MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION CERTIFICATE EXAMINATION
INTERMEDIATE LEVEL

MAY 2012

SUBJECT:	PURE MATHEMATICS
DATE:	9th May 2012
TIME:	9.00 a.m. to 12.00 noon

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. (a) By taking logarithms, or otherwise, find the value of x if

$$4^{x+1}3^{2x-1} = 8.$$

(b) Find the positive value of x if $\log(x+1) + \log(3x+1) = \log 65$.

(c) Without using a calculator, simplify the expression

$$\frac{1}{(1+\sqrt{5})^2},$$

and write it in the form $\dfrac{a - b\sqrt{5}}{c}$, where a, b and c are positive integers.

[4; 3; 3 marks]

2. The function $f(x)$ is given by $f(x) = x^3 - 3x^2 - 2x + 6$.

   (i) Using the remainder theorem, show that $(x-3)$ is a factor of $f(x)$. Hence, obtain by division or otherwise the other factors of $f(x)$.

   (ii) Solve the equation $f(x) = 0$. Hence deduce the solutions of $f(x+1) = 0$.

   (iii) What is the behaviour of $f(x)$ as x takes large negative values.

   (iv) Find the integral $\int_0^1 f(x)dx$.

[4, 2, 1, 4 marks]

3. In a fun fair, children can sit on the circumference of a big ferris wheel which is rotating slowly with a constant speed. The height, in metres, of a child above the ground is given by

$$h(t) = 15 + 10 \sin t,$$

   where t represents time in minutes.

   (i) Find the maximum and minimum values of $h(t)$ for the time range $0 \leq t \leq 2\pi$.

   (ii) Draw a sketch of $h(t)$ for the time range $0 \leq t \leq 2\pi$.

   (iii) Find the time for the ferris wheel to complete one revolution.

   (iv) Find the radius of the ferris wheel and the speed of the child in metres/minute.

   (v) Calculate the rate of increase of the child's height at time $t = \pi/4$ minutes.

[3, 3, 1, 3, 2 marks]

4. (a) An athlete plans a training schedule which involves running 20 km in the first week of training. In each subsequent week, the distance is to be increased by 10% over the previous week.

   (i) Find the distance run in the nth week.

   (ii) Find the total distance run by the athlete in the first n weeks.

   (iii) After how many weeks would the distance calculated in part (ii) exceed 1000km.

   (b) Using arithmetic progressions, find the number of integers between 200 and 350, which are exactly divisible by 9. Hence find their sum.

[2, 2, 2; 5 marks]

5. A function is defined by $f(x) = x^2 + 4x + 5$.

   (i) Show by differentiation that the gradient of the tangent to the graph of this function at $x = -1$, is 2. Hence find the equation of the tangent at $x = -1$.

   (ii) Find the point of intersection of this tangent with the line $x + 2y = 5$.

   (iii) Show that the two lines are at right angles to each other.

   (iv) By completeing the square, or otherwise, show that the equation $f(x) = 0$ does not have real roots.

[5, 4, 1, 2 marks]

6. (a) Using the binomial expansion, expand $(1 + \frac{x}{2})^8$ in ascending powers of x up to and including the term in x^3. Using an appropriate value of x in this expansion, obtain an approximate estimate of 1.05^8.

   (b) The word *MATRICES* has eight different letters of the alphabet, five consonants and three vowels.

   (i) Find the number of different *permutations* of the eight letters of the word *MATRICES*.

   (ii) In how many of these permutations do the three vowels appear at the start of the permutation?

   (iii) Find the probability that a permutation has the three vowels at the start of the permutation.

[5; 1, 3, 1 marks]

7. (a) Differentiate the following functions with respect to x:

$$g(x) = \frac{x}{1 + x^2}, \qquad h(x) = e^{-3x+1}.$$

   (b) A farmer wishes to fence a rectangular area with 100 metres of fencing. One side of the area is a fixed wall, the fencing being only used for the other three sides.

   Using differentiation, find the dimensions of the rectangle that give maximum area. Find the maximum area.

[2; 4, 1 marks]

8. (a) The population $y(t)$ of a small tropical island at time t is given by

$$\frac{dy}{dt} = 10t - t^2,$$

where t is in years. The population is 500 when $t = 10$ years.

(i) Find by integration the population $y(t)$ as a function of t.

(ii) Find by trial and error, to the nearest integer, the value of t for which the population decreases to 0.

(b) A function is defined by $f(x) = 1 - e^{-x}$.

(i) Find $\int_0^{10} f(x)dx$ to the nearest integer.

(ii) Draw a sketch of the area represented by this integral.

[3, 2; 3, 2 marks]

9. A 2×2 matrix $\mathbf{A}$ represents a reflection in the line $y = x$, whilst a 2×2 matrix $\mathbf{B}$ represents a reflection in the line $y = -x$.

(i) Write down the matrices $\mathbf{A}$ and $\mathbf{B}$.

(ii) Find the matrix product $\mathbf{AB}$.

(iii) Show that $\mathbf{AB}$ represents a rotation through an angle θ. Find θ.

[4, 1, 2 marks]

10. A matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$.

(i) Show that $\mathbf{A}^2 - 5\mathbf{A} + 5\mathbf{I} = \mathbf{0}$.

(ii) Find the determinant of $\mathbf{A}$. Deduce that $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$, exists. Hence find $\mathbf{A}^{-1}$.

(iii) By formally multiplying the equation in (i) by $\mathbf{A}^{-1}$, obtain an expression for $\mathbf{A}^{-1}$ in terms of $\mathbf{I}$ and $\mathbf{A}$.

(iv) Calculate $\mathbf{A}^{-1}$ using the expression obtained in (iii), and show that this is equal to the inverse obtained in (ii).

[3, 4, 2, 1 marks]