the coefficient of restitution between the ball and the wall is $1/2$, find the distance from the foot of the wall of the point where the ball hits the ground. (3)

(Total: 10 marks)

4. A cyclist and his bicycle have a total mass of 100 kg. When travelling up a hill inclined at $\sin^{-1}(1/50)$ to the horizontal against a resistance to motion of 20 N, the cyclist can maintain a speed of 12 km/hr.

   (a) Find the rate at which he is working. (4)

   (b) If the resistance to motion is unchanged, find, in ms^{-2}, the acceleration of the cyclist when travelling at 10 km/hr on a level road, and working at the same rate. (4)

(Total: 8 marks)

5. A block of wood of weight W rests on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the plane and the block is $1/2$. A horizontal force P acts on the block, which is on the point of moving up the plane.

   Find P in terms of W. **(Total: 8 marks)**

6. A framework consists of three identical light rods AB, BC and CA smoothly jointed together to form an equilateral triangle ABC. The framework rests in a vertical plane on supports at A and B, with AB horizontal, and with C above AB. The framework carries a weight W at C.

   Find the reactions at A and B, and the forces in the rods, stating whether they are in tension or in compression.

   (Total: 9 marks)

7. Two light inextensible strings each of length 1 m are attached to a particle of mass 2 kg at B. The other ends A and C are fixed to two points in a vertical line, such that A is distant 1 m above C. The particle describes a horizontal circle with $\omega = 50$ rad/s. Find:

   (a) the tension in the strings; (8)

   (b) the least value of ω such that both strings shall be taut. (3)

   (Total: 11 marks)

8. A light elastic string has modulus of elasticity 20 N and natural length L metres. One end of the string is attached to a fixed point A, whilst a particle of mass 2 kg is attached to the other end of the string. The mass is placed at A, and released from rest. The maximum depth reached by the particle below A is 5 m. Using conservation of mechanical energy, find:

   (a) the natural length L of the string; (6)

   (b) the velocity of the particle when it is 4 m below A. (6)

   (Total: 12 marks)

9. A particle of mass 10 kg moves under the action of a force **F** with velocity

$$\mathbf{v} = (3t + 2)\mathbf{i} + (4 - t^2)\mathbf{j} \quad \text{ms}^{-1}$$

where t is the time in seconds.

(a) Find **F** in terms of t. (2)

(b) Find the displacement $\mathbf{r}(t)$ of the particle in terms of t, given that

$$\mathbf{r}(0) = \mathbf{j} \quad \text{when } t = 0. \tag{3}$$

(c) Find the work done by the force over the interval $0 \le t \le 1$. (3)

(d) Show that this work is equal to the kinetic energy gained by the particle over this interval. (2)

(Total: 10 marks)

10. A gun of mass 2000 kg fires a shell of mass 25 kg. The shell travels along the barrel of the gun with a velocity of 500 ms^{-1} relative to barrel. Find the speed with which the gun recoils if:

(a) (i) the barrel is horizontal; (5)

(ii) the barrel is inclined at 30° to the horizontal. (5)

(b) In each case, find the constant force required to bring the gun to rest in 1.5 s. (2)

Hints: It can be assumed that the gun and barrel recoil with the same velocity.
The velocity of B relative to A is defined as velocity of B − velocity of A.
In this problem, consider the conservation of horizontal momentum.

(Total: 12 marks)