MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2013

SUBJECT:	**PURE MATHEMATICS**
DATE:	**7th September 2013**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. (a) Without using calculators, find x if

$$\log x = \frac{4}{3}\log 27 + \frac{3}{2}\log 49 - \log 63.$$

(b) Using surds, simplify the expression

$$\frac{2\sqrt{3}+1}{(4\sqrt{3}-5)},$$

writing your answer in the form $(a+b\sqrt{3})/c$, where a, b, c are integers.

(c) The function $f(x) = \dfrac{50}{x^2(x+5)}$ has a partial fraction representation as follows:

$$f(x) \equiv \frac{Ax+B}{x^2} + \frac{C}{x+5}.$$

Find the constants A, B and C in this relation.

[4; 3; 4 marks]

2. (a) If α and β are the roots of the quadratic equation $x^2 - 8x + 2 = 0$, find the quadratic equation whose roots are α^2 and β^2.

   (b) The cubic polynomial $p(x)$ is defined by $p(x) = x^3 + ax^2 + bx - 3$. This polynomial is exactly divisible by the factor $x + 1$, and gives a remainder of 15 when divided by $x - 2$.

      (i) Find a and b.

      (ii) Factorise the polynomial $p(x)$.

      (iii) Draw a rough sketch of $p(x)$.

      (iv) Find the range of $p(x)$ for $x \geq 1$.

[5; 4, 2, 1, 1 marks]

3. (a) Evaluate the arithmetic series

$$\sum_{r=1}^{100} (5r + 2).$$

   (b) In a geometric progression, the third term is 14, whilst the sixth term is 7/4. Find the common ratio, the first term and the sum to infinity.

[4; 5 marks]

4. (a) Find the coefficient of x^4 in $(1 - 2x)^9$.

   (b) In a small class of 5 students, the probability that a student passes a certain examination is 0.6.

      (i) Find the probability that all 5 students pass from this examination.

      (ii) Find the probability that all 5 students fail in this examination.

      (iii) Find the probability that at least one student passes from this examination.

   (c) Tom tosses *three* fair dice. Find the probability that he obtains a total score of 5.

[3; 2, 1, 2; 5 marks]

5. (a) Draw a sketch of the the function $f(x) = 1 + \sin 2x$ over the range $0 \leq x \leq 2\pi$.

   (b) If y is an acute angle such that $\sin y = 12/13$, find, without using a calculator, $\cos y$ and $\tan y$ in the form m/n where m and n are integers. Hence find

$$26 \sin y + 13 \cos y + 10 \tan y.$$

[5, 5 marks]

6. (a) A straight line joins the points $(1, 2)$ and $(2, 1)$.
   (i) Find the equation of this line.
   (ii) If this line cuts the x- and y- axes at A and B respectively, and O is the origin of coordinates, find the area of the triangle OAB.

   (b) The following is a table of x and y values from a certain experiment:

x:	1	2	3	4	7
y:	3	13	30	51	157

   The relation between x and y is given by $y = ax^b$. By taking logarithms of x and y, and plotting a suitable graph, find the constants a and b to one decimal place.

[3, 1; 6 marks]

7. (a) Differentiate the following functions with respect to x:
   (i) $f(x) = \sin^3 x$,
   (ii) $g(x) = \dfrac{\ln x}{2x + 3x^2}$.

   (b) ABCD is a thin straight wire of length 1 metre, with BC $= x$ metres and with AB $=$ CD. The wire is bent at B and C to form the three sides of a trapezium, AB, BC and CD, with the angles ABC and BCD both equal to 120°.
   (i) Find the area of this trapezoidal shape in terms of x.
   (ii) Find the value of x for which this area is maximum.

[1, 2; 5, 2 marks]

8. (a) Evaluate and draw a sketch of the following integral:

$$\int_1^4 (2 - \frac{1}{x})dx.$$

   (b) In a convection problem, the temperature $y(t)$ of a solid at time t is given by the differential equation

$$\frac{dy}{dt} = -0.05(y - 15).$$

   Initially, when $t = 0$, $y(0) = 90$. Using separation of variables, solve this differential equation to obtain $y(t)$ in terms of t.

[3, 2; 5 marks]

9. The matrix $\mathbf{A}$ represents a reflection in the line $y = -x$, whilst the matrix $\mathbf{B}$ represents an anticlockwise rotation through $3\pi/2$ radians.

   (i) Write down the matrices $\mathbf{A}$ and $\mathbf{B}$.

   (ii) Find $\mathbf{AB}$ and $(\mathbf{AB})^2$.

   (iii) Interpret the matrices obtained in (ii) geometrically.

[2, 2, 2 marks]

10. The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$.

   (i) Find the determinant of the matrix $\mathbf{A}$. Hence, show that the inverse matrix, $\mathbf{A}^{-1}$, exists and find it.

   (ii) Find the matrix expression $\mathbf{A}^2 - 3\mathbf{A} + 5\mathbf{I}$, where $\mathbf{I}$ is the identity 2×2 matrix.

   (iii) Using the method of the matrix inverse, solve the system of equations

$$\begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 80 \end{pmatrix}.$$

[4, 2, 2 marks]