MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2016

SUBJECT:	**PURE MATHEMATICS**
DATE:	**30th April 2016**
TIME:	**4.00 p.m. to 7.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

1. (a) By taking logarithms, find the value of t if

$$20 = 30e^{-5t}.$$

(b) Find the values of x if

$$\log_3(9x - 2) - 2\log_3 x = 2.$$

(c) Using surds, express the fraction

$$\frac{1}{\sqrt{5}} - \frac{\sqrt{5}}{1 - \sqrt{5}}$$

in the form $\dfrac{a + b\sqrt{5}}{c}$, where a, b and c are integers.

[3; 3; 3 marks]

2. (a) A quadratic equation is given by $(2k-1)x^2 + (2k+1)x + (k+1) = 0$.
If the sum of the roots is 7, find:

   (i) the value of k;

   (ii) the product of the roots.

   (b) A cubic polynomial is given by $p(x) = x^3 + x^2 - x - 1$.

   (i) Show that $x - 1$ is a factor of $p(x)$.

   (ii) Using division, or otherwise, factorise $p(x)$.

   (iii) Draw a sketch of the graph $y = p(x)$.

   (iv) Find the range of $p(x)$. **[3, 2; 1, 2, 2, 1 marks]**

3. The function $f(x)$ is given by $f(x) = \dfrac{2x^2 + 4x - 2}{(x-1)(x+1)^2}$.

   (i) Using partial fractions, find the constants A, B and C in the expression

$$f(x) \equiv \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}.$$

   (ii) Using the result obtained in (i), find the integral $\int_2^3 f(x)dx$. **[4, 5 marks]**

4. (a) Find the sum of the integers from 1 to 500, excluding all integers divisible by 5.

   (b) In a geometric progression, the 7th term is 27, whilst the 10th term is 1.

   (i) Find the common ratio and initial term of this series.

   (ii) Find the sum to infinity of the *even* terms in this series.

[5; 3, 2 marks]

5. (a) Using the identity $\sin^2 x + \cos^2 x = 1$, or otherwise, find two values of x in the range $0 \leq x \leq \pi$, which satisfy the equation $2\cos^2 x = 5\sin x - 1$.

   (b) A circle has centre O and radius r. A and B are two points on the circumference of this circle, such that the chord AB divides the circle into two segments, the minor segment having an area 1/6 of the circle.

   (i) Show that if $\angle$ AOB $= \theta$ radians, then $\theta = \pi/3 + \sin\theta$.

   (ii) Using radians, and starting from $\theta = 2.00$, find by trial and error or otherwise, the value of θ to two places of decimals.

[5; 3, 3 marks]

6. (a) In a certain town, 50% of families own a cat, 40% own a dog, and 76% own a cat and/or dog. Find the probability that

   (i) a family owns both a cat and a dog;

   (ii) a family owns a cat only;

   (iii) a family owns neither a cat nor a dog.

   (b) In a class of 18 students, there are 4 British students, 6 Germans, 5 French and 3 Italians. A committee of 5 students is to be selected from this class, with at least one member selected from each nationality. In how many ways can the committee be seelcted?

   (c) Expand the function $(3+2x)^{10}$ in ascending powers of x up to and including the term in x^2. Using this approximation, evaluate 3.02^{10}.

[1,1,1; 4; 4 marks]

7. (a) Differentiate the following functions with respect to x:

$$f(x) = \frac{x}{1+x^2}, \qquad g(x) = e^x(1+x)^5.$$

   (b) A function is given by the equation $y = x^2 - 2x + 2$.

   (i) Find the coordinates of the minimum of this function.

   (ii) Find the equation of the tangent to this curve at $x = 0$.

[2, 2; 3, 3 marks]

8. (a) A differential equation is given by $\dfrac{dy}{dx} = 2xe^{-y}$, with the condition that when $x = 1$, $y = 0$.

   Using separation of variables and integrating, find y in terms of x.

   (b) Using integration, find the area enclosed by the curves $y = x$ and $y = 3x - x^2$. Draw a sketch of this area.

[4; 6 marks]

9. (a) A 2×2 matrix $\mathbf{A}$ represents a reflection in the x-axis, whilst a 2×2 matrix $\mathbf{B}$ represents a reflection in the y-axis.

   (i) Write down the matrices $\mathbf{A}$ and $\mathbf{B}$.

   (ii) Find the matrix products $\mathbf{BA}$ and $(\mathbf{BA})^2$.

   (iii) Interpret geometrically the matrices obtained in (ii).

   (b) Using the method of the matrix inverse, solve the system of linear equations:

$$\begin{pmatrix} 6 & -1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 23 \\ 46 \end{pmatrix}.$$

[2, 2, 2; 4 marks]

10. The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$.

   (i) Show that $\mathbf{A}^2 - 8\mathbf{A} + 15\mathbf{I} = \mathbf{0}$, where $\mathbf{I}$ is the 2×2 identity matrix.

   (ii) By multiplying the equation in (i) by $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$, obtain a relation for $\mathbf{A}^{-1}$ in the form

$$\mathbf{A}^{-1} = \frac{a\mathbf{I} - b\mathbf{A}}{c},$$

   where a, b and c are integers.

   (iii) Using the relation obtained in (ii), or otherwise, find $\mathbf{A}^{-1}$.

[4, 3, 2 marks]