MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL
2023 FIRST SESSION

SUBJECT:	**Applied Mathematics**
DATE:	2nd May 2023
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, ***i***, ***j*** are unit vectors along the x- and y- axes of a Cartesian system.

(take g = 10 ms^{-2})

1. A regular hexagon *ABCDEF* of side a, is in the x-y plane with A at the origin. The side AB lies on the positive x-axis and unit vector ***i-*** direction, whilst AE lies on the positive y-axis and unit vector ***j***- direction. Forces act on the hexagon as follows:

   6 N along AB, 8 N along BC, 10 N along BD and 12 N along FA.

   All forces act in the direction implied by the letters.

   a) Express **each** force in the ***i, j*** notation. (5)
   b) Find the additional force ***F*** required to reduce the system to equilibrium. (2)
   c) By taking moments about A, find the point on the x-axis where the force ***F*** acts. (3)

(Total: 10 marks)

2. Two points *A* and *B* on the same horizontal line are *2L* apart. A mass of *6M* is attached at *C* to two identical elastic strings *AC* and *BC* of natural length *L*. The mass rests in equilibrium at *3L/4* below the level of *AB*. Determine:

   a) the tension in either string; (6)
   b) the modulus of elasticity of either string. (4)

(Total: 10 marks)

3. A rod *AB* of mass 4 kg is non-uniform, i.e. the centre of mass does not act at the midpoint of the rod *AB*. The 4 kg mass of the rod acts at *G*, *s* m away from the side *A*. The rod *AB* is of length 6 m and rests horizontally in equilibrium. The rod *AB* is supported by two strings attached at the ends *A* and *B* of the rod. The strings make an angle of 30° and 45° with the horizontal with the ends *A* and *B* respectively.

   a) Sketch the free body diagram. (2)
   b) Find the tensions in **each** of the strings. (5)
   c) Determine the position of the centre of mass of the rod. (3)

(Total: 10 marks)

4. A uniform lamina *ABCEF* is obtained from a rectangle *ABCD*, with *AB* = *CD* = 4 cm and *BC* = *AD* = 3 cm, by removing the triangle *EDF*, where *E* and *F* lie on *CD* and *AD* respectively, with *CE* = 1 cm and *AF* = 1.5 cm.

   a) Find the distances of the centre of mass of the lamina *ABCEF* from *AB* and *AD*. (8)

   The lamina is freely suspended from *F* and hangs in equilibrium under gravity.

   b) Find the angle which *AF* makes with the vertical. (2)

(Total: 10 marks)

5. A truck and a car are at rest and level with each other at a set of traffic lights on a straight road. When the lights change, the car and truck move off at the same time.

   The truck accelerates with a constant acceleration until it reaches a top speed of 20 ms^{-1}. It then continues at this constant speed.

   The car accelerates with a constant acceleration for 25 s until it reaches a top speed of *V* ms^{-1}, where $V > 20$ ms^{-1}. It then continues at the constant speed. The car draws level with the truck when the truck has been travelling for 25 s at its top speed.

   The distance travelled by each vehicle is 600 m.

   a) Find the time taken for which the truck is accelerating. (3)
   b) Find the top speed, *V* of the car. (2)
   c) On the same diagram sketch the velocity-time graphs to illustrate the motion of the two vehicles from the time they start moving to the time when the car overtakes the truck. (5)

(Total: 10 marks)

6. Two particles P and Q of masses 7 kg and 4 kg respectively are connected by a light inextensible string. Particle P rests on a rough horizontal table. The string passes over a light smooth pulley fixed at the edge of the table and Q hangs vertically. The coefficient of friction between P and the table is 1/4. The system is released from rest. Find in terms of g:

a) the acceleration of Q; (4)
b) the tension in the string; (3)
c) the force exerted on the pulley. (3)

(Total: 10 marks)

7. A particle of mass 6 kg is pulled at constant speed a distance of 30 m up a rough plane which is inclined at 40° to the horizontal. The coefficient of friction between the particle and the surface is 0.3. Assuming the particle moves up the slope, find:

a) the work done against friction; (6)
b) the work done against gravity. (4)

(Total: 10 marks)

8. Two particles A and B of mass 0.3 kg and 0.7 kg respectively are moving towards each other along the same straight line on a smooth horizontal table. Particle A has a speed of 15 ms^{-1} and particle B has a speed of 4 ms^{-1}. Given that the coefficient of restitution between the particles is ½, find:

a) the speed of the particles A and B after impact; (8)
b) the magnitude of the impulse given to each particle. (2)

(Total: 10 marks)

9. A car of mass 1200 kg is rounding a bend on a road which is banked at 15° to the horizontal. The car is assumed to be moving at a constant speed in a horizontal circle of radius 250 m. The coefficient of friction between the car's tyres and the road is 0.3. Calculate the maximum speed at which the car can travel if slipping outwards is not to occur. (10)

(Total: 10 marks)

10. A particle P is projected from a point O on horizontal ground, with a speed V ms^{-1} and angle of elevation α. The particle moves freely under gravity. After 5 s the components of velocity of P are 20 ms^{-1} horizontally and 14 ms^{-1} *vertically downwards*.

a) Show that $\tan \alpha = 9/5$. (3)
b) Calculate the value of V. (3)
c) Find the greatest height above the ground reached by P. (2)
d) Calculate the speed of P at 7 s after leaving O. (2)

(Total: 10 marks)