MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2023 SECOND SESSION

SUBJECT:	**Applied Mathematics**
DATE:	5^{th} September 2023
TIME:	4:00 p.m. to 7:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, ***i***, ***j*** are unit vectors along the *x*- and *y*- axes of a Cartesian system.

(take g = 10 ms^{-2})

1. A triangle *ABC* has sides of length, *AB* = *AC* = *5 m* and *BC* = 8 m. The triangle lies on the *x*-*y* plane, with BC lying on the positive *x*-axis, the vertex *B* lying at the origin, and with the vertex *A* having positive values for both its *x*- and *y*- coordinates. Taking the unit vectors ***i-*** and ***j-*** along the positive *x*- and *y*- axis direction.

   Forces of 5 N, 15 N and 10 N act along *AB, BC* and *CA* respectively.

   All forces act in the direction implied by the letters.

   a) Express each force in the ***i, j*** notation. (4)
   b) Find the resultant **F** of these forces. (1)

   By taking moments about *B*, or otherwise, find:

   c) the point on the *x*-axis where the force ***F*** acts. (5)

   (Total: 10 marks)

Please turn the page.

2. Two points *A* and *B* are 1.56 m apart in a horizontal line. A mass of 1.3 kg is suspended from *A* and *B* by two light elastic strings *AC* and *BC*. In the equilibrium position *AC* is perpendicular to *BC* and *tan ABC* = 5/12.

   a) Calculate, by resolving horizontally or otherwise, the ratio of the tensions in the two strings. (4)
   b) Find the magnitude of the tension in the string AC. (4)

   Given that the unstretched length of the string *AC* is 54 cm,

   c) calculate the value of the modulus of elasticity of the string *AC.* (2)

(Total: 10 marks)

3. A uniform rod *AB* of mass 25 kg and length 10 m rests with the end *A* on rough horizontal ground. The rod rests against a smooth peg *C*, where *AC* = *(5 + x)* m. The rod is in limiting equilibrium inclined at an angle of 45°. Given that the coefficient of friction between the rod and ground is ½, find:

   a) the normal reaction and frictional forces at the ground; (4)
   b) the magnitude of the reaction at *C*; (3)
   c) the value of *x*. (3)

(Total: 10 marks)

4. A uniform lamina is made from two shapes. *ABCD* is a rectangle of sides 10 cm by *x* cm. A semicircle of radius 5 cm is attached to the rectangle with the diameter of the semicircle coinciding with the side AB of the rectangle. *The centre of mass of the composite lamina lies on AB and the line of symmetry of the lamina.*

   a) Calculate the value of *x*. (7)

   The lamina is suspended from the point *C* and hangs freely under gravity.

   b) Calculate the angle between *CD* and the vertical. (3)

   (The centre of gravity of a semicircle of radius *r* is at $\frac{4r}{3\pi}$ from the diameter).

(Total: 10 marks)

5. A boat of mass 400 kg has an outboard motor with a propulsive force of 450 N. The boat is initially at rest. The motor is started and runs for 7 s. During this time a constant force of resistance acts on the boat. At the end of the 7 s the velocity of the boat is 5.5 ms^{-1}. Find:

   a) the acceleration of the boat; (2)
   b) the force of resistance of the boat. (2)

   At time, t = 7 s, the motor is switched off. The force of resistance remains the same. Find:

   c) the deceleration of the boat; (2)
   d) the time taken for the boat to come to rest; (2)
   e) the distance travelled by the boat in this time. (2)

(Total: 10 marks)

6. A mass of 15 kg is pulled up a rough plane inclined at 30° to the horizontal, by a force P parallel to the plane. The coefficient of friction between the mass and the plane is ¼.

a) Sketch a diagram showing **all** the forces acting on the mass. (2)
b) Show that the normal reaction of the plane on the mass is 130 N. (2)
c) Find the frictional force on the mass. (2)
d) Find the value of P when the mass is:
   i. moving with a constant speed up the slope; (2)
   ii. accelerating at 3 ms^{-2} up the slope. (2)

(Total: 10 marks)

7. A sports car of mass 1200 kg is travelling along a level road against a constant resistance of magnitude 500 N. The engine of the car is working at 6 kW. Calculate:

a) the acceleration of the car when the car is moving at 6 ms^{-1}; (7)
b) the maximum speed of the car. (3)

(Total: 10 marks)

8. A particle A of mass 0.6 kg is moving with a speed of 2 ms^{-1} on a smooth horizontal table of height 1 m above the horizontal floor. Another particle B of mass M kg is at rest on the edge of the table top. Particle A strikes particle B and the two particles coalesce. The single coalesced particle then falls from the table with a horizontal speed of V. From the point of leaving the table to the point of hitting the floor, the horizontal displacement is 0.4 m.

a) Show that the coalesced particle takes $\sqrt{5}/_{5}$ s to fall to the floor. (5)
b) Find the value of V and M. (5)

(Total: 10 marks)

9. A particle P of mass 0.7 kg rests on a rough horizontal disc at a distance 0.7 m from the centre. The disc is rotating about its centre. The coefficient of friction between the particle and the disc is ¼. Given that the particle is on the point of slipping, find the angular speed of the disc.

(Total: 10 marks)

10. A golf ball is struck from a point P, at the top of a cliff 60 m above sea level, with a speed of 15 ms^{-1} at an angle of 45° to the horizontal. The point O is at sea level vertically below P. The point A is the highest point reached by the ball in its motion. The ball strikes the sea at point B. Neglecting air resistance, find:

a) the height of A above sea level; (4)
b) the distance OB; (3)
c) the velocity in magnitude and direction of the golf ball when hitting the sea at B. (3)

(Total: 10 marks)