L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2022 SECOND SESSION

SUBJECT:	**Applied Mathematics**
DATE:	2nd September 2022
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, ***i***, ***j*** are unit vectors along the *x*- and *y*- axes of a Cartesian system.

(take $g = 10\ \text{ms}^{-2}$)

1. A square *ABCD* has sides of length 3 m with *AB* along the x-axis and *AD* along the y-axis, in the ***i*** and ***j*** direction respectively. Forces acting along the sides are 2 N along *BA*, 3 N along *BC*, $5\sqrt{2}$ N along *BD*, 1 N along *DA,* and 4 N along *CD.*

   a) Express each force in the (***i, j***) notation and find the magnitude of the resultant and the angle it makes with the *x*-axis. (3)

   b) Taking moments about *A*, or otherwise, find where the resultant intersects the *x*-axis. (3)

   A force ***F*** is applied to reduce the system to equilibrium.

   c) Find the cartesian equation of the line of action of ***F*****.** (4)

(Total: 10 marks)

2. A block of mass m rests on a smooth plane inclined at an angle θ to the horizontal. The block is attached to an elastic string AB, of natural length l and modulus of elasticity, λ. at A. The end B is attached to a wall further up the inclined plane. The elastic string extension is x. The system is in equilibrium.

   a) In terms of l, m, λ and θ, show that the string has a stretched length given by: (7)

   $$l\left(\frac{\lambda + mg\sin\theta}{\lambda}\right)$$

   b) The mass is 2 kg, the natural length l of the elastic string is 2.1 m, the modulus of elasticity is 20 N and the plane is inclined at an angle of 27^o to the horizontal. Find the extension. (3)

(Total: 10 marks)

3. A uniform ladder of length 4 m and mass of 15 kg leans at rest in limiting equilibrium against a rough vertical wall so that the ladder is at an angle θ to the horizontal. The ladder remains at rest with its base on the rough horizontal ground. The coefficient of friction at the wall and ground of the ladder are 0.6 and 0.25 respectively. By taking moments or otherwise, find:

   a) the angle which the ladder makes with the ground; (6)

   b) the reaction and frictional forces acting on the ground and wall. (4)

(Total: 10 marks)

4. An equilateral triangle ABC of side 2 m, has AB lying on the x-axis and the y-axis passes through the midpoint of AB and point C. A semi-circle of radius 0.5 m and centre at the midpoint of AB is cut out from the triangle. Find:

   a) the position of the centroid of the resultant lamina. (7)

   The lamina is suspended from point A. Find:

   b) the angle that the side AB makes with the vertical. (3)

   Note: Centroid of a semi-circle of radius r from the straight edge is given by $\frac{4r}{3\pi}$.

(Total: 10 marks)

5. A ball A is thrown vertically upwards with a speed of 20 ms^{-1}. After 3 s a second ball B is thrown upwards from the same point at a speed of 15 ms^{-1}. Find:

   a) the time which passes between the second ball being thrown and the collision between the balls; (6)

   b) the maximum height reached by the ball A and the time to reach this maximum height. (4)

(Total: 10 marks)

6. A particle *A* of mass 3 kg lies on a rough horizontal table of coefficient of friction of 0.3. Particle *A* is connected to a second particle *B* of mass 7 kg with a light inextensible string and hangs over a frictionless pulley at the edge. All lie in the same vertical plane. The system is released and starts to move. Find:

   a) the acceleration of both masses; (7)

   b) the tension in the string. (3)

(Total: 10 marks)

7. A car of mass 1000 kg pulls a caravan of mass 600 kg. The resistance to motion of the car and caravan are 200 N and 300 N respectively. Find:

   a) the power output of the car engine when travelling at maximum speed of 18 ms^{-1} on level ground. (3)

   The car and caravan are now travelling at 8 ms^{-1} up a hill inclined at $\theta = \sin^{-1}\left(\frac{1}{50}\right)$. If the resistance is constant and the engine is working at full power, find:

   b) the acceleration; (4)

   c) the tension in the coupling between the car and caravan. (3)

(Total: 10 marks)

8. Particles *A* and *B* are in line at rest on a smooth horizontal surface in front of a wall. The masses of *A* and *B* are 5 kg and 2 kg respectively. The coefficient of restitution between the particles *A* and *B* is 0.4 and between *B* and the wall is 0.6. Particle *A* is projected towards *B* at a speed of 4 ms^{-1}. Particle *B* goes on to hit the wall directly. Find:

   a) the velocity of *B* after each impact; (6)

   b) the loss of kinetic energy after each impact. (4)

(Total: 10 marks)

9. A car is travelling at speed on a circular track of radius *r* and banked at an angle θ to the horizontal.

   a) If *v* is the maximum speed that would tend the car to slip up the rough incline of coefficient of friction μ, show that:

$$\mu = \frac{v^2 - r\,g\,\tan\theta}{r\,g + v^2\,\tan\theta} \quad (7)$$

   b) The car is travelling at a speed of 50 $kmhr^{-1}$ on a banked track of radius 30 m inclined at $\tan^{-1}\left(\frac{5}{12}\right)$ without slipping up the track. Find the coefficient of friction between the tyres and the track. (3)

(Total: 10 marks)

Questions continues on next page.

10. A particle is projected from the top of a tower with a velocity of 40 ms^{-1} at an angle of $\sin^{-1}\left(\frac{3}{5}\right)$ above the horizontal. The height of the tower is 50 m. Find:

a) the time taken for the particle to hit the ground; (5)

b) the horizontal distance travelled from the base of the tower where it hits the ground; (2)

c) the speed of the particle as it hits the ground and the angle it makes to the horizontal. (3)

(Total: 10 marks)