L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2021 SECOND SESSION

SUBJECT:	**Applied Mathematics**
DATE:	11^{th} October 2021
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, ***i***, ***j*** are unit vectors along the *x*- and *y*- axes of a Cartesian system.

(take g = 10 ms^{-2})

1. Three forces, $\boldsymbol{F_1}$ = (3$\boldsymbol{i}$ + 4$\boldsymbol{j}$) N, $\boldsymbol{F_2}$ = (-5$\boldsymbol{i}$ + 7$\boldsymbol{j}$) N, $\boldsymbol{F_3}$ = (4$\boldsymbol{i}$ - 9$\boldsymbol{j}$) N act at $\boldsymbol{r_1}$= (2$\boldsymbol{i}$ + 4$\boldsymbol{j}$) m, $\boldsymbol{r_2}$ = (-3$\boldsymbol{i}$ + $\boldsymbol{j}$) m and $\boldsymbol{r_3}$ = (7$\boldsymbol{i}$ - 2$\boldsymbol{j}$) m respectively.

   a) Find the magnitude of the resultant force and its line of action. (7)

   b) Find the moment of the couple necessary which would make the line of action of the resultant pass through the origin. (3)

(Total: 10 marks)

2. A framework consists of two light rods *AB* and *BC* of length *2a* and *a* respectively smoothly jointed at *B*. The ends *A* and *C* are smoothly hinged to a vertical wall. When a weight *3W* is attached at *B*, the framework is in equilibrium with *BC* horizontal and *A* above *C*. Find:

   a) the reaction forces at *A* and *C* in magnitude and direction; (6)

   b) the forces in **each** rod, stating if in tension or compression. (4)

(Total: 10 marks)

3. An elastic string of natural length 3 m is fixed at one end and hangs vertically supporting a particle of mass 5 kg at the other end. The string stretches to a length of 4.2 m.

   a) Calculate the modulus of elasticity, λ of the elastic string. (4)

   A horizontal force of 37 N is then applied gradually to the particle until it is once again in equilibrium. Calculate:

   b) the angle of inclination to the vertical of the string in this position; (3)

   c) the final length of the string. (3)

(Total: 10 marks)

4. A uniform rectangular lamina *ABCD* has sides *AB* of 5 cm and *BC* of 3 cm. The sides *AB* and *AD* lie on the *x*- and *y*- axis respectively, with *A* as origin. A circle of radius 1 cm, centre $\left(\frac{7}{3}, 1\right)$ and square with vertices at the points (1,2), (2,2), (2,3) and (1,3) are cut out from the rectangle.

   a) Find the position of the centroid of the remaining lamina. (7)

   b) If the lamina is suspended from point *A*, find the angle which *AB* makes with the vertical. (3)

(Total: 10 marks)

5. Two particles *A* and *B* are moving along the same line in the same direction. *A* is 12 m behind *B*. *A* starts from rest and has an acceleration of 3 ms^{-2}. *B* has a uniform velocity of 3 ms^{-1}.

   a) How long does *A* take to reach *B*? (4)

   b) How far has *A* travelled in doing so? (2)

   c) Sketch the displacement-time and velocity-time graphs for the motion of *A* and *B*. (4)

(Total: 10 marks)

6. Two particles of masses 6 kg and 3 kg are connected by a light inextensible string which passes over a fixed light frictionless pulley. If the system is released from rest, find:

   a) the acceleration of **each** mass; (7)

   b) the tension in the string when the system is moving freely. (3)

(Total: 10 marks)

7. A cyclist and cycle of combined mass of 75 kg freewheels down a hill of slope of angle of $\sin^{-1}\left(\frac{1}{12}\right)$. The speed of the cyclist increases from 4 ms^{-1} at the top of the hill to 10 ms^{-1} at the bottom whilst travelling a distance of 120 m. Find:

a) the difference in the total energy of the cyclist and cycle between the top and bottom of the hill; (3)

b) the work done against the frictional forces whilst the cyclist freewheels and the magnitude of the average frictional force; (3)

c) the speed of the cyclist and cycle when s/he travelled 50 m from the top of the hill. (4)

(Total: 10 marks)

8. A bullet of mass 50 g is fired horizontally at a wooden block of mass 4 kg, which rests on a rough horizontal surface. The coefficient of friction between the block and surface is 0.4. As a result of the collision, the block with the bullet embedded, moves a distance of 10 m along the surface before coming to rest. Find the speed at which the bullet enters the block. (10)

(Total: 10 marks)

9. Two particles A and B of mass $2m$ and m respectively are connected by a light inextensible string of length $5l$ which passes through a small smooth hole in a smooth horizontal table. Particle A lies on the table and moves in a circular path with a constant angular speed ω around the hole, such that the string is taut and particle B hangs at rest at a depth of $2l$ below the hole. Find ω in terms of g and l. (10)

(Total: 10 marks)

10. Two particles A and B are projected simultaneously. A is projected from the top of a vertical cliff and B from the base of the cliff. Particle A is projected horizontally with a speed of $6u$ ms^{-1} and B is projected at an angle θ above the horizontal with a speed of $10u$ ms^{-1}. The height of the cliff is 56 m and the particles collide after 2 s. Find:

a) the angle of projection of particle B from the horizontal; (4)

b) the vertical distance above the base of the cliff when the particles collide; (2)

c) the value of u; (2)

d) the horizontal distance travelled from the base of the cliff by both particles at the point of collision. (2)

(Total: 10 marks)