MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2022 FIRST SESSION

SUBJECT:	**Applied Mathematics**
DATE:	3rd May 2022
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, $\boldsymbol{i}$, $\boldsymbol{j}$ are unit vectors along the x- and y- axes of a Cartesian system.

(take $g = 10\text{ ms}^{-2}$)

1. A rectangle $ABCD$ has sides $AB = 4$ cm, $BC = 3$ cm, with unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ pointing along AB and AD respectively. Forces act along the sides of the rectangle as follows: 3 N along AB, 4 N along BC, 5 N along CA, 1 N along CD and 6 N along AD.

   a) Express each force in the $(\boldsymbol{i}, \boldsymbol{j})$ notation and find the resultant of these forces. (3)

   b) Find the point where the line of action of the resultant force intersects the line AB. (3)

   c) This system of forces is equivalent to a force acting at A together with a couple. Find the magnitude and direction of this force and the moment of the couple. (4)

(Total: 10 marks)

2. A block of mass 3 kg is placed on a rough surface inclined at 43° to the horizontal. The block is in limiting equilibrium.

   a) Find the coefficient of friction between the block and the surface of the slope. (3)

   The slope is now inclined at 50° to the horizontal, and a force P is applied parallel to the slope surface. Find:

   b) the least value of P to prevent the block from slipping down the slope; (4)

   c) the greatest value of P such that the block is about to move up the slope. (3)

(Total: 10 marks)

3. A triangular framework is formed from 3 light rods, where *AB*, *BC* and *CA* are each of length *l*. *AB* is horizontal and point *C* is above *AB*. Point *A* is freely hinged to a vertical wall, and is prevented from rotating at *A* by another light rod *CD*, parallel to *AB*. *CD* is freely hinged at *D* to the same wall, vertically above point *A*. A mass of 60 kg hangs from point *B*. Find:

   a) the magnitude and direction of the reaction force, exerted on the rods by the wall at *A* and *D*; (6)

   b) the forces in **each** rod, stating whether in tension or compression. (4)

(Total: 10 marks)

4. A square *ABCD* of side 4 cm has a triangle *BCE* attached on the side *BC*. The sides *BE* and *CE* are of equal length $2\sqrt{5}$ cm. Find:

   a) the position of the centroid on the axis of symmetry and from the edge *AD*. (7)

   The whole shape is suspended from point *C*. Find:

   b) the angle between *BC* and the vertical. (3)

(Total: 10 marks)

5. A car travels along a straight horizontal road, passing two houses, *P* and *Q*. The car passes *P* at u ms^{-1} and maintains this speed for 40 s, during which time it travels 800 m. Approaching a corner, the car slows down at a uniform rate of a ms^{-2} over the next 150 m to reach a speed of 10 ms^{-1}. At this instant the car accelerates at a uniform rate of 1.25 ms^{-2}. This acceleration is maintained for 12 s until the car has reached a speed of v ms^{-1}, which is then kept constant. The car passes house *Q*, 50 s after its speed reaches v ms^{-1}.

   a) Calculate the value of *u, a* and *v*. (4)

   b) Sketch the velocity-time graph for the motion of the car between P and Q. (3)

   Find:

   c) the time taken by the car to travel this distance; (1)

   d) the distance between the houses *P* and *Q.* (2)

(Total: 10 marks)

6. A mass of 6 kg is initially at rest at the bottom of a smooth slope which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. The mass is pushed up the slope by a horizontal force of 50 N. Find:

   a) the reaction between the mass and the slope; (3)

   b) the acceleration up the slope; (3)

   c) the distance the mass will travel in the first 5 s; (2)

   d) the velocity of the mass after 5 s. (2)

(Total: 10 marks)

7. A smooth heavy bead of mass *m* is threaded on a smooth wire in the shape of a circle of radius 1.2 m and centre *C*. The circular wire is in a vertical plane with the bead at rest at the lowest point *A* of the circle. The bead is given a speed of 6 ms^{-1}. Find:

a) the height above *A* when it first comes to rest; (6)

b) the velocity of the bead when it is in line horizontally with the centre of the circle *C*. (4)

(Total: 10 marks)

8. Two particles each of mass *m* kg are projected towards each other with speeds of *6u* and *2u*. The coefficient of restitution for the impact is *e*.

a) Find the velocities of the particles after impact in terms of *u* and *e*. (5)

b) If $e = \frac{3}{5}$, show that the particles reverse their direction. (2)

c) Find the impulse of the particles in terms of *m* and *u*. (3)

(Total: 10 marks)

9. A light inextensible string *AB* of length 6 m has a particle *P* of mass 2 kg attached to its midpoint. The ends *A* and *B* are attached to two fixed points in a vertical plane. *A* is 3 m vertically above *B*. The particle *P* is rotating in a horizontal circle. If the strings are to remain taut, find the minimum angular speed of particle *P*. (10)

(Total: 10 marks)

10. A particle is projected from a point A on a horizontal plane and has an initial velocity *u* at an angle θ above the plane. Show that:

a) the time of flight is given by: $\frac{2u\ \sin\theta}{g}$; (3)

b) the horizontal range is given by: $\frac{u^2 \sin 2\theta}{g}$. (3)

If the particle is projected at 70 ms^{-1} at an angle of 10° above the plane, find:

c) the time of flight and range; (2)

d) the maximum horizontal range and the angle of projection for maximum range. (2)

(Total: 10 marks)