IM 27.14s

SUBJECT:	PURE MATHEMATICS
DATE:	6th September 2014
TIME:	9.00 a.m. to 12.00 noon

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. (a) Find the values of x and y if
$$2^{x+1} + 3^y = 5,$$
$$2^x + 2(3^y) = 7.$$

(b) Without using calculators, find x if
$$\log x = \log 45 + 4\log 2 - 0.5\log 81 - \log 10.$$

(c) Using surds, simplify the expression
$$\frac{3\sqrt{5}+2}{\sqrt{5}+2} + \frac{2\sqrt{5}+1}{\sqrt{5}-1}$$
writing your answer in the form $(a - b\sqrt{5})/c$, where a, b, c are integers.

[4; 4; 4 marks]

2. (a) Obtain a partial fraction representation of the function

$$f(x) = \frac{4x - 1}{(2x + 1)(x + 2)}$$

and hence find $\int_0^1 f(x)dx$.

(b) A fisherman casts her fishing line a distance of 80 metres, during which the reel spins for 1.8 seconds. The reel has a diameter of 4 cm. Find in radians/second the rate of rotation of the reel, stating any assumptions made in your calculation.

[4, 4; 4 marks]

3. (a) How many terms of the series $2 + 5 + 8 + ...$ are required to make a sum of 13872?

(b) If the sixth term of a geometric series is -32 and the eleventh term is 1, find the first term, the common ratio and the sum to infinity.

[5; 4 marks]

4. (a) A quadratic polynomial $p(x)$ is defined by $p(x) = x^2 + 4x + 7$.

   (i) If α and β are the roots of the quadratic equation $p(x) = 0$, find the quadratic equation whose roots are α^2 and β^2.

   (ii) The polynomials $p(x)$ and $q(x) = x^3 + 2x^2 - 6x + a$ give the same remainder when divided by $x + 3$. Find the value of the constant a.

(b) Find the range of values of x for which $\dfrac{x - 3}{x - 2} > 2$.

[4, 3; 4 marks]

5. The line $y = 4x + c$ is a tangent to the curve $y = (x + 3)(x + 1)$ at the point P.

   (i) Using derivatives, or otherwise, find the coordinates of P and the value of c.

   (ii) Find the equation of the line which passes through $(2, 1)$ and which is perpendicular to this tangent.

   (iii) Find the shortest distance of the point $(2, 1)$ from this tangent.

[3, 3; 3; 2 marks]

6. (a) The function $f(x)$ is defined by $f(x) = (3 + 4x)^5$.

    (i) Expand $f(x)$ in ascending powers of x up to the term in x^2.

    (ii) Using this series, obtain an approximate value for 3.04^5.

    (iii) By evaluating the term in x^3 in the expansion, find the approximate error in the estimate obtained in (ii).

(b) A survey is conducted among families of *six* children.

It can be assumed that the probability that a boy is born is 0.55, and that multiple births (eg. twins, triplets etc.) are very rare.

Find the probability that in a family of six children:

    (i) all 6 children are boys;

    (ii) at least one of the children is a girl;

    (iii) both girls and boys are present.

[3, 1, 2; 2, 1, 2 marks]

7. (a) Differentiate the following functions with respect to x:

    (i) $f(x) = \dfrac{x}{3 + \sin x}$,

    (ii) $g(x) = (2 + e^{-x})^5$.

(b) Using derivatives, find the turning points of the curve $y = x^3 - 3x$. Draw a sketch of this curve showing the turning points and the points where the curve cuts the coordinate axes.

[1, 1; 3, 3 marks]

8. (a) Evaluate to four places of decimals the integral:

$$\int_0^1 (x + e^{-2x})dx.$$

(b) A physical problem is modelled with the differential equation

$$\frac{dx}{dt} = \cos t.$$

Initially, when $t = 0$, $x(0) = 1$.

Using separation of variables, solve this differential equation to obtain $x(t)$ in terms of t. Draw a sketch of $x(t)$ as a function of t over the range $0 \le t \le 4\pi$.

[4; 3, 3 marks]

9. The matrix **A** represents an anticlockwise rotation through $3\pi/2$ radians, whilst the matrix **B** represents a reflection in the line $y = x$.

   (i) Write down the matrices **A** and **B**.

   (ii) Find **AB** and **BA**.

   (iii) Interpret the matrices obtained in (ii) geometrically.

[2, 2, 2 marks]

10. (a) The matrices **A**, **B**, **C**, **D** given by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x & y \\ 2y & 2x \\ 1 & 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 7 & 6 \\ 19 & 15 \end{pmatrix}$$

are related by the equation $\mathbf{ABC} = \mathbf{D}$.

By equating any *two* entries of **D** with the corresponding entries of the matrix product **ABC**, calculate the values of x and y.

(b) Using the method of the matrix inverse, solve the equation

$$\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}.$$

[6, 4 marks]