MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION CERTIFICATE EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2012

SUBJECT:	PURE MATHEMATICS
DATE:	10th September 2012
TIME:	9.00 a.m. to 12.00 noon

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

---

1. (a) Showing all your working, find the value of x if

$$(27)^x 9^{2-x} = \sqrt{3}.$$

(b) Showing all your working, find the value of x if,

$$2\log(x-1) - \log(2-x) = \log(3-x).$$

(c) Using surds, and showing all your working, simplify the expression

$$\frac{1}{(3-\sqrt{2})^2},$$

writing your answer in the form $(a + b\sqrt{2})/c$, where a, b, c are integers.

[4; 4; 3 marks]

2. (a) The function $f(x) = \dfrac{7(x+1)}{(2x+1)(x-3)}$ has a partial fraction representation as follows:

$$f(x) \equiv \frac{A}{2x+1} + \frac{B}{x-3}.$$

Find the constants A and B in this relation. Hence find $\int f(x)dx$.

(b) If α and β are the roots of the quadratic equation $x^2+3x+6=0$, find the equation whose roots are $1/\alpha$ and $1/\beta$.

(c) The quadratic polynomial $p(x) = x^2+ax+7$ gives a remainder of -5 when divided by $x-2$. Find a, and hence determine the values of x for which $p(x) \leq 0$.

[3, 2; 3; 2, 3 marks]

3. (a) The nth term of a series is $\dfrac{2n+1}{3}$.

(i) Show that the series is an arithmetic progression. Find the common difference and the first term.

(ii) Find the sum of the first 100 terms of this series.

(b) A real number s has a decimal representation $s = 0.1212121212...$, where the digit combination '12' recurs an infinite number of times. Using geometric progressions, express s as a fraction m/n in its lowest terms, m and n being appropriate integers.

[3, 2; 4 marks]

4. (a) In the expansion in ascending powers of x of the function

$$(1+\lambda x+\mu x^2)(1+2x)^6,$$

the coefficients of x and x^2 are 14 and 87 respectively. Find the values of the constants λ and μ.

(b) In a juvenile chess club there are 15 members, of which 10 are boys and 5 are girls. A team of four children is to be selected from this club.

(i) Find the number of teams of 4 children that can be selected, irrespective of gender.

(ii) How many of these teams consist of exactly 3 boys and 1 girl?

(iii) Using (i) and (ii) above, find the probability that a team consists of 3 boys and 1 girl.

(iv) If one of the teams in (i) is selected at random, find the probability that it contains the youngest child in the club.

[5; 1, 2, 1, 2 marks]

5. A function $f(x)$ is given by $f(x) = \ln x \quad$ where $x > 0$.

   (i) Draw a sketch of the graph of $y = f(x)$, showing the asymptote and the point of intersection of the graph with the x-axis.

   (ii) On the same sketch draw a graph of $y = f(x-1)$.

   (iii) Find the values of x for which $f(x) \geq 3$.

   (iv) Find the equation of the tangent to the graph of $y = f(x)$ at $x = 1$.

[3, 1, 2, 4 marks]

6. (a) A small mass is attached to a light elastic spring and performs vertical oscillations according to the equation

$$y(t) = 3\cos t,$$

   where $y(t)$ represents the vertical position of the mass in cm, and t is the time elapsed in seconds.

   (i) Draw a sketch of $y(t)$ for $0 \leq t \leq 4\pi$.

   (ii) Find the maximum and minimum values of $y(t)$.

   (iii) Find the times at which the mass is at its highest position. Deduce the periodic time of the mass.

   (iv) Find the velocity, dy/dt, of the mass at time $t = \pi/2$.

   (b) If x is in the range $0 \leq x \leq \pi/2$, find in radians the value of x which satisfies the equation

$$2\cos^2 x = 1 + \sin x.$$

[2, 2, 2, 2; 4 marks]

7. (a) Differentiate the following with respect to x: $\quad f(x) = xe^{-2x}, \quad g(x) = \dfrac{\sin x}{1+2x}$.

   (b) An equilateral triangle of side $5\sqrt{3}$ cm has its sides increasing in length at the same rate. If the area of the triangle is increasing at 6 cm^2/minute, find the rate at which a side is increasing in length.

[2, 2; 5 marks]

8. (a) A function is defined by $f(x) = 1 + e^{-x}$. Evaluate the integral $\int_0^5 f(x)dx$, and explain why it should be positive.

   (b) Using separation of variables, solve the differential equation

$$\frac{dp}{dt} = 0.03p,$$

   given that when $t = 0$, $p = p_0$, where p_0 is a constant. Find the value of t at which p reaches the value of $2p_0$.

[3, 1; 3, 1 marks]

9. (a) The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix}$. Calculate:

   (i) the inverse matrix, $\mathbf{A}^{-1}$ and (ii) the matrix expression $\mathbf{A}^2 - 6\mathbf{A} + \mathbf{I}$, where $\mathbf{I}$ is the identity 2×2 matrix.

   (b) Write down a matrix $\mathbf{C}$ which represents a reflection in the line $y = x$. Find $\mathbf{C}^2$, and comment on your answer.

[2, 3; 2, 1, 1 marks]

10. Using matrix multiplication, find the constants a, b, c if

$$\begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Check your working by working out the matrix product in the left hand side of this equation using the values you obtained for a, b, c.

Write down the inverse of the matrix on the left. Using this inverse, solve the system of equations

$$\begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}.$$

[4, 1, 3 marks]