MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION CERTIFICATE EXAMINATION
INTERMEDIATE LEVEL
MAY 2017

SUBJECT:	PURE MATHEMATICS
DATE:	6^{th} MAY 2017
TIME:	09:00 to 12:05

Directions to Candidates

Attempt all questions. There are 10 questions in all and each question carries 10 marks.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

1. (a) Find the equation of the normal to the curve $y = x\ln(2x-1)$ at the point on the curve with x-coordinate 1.

[5 marks]

(b) If $f(x) = \dfrac{e^x}{2x+1}$, find $f'(x)$. Find also the coordinates of the stationary point on the curve $y = f(x)$.

[5 marks]

2. (a) Split $\dfrac{12x}{(x+1)(2x+3)(x-3)}$ into partial fractions. Hence, or otherwise, find the values of p and q if

$$\int \frac{12x}{(x+1)(2x+3)(x-3)}\,\mathrm{d}x = \ln\left| c\,\frac{(x+1)^p(x-3)}{(2x+3)^q}\right|,$$

where c is the constant of integration.

[5 marks]

(b) Show that the graphs of $y = f(x)$ and $y = g(x)$, where $f(x) = x^3 - x^2 - 6x + 8$ and $g(x) = x^3 + 2x^2 - 1$, intersect at two points. Show also that $f(x) - g(x) > 0$ for $-3 < x < 1$ and hence, find the area of the region enclosed between the two curves.

[5 marks]

3. (a) How many terms of the geometric series $1 + 1.02 + 1.02^2 + \cdots$ must be taken to give a sum greater than 3 million?

[5 marks]

   (b) The first term of an arithmetic progression is 100 and the common difference is −4. Find the greatest possible sum of the first n terms of this sequence and the value(s) of n when it occurs?

[5 marks]

4. The function $f(x)$ is given by $f(x) = 2x^3 + ax^2 - 43x + b$.

   (a) Given that $x + 2$ is a factor of $f(x)$, and that when divided by $x + 1$ the function $f(x)$ leaves a remainder 10, find the values of a and b.

[4 marks]

   (b) Factorize $f(x)$ completely.

[3 marks]

   (c) Draw a sketch of $y = f(x)$ and find the set of values of x for which $f(x) \leqslant 0$.

[3 marks]

5. (a) Find a and b, given that a and b are integers satisfying the equation

$$\frac{2\sqrt{5}-1}{\sqrt{5}+3}(a\sqrt{5}+b) = \frac{13-7\sqrt{5}}{11+5\sqrt{5}}.$$

[4 marks]

   (b) A wire of length 4 metres is cut into two pieces, and each piece is bent into a square. Show that the squares together have an area A given by

$$A = \frac{1}{8}(x^2 - 4x + 8),$$

where x is the length of one of the pieces.
Find x if:

   (i) $A = 0.75$ metres squared.
   (ii) A has the smallest value.

[2, 2, 2 marks]

6. The points A, B and C have coordinates $(1, -1)$, $(5, 2)$ and $(-9, 4)$, respectively.

   (a) Find the equation of the line ℓ_1 passing through A and B.

[3 marks]

   (b) Find the equation of the line ℓ_2 passing through C and perpendicular to ℓ_1.

[3 marks]

   (c) Find the point of intersection D of the lines ℓ_1 and ℓ_2.

[2 marks]

   (d) Find the lengths AD and CD, and hence the area of the triangle $\triangle ADC$.

[2 marks]

7. (a) Use the identity $\cos^2 x + \sin^2 x = 1$ to find the value of x between 0 and π such that $1 + 3\cos x - \sin^2 x = 0$.

[3 marks]

(b) The function $f(x)$ is defined by $f(x) = 3 + 2\cos x$.

(i) What are the maximum and minimum values of the function $f(x)$ for $0 \le x \le 2\pi$?

(ii) Sketch the graph of the function $f(x)$ for $0 \le x \le 2\pi$.

(iii) Write down an expression for the rate of change of the function $f(x)$. At which value between 0 and 2π does the function $f(x)$ have the fastest rate of decrease?

[2, 2, 3 marks]

8. (a) (i) The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Interpret **A** geometrically.

(ii) Write down the 2×2 matrix **B** representing a clockwise rotation through $\frac{\pi}{2}$.

[2, 2 marks]

(b) The matrices **P** and **Q** are given by

$$\mathbf{P} = \begin{pmatrix} -1 & 0 & 2 \\ -3 & 1 & 5 \\ -2 & 1 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}.$$

(i) Find the matrix product **PQ**.

(ii) Hence, solve the system of equations

$$\begin{pmatrix} 1 & -2 & 2 \\ -2 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

[3, 3 marks]

9. The area of a circle of radius r metres is $A\,\mathrm{m}^2$.

(a) Find $\dfrac{\mathrm{d}A}{\mathrm{d}r}$. By writing r in terms of A, find also an expression for $\dfrac{\mathrm{d}r}{\mathrm{d}A}$ in terms of r, showing that $\dfrac{\mathrm{d}A}{\mathrm{d}r} = \dfrac{1}{\mathrm{d}r/\mathrm{d}A}$.

(b) The area increases with time t seconds in such a way that $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{2}{(t+1)^3}$. Find an expression, in terms of r and t, for $\dfrac{\mathrm{d}r}{\mathrm{d}t}$.

(c) Solve the differential equation $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{2}{(t+1)^3}$ to obtain A in terms of t, given that $A = 0$ when $t = 0$.

(d) Show that, when $t = 1$, $\dfrac{\mathrm{d}r}{\mathrm{d}t} = 0.081$ correct to 2 significant figures.

[3, 2, 3, 2 marks]

10. (a) Francesca has 10 different music DVDs. Of these, 5 are rock music, 3 are classical music and 2 are folk music. In how many different ways can she arrange all the 10 DVDs on a shelf such that the rock DVDs are all next to each other?

[4 marks]

(b) Events A and B are such that $P(A) = 0.36$, $P(B) = 0.25$ and $P(A \cap B') = 0.24$. Using a Venn diagram find:

(i) $P(A \cap B)$;

(ii) $P(A' \cap B)$;

(iii) the probability that exactly one of A or B occurs.

[2, 2, 2 marks]