MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2016

SUBJECT:	PURE MATHEMATICS
DATE:	30th August 2016
TIME:	4.00 p.m. to 7.05 p.m.

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Non-programmable scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. (a) Find the values of x and y if

$$3\log_3 x = y,$$
$$\log_3(3x) = y - 5.$$

(b) Two variables x and y are related by a law of the type $y = ax^n$, where a and n are constants. The following values of x and y were recorded in a physical experiment:

x:	2	3	4	5
y:	13	28	49	74

By plotting $\log_{10} y$ against $\log_{10} x$, find the values of a and n to one significant figure.

[4; 7 marks]

2. (a) A function $f(x)$ is defined by

$$f(x) = \frac{x+8}{2x^2+5x+2} \equiv \frac{A}{2x+1} + \frac{B}{x+2}.$$

   (i) Using partial fractions, find the value of the constants A and B.

   (ii) Find the integral:

$$\int_0^1 f(x)dx.$$

   (b) Using the identity $\sin^2 x + \cos^2 x = 1$, or otherwise, find, in the range $0 \le x \le 2\pi$, the values of x which satisfy the equation

$$1 - \cos x = \sin^2 x.$$

[3, 4; 4 marks]

3. (a) A quadratic polynomial $p(x)$ is defined by $p(x) = 3x^2 + 5x + 1$.

   (i) If α and β are the roots of the quadratic equation $p(x) = 0$, find the quadratic equation whose roots are α^2 and β^2.

   (ii) By completing the square, or otherwise, find the minimum value of $p(x)$ and the value of x where it is attained.

   (iii) Find the range of $p(x)$.

   (b) The cubic polynomial $q(x)$ is defined by $q(x) = 2x^3 + ax^2 + x - 12$, where a is a constant. Find the value of a if one obtains a remainder of 42 when $q(x)$ is divided by $x - 2$.

[4, 3, 1; 3 marks]

4. (a) Find the sum, S_n, of the arithmetic series

$$\Sigma_{i=1}^{n}(2+5i).$$

   Find the smallest value of n for which $S_n > 500$.

   (b) The real number x is defined by $x = 0.2929292929....$, where the digits 29 recur infinitely often. By using the formula for the sum of a geometric series, or otherwise, express the real number x as a fraction in its lowest terms.

[6; 4 marks]

5. A straight line L has the equation $3y + 5x - 22 = 0$.

   (i) Find the equation of the line perpendicular to L and passing through $(2, 7)$.

   (ii) Find the distance from the point $(1, 1)$ to the line L.

[4, 3 marks]

6. (a) The function $f(x)$ is defined by $f(x) = (1 + 2x)^6$.

   (i) Using the binomial expansion, expand $f(x)$ in ascending powers of x up to and including x^2.

   (ii) Using this approximation, estimate 1.04^6.

   (iii) Give an estimate of the error in your estimate in (ii), by calculating the term in x^3 in the binomial expansion.

   (b) A four digit number is written down using only the digits 1, 2, 3, 5, 6 and 8 without replication. Find:

   (i) the number of different numbers which can be formed;

   (ii) the number of different odd numbers which can be formed;

   (iii) the probability that a number chosen randomly from those in (i) is odd.

[3, 2, 1; 2, 2, 1 marks]

7. (a) Differentiate the following functions with respect to x:

   (i) $f(x) = (1 - x)\sin x$;

   (ii) $g(x) = e^{2x}/(2 + \cos x)$;

   (iii) $h(x) = (x - e^x)^6$.

   (b) A sector of a circle with radius r and angle θ radians has a perimeter of 30 cm.

   (i) Show that $\theta = \dfrac{30}{r} - 2$.

   (ii) Find the values of r and θ which maximise the area.

[2, 2, 2; 2, 3 marks]

8. (a) Using separation of variables, solve the differential equation

$$\frac{dy}{dx} = 0.1y,$$

given that $y = 1000$ when $x = 0$. Find the value of x when $y = 2000$.

(b) Find the integral:

$$\int_0^1 (1 - e^{-x})dx.$$

Draw a sketch of this integral.

[4, 1; 3, 2 marks]

9. The 2×2 matrix $\mathbf{P}$ represents a reflection in the line $y = x$, whilst the 2×2 matrix $\mathbf{Q}$ represents a reflection in the line $y = -x$.

(i) Write down the matrices $\mathbf{P}$ and $\mathbf{Q}$.

(ii) Find the matrix products $\mathbf{QP}$, and $(\mathbf{QP})^2$.

(iii) Interpret these two products geometrically.

[2, 2; 2; 2 marks]

10. (a) The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ -2 & 7 \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

(ii) Find $\mathbf{A}^2 - 6\mathbf{A} - \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix.

(b) The matrices $\mathbf{F}$ and $\mathbf{G}$ are given by

$$\mathbf{F} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -2 & 3 \\ -1 & -2 & 1 \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} 4 & -2 & 2 \\ -4 & 3 & -5 \\ -4 & 4 & -4 \end{pmatrix}.$$

Find the matrix product $\mathbf{FG}$, and hence deduce $\mathbf{F}^{-1}$.

[3, 3; 2, 2 marks]