L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2020 SECOND SESSION

SUBJECT:	**Applied Mathematics**
DATE:	12th December 2020
TIME:	16:00 to 19:05

Directions to Candidates

Answer **ALL** questions. There are 10 questions in all.
Each question carries 10 marks.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are **not** allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.
In the paper, **i**, **j** are unit vectors along the x- and y-axis of a Cartesian system.

1. (a) The edges of a uniform square lamina $ABCD$ are each of length $3l$. A portion of the lamina in the form of a square $APQR$, where P lies on AB, R lies on AD, and AP is of length l, is removed. Find the distances of AB and BC from the centroid of the remainder of the lamina.

[8 marks]

(b) This remainder of the lamina is suspended freely from B. Show that, if α is the angle of inclination of BC to the vertical, then $\tan\alpha = 11/13$.

[2 marks]

2. Let $OABC$ be a rectangle, where O is the origin, and OB and OC are the x-axis and y-axis respectively. Let A be the point $(2,0)$ and B the point $(2,1)$. Forces of P, Q and R newtons act along OA, AB and BC in the directions $\overrightarrow{OA}$, $\overrightarrow{AB}$ and $\overrightarrow{BC}$, respectively. Their resultant lies along the line $x+2y=7$. Find:

(a) the magnitude of the resultant in terms of P;

[6 marks]

(b) the moment of a couple which when added to the system would transfer the resultant to the line $x+2y=9$.

[4 marks]

3. Three particles A, B and C of mass m_1, m_2 and m_3 respectively, lie in that order in a straight line on a smooth horizontal plane. The coefficient of restitution between each pair of particles is e. Particle A is projected directly towards B with velocity u and comes to rest after striking it. Particle B in turn strikes C and also comes to rest. Find:
   (a) the masses m_2 and m_3 in terms of m_1 and e;

   [4 marks]

   (b) the total kinetic energy lost after C has been set in motion, in terms of m_1, u and e;

   [4 marks]

   (c) the impulsive action between each pair of particles.

   [2 marks]

4. (a) A particle has an initial velocity u at an angle α to the horizontal. Show that the equation of its trajectory can be expressed in the form

$$y = x\tan\alpha - \frac{g\,x^2\sec^2\alpha}{2u^2}.$$

   [2 marks]

   (b) Two particles are projected in the same vertical plane from the point O with the same speed of $\sqrt{ag}$ in directions making acute angles β and $45°$. If their paths cross at the point P, show that:
      (i) the horizontal distance from O to P is $2a/(1+\tan\beta)$;

      [4 marks]

      (ii) the point P is at a higher level than O if $\tan\beta > 1$.

      [4 marks]

5. A uniform rod BC of weight W and length $4l$, has the end B smoothly hinged at a fixed point. A light inextensible string has one end attached at a point which is at a distance $4l$ vertically above B, and the other end tied to a light ring D, which is threaded on the rod BC. When $BD = 3l$, the rod BC is horizontal and the frictional force between the ring and the rod is limiting.
   (a) Draw a diagram showing the forces acting on the ring D and find the coefficient of friction between the rod and the ring.

   [4 marks]

   (b) Draw a second diagram showing the forces acting on the rod BC, and find the magnitude and direction of the reaction at the hinge and the tension in the string.

   [6 marks]

6. (a) A particle P of mass m is connected by a light elastic string of natural length $4a$ and modulus of elasticity λ to a fixed point Q. A horizontal force kmg acts on the particle maintaining it in equilibrium with $QP = 5a$ and the string inclined at an angle $\theta = \cos^{-1}(3/5)$ to the downward vertical. Show that $\lambda = 20mg/3$, and find the tension in the string and the value of k.

[5 marks]

(b) The horizontal force kmg is removed and the particle is made to rotate with constant angular speed in a horizontal circle with the string inclined at an angle ϕ with the downward vertical. Find the radius of this circle in terms of a, and show that the particle will complete one revolution in time $2\pi\sqrt{(3a/g)}$.

[5 marks]

7. (a) A car is moving along a level road at a steady speed of $72\,km/h$ against constant resistances which total $1250\,N$. Calculate the power, in kW, at which the engine of the car is working.

[3 marks]

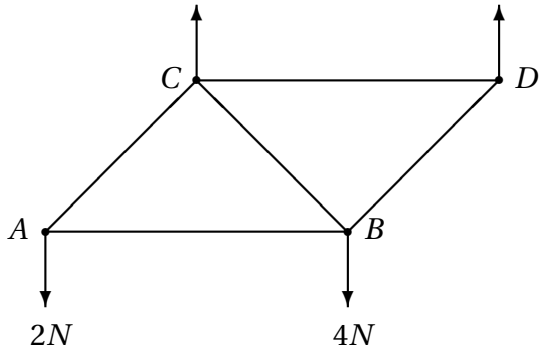
(b) The car climbs a hill whose inclination to the horizontal is $\sin^{-1}(1/n)$ against the same total non-gravitational resistance. The mass of the car is $1500\,kg$ and its engine is working at $30\,kW$. When the speed is $36\,km/h$, the acceleration of the car is $\frac{1}{3}\,m/s^2$. Find the value of n.

[4 marks]

(c) Find the maximum steady speed possible when the car is moving up the hill with the engine working at $30\,kW$.

[3 marks]

8. The framework shown in the diagram consists of five equal light rods freely jointed at their ends. Loads of $2\,N$ and $4\,N$ are carried at A and B and the framework is kept in equilibrium, with AB horizontal, by vertical supporting forces at C and D.
Calculate these supporting forces and determine the forces in the rods, stating which rods are in tension and which in compression.



[10 marks]

9. (a) The variable force **F** acting at time t $(0 \leq t \leq 2)$ on a particle of unit mass is given by $\mathbf{F} = 24t^2\mathbf{i} + 6\mathbf{j}$. At time $t = 0$ the particle is at rest at the point with position vector $-2\mathbf{i} + 3\mathbf{j}$. Find the velocity and position vector of the particle at time $t = T$ $(0 \leq T \leq 2)$.

[5 marks]

(b) For time $t > 2$ the force $\mathbf{F} = 6\mathbf{j}$. Find the position vector of the particle at $t = 3$.

[5 marks]

10. (a) A particle Q of mass $2m$ is initially at rest on a smooth plane inclined at an angle θ to the horizontal. It is supported by a light inextensible string which passes over a smooth light pulley P at the top of the plane. The other end of the string supports a particle R, of mass m, which hangs freely. Given that the system is in equilibrium, find θ, and the magnitude and direction of the resultant force exerted by the string on the pulley.

[5 marks]

(b) A further particle of mass m is now attached to R and the system is released. Find, for the ensuing motion, the acceleration of R, the tension in the string and the magnitude and direction of the resultant force exerted by the string on the pulley.

[5 marks]