MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL
2021 FIRST SESSION

SUBJECT:	**Applied Mathematics**
DATE:	5^{th} June 2021
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, $\boldsymbol{i}$, $\boldsymbol{j}$ are unit vectors along the x- and y- axes of a Cartesian system.

(take g = 10 ms^{-2})

1. $PQRS$ is a square. Forces of 5 N, 2 N, 3 N, 1 N and F N act along $\overrightarrow{PQ}$, $\overrightarrow{RQ}$, $\overrightarrow{RS}$, $\overrightarrow{PS}$ and $\overrightarrow{SQ}$ respectively in the direction indicated by the order of the letters.

   a) Find F if the system reduces to a single force through P. (6)

   b) For this value of F, find the magnitude of the resultant and the angle it makes with PQ. (4)

(Total: 10 marks)

2. A mass of 3 kg is attached to two elastic strings AB and BC at B. The string AB is fixed at A and string BC is fixed at C, 2.50 m vertically below A. The strings AB and AC have natural lengths of 0.75 m and 0.50 m and modulus of elasticity, λ of 30 N and 10 N respectively. The mass is in equilibrium. Find the final lengths of strings AB and BC. (10)

(Total: 10 marks)

3. A uniform rod *AB* of mass 8 kg and length 4 m rests with *A* on a smooth horizontal ground and *B* on a smooth peg 2 m above the ground. The rod is held in this position by a horizontal force *P* at *A*. Find:

   a) the magnitude of the force *P*; (6)

   b) the magnitude of the reactions at *A* and *B*. (4)

(Total: 10 marks)

4. A square *ABCD* of side 3 cm, and uniform density of 2g cm^{-2} is placed with point *A* at the origin. Sides *AB* and *AD* are placed on the x- and y- axis respectively. A square *DEFG* of side 1 cm of density 3g cm^{-2} is glued on the square *ABCD*. Both sheets have the same thickness.

   a) Find the position of the overall centre of gravity. (7)

   b) The resulting lamina is suspended at the point *A*. Find the angle *AD* makes with the vertical. (3)

(Total: 10 marks)

5. A car is moving at a speed u ms^{-1}. The brakes of the car can produce a constant retardation of 6 ms^{-2}. When the driver decides to stop, s/he has a reaction time of $\frac{2}{3}$ s before the brakes are applied. As the car passes a point *A* the driver decides to stop.

   a) Show that the minimum distance of the car from *A* when the car comes to rest is given by:

$$\frac{u}{12}\,(u+8). \qquad (6)$$

   The driver is approaching the traffic lights and is 90 m from the traffic lights when the lights change from green to amber. The light remains amber for 3 s before changing to red. Find:

   b) the speed u; (2)

   c) the time the lights have been red before the car comes to a stop. (2)

(Total: 10 marks)

6. An elastic string has one end fixed to a point *A*. The other end *B*, which is attached to a particle of mass 3 kg is pulled vertically down from *A* until *AB* is 3.0 m and then released. The modulus of elasticity of the string is 50 N and the natural length is 1.2 m.

   a) Using the principle of conservation of energy, find the velocity of the particle when the string first becomes slack. (7)

   b) Find the distance from *A* of the particle when it first comes to rest. (3)

(Total: 10 marks)

7. A small block of mass 4 kg moves 6 m up a rough slope inclined at an angle of 25^{o} to the horizontal by means of a light inextensible string which passes over a frictionless pulley at the top of the slope. The coefficient of friction of the mass and slope is 0.25. A mass of 7 kg is hanging vertically freely on the other side of the string. The system is released from rest. Find:

a) the frictional force acting on the 4 kg mass; (3)

b) the acceleration up the slope of the 4 kg mass; (2)

c) the tension in the string; (3)

d) the time taken for the 4 kg block to move 6 m up the slope, assuming that the block does **not** reach the pulley. (2)

(Total: 10 marks)

8. Two particles *A* and *B* of masses *2m* and *m*, respectively, are moving towards each other with velocities *u* and *3u* respectively. The collision is perfectly inelastic. Find:

a) the final velocity; (5)

b) loss of kinetic energy. (5)

(Total: 10 marks)

9. A car of mass 900 kg is travelling around a circular bend of radius 40 m. The coefficient of friction between the tyre and road is 0.4.

a) If the bend is unbanked, find the maximum safe speed in km/hr to prevent slipping outwards. (4)

b) The road is now banked at 20^{o} to the horizontal. Find the new maximum safe speed for the new construction to prevent slipping outwards. (6)

(Total: 10 marks)

10. A particle is projected at an angle θ with a velocity of 28 ms^{-1} and is to hit a target 64 m away on the same horizontal level. Find:

a) at time *t*, the horizontal and vertical distance travelled in terms of *t* and θ; (3)

b) the trajectory equation of the particle; (4)

c) the **TWO** possible angles of projection. (3)

(Total: 10 marks)