MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL

2018 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	5^{th} May 2018
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. The line ℓ_1 is defined by the relation $y = 2x + 1$.

   (a) Find the equation of the line ℓ_2 which passes through $(1, 2)$ and is perpendicular to ℓ_1. (3)

   (b) Find the point of intersection of ℓ_1 and ℓ_2. (2)

   (c) Find the area of the triangle bounded by ℓ_1, ℓ_2 and the x-axis. (2)

   (d) The line ℓ_1 is tangent to the curve $y = x^2 + c$, where c is a constant. By comparing the gradients of the line and the curve, or otherwise, find the value of c. (3)

[Total: 10 marks]

2. The quadratic $p(x)$ is defined by $p(x) = 3x^2 + x + 2$.

   (a) By completing the square, or otherwise, find the coordinates of the minimum of the graph of $y = p(x)$. (3)

   (b) Find the range of $p(x)$. (1)

   (c) The roots of the quadratic equation $p(x) = 0$ are denoted by α and β. Find the quadratic equation whose roots are $\alpha + 1$ and $\beta + 1$. (4)

   (d) Find the values of x where $p(x) \leq 4$. (3)

[Total: 11 marks]

3. (a) An arithmetic progression is given by $1, 4, 7, 10, 13, 16, ..., 1000$.
   Every third term of the above progression, namely $7, 16, ...$ is removed. Find the sum of the remaining terms. (4)

   (b) In a geometric progression, the sum of the first two terms is 7, whilst the sum of the second and third term is 28/3. Find the initial term and the common ratio. Find the sum of the first thirty terms to the nearest integer. (4)

   (c) The variable x satisfies the equation $3e^{-x} = 2$. By taking logarithms, find the value of x, giving your answer to four places of decimals. (3)

[Total: 11 marks]

4. (a) A binomial series is given by $(1 + ax)^n = 1 + 36x + 576x^2 + \dots$.
   Find the integer values of n and a, and the coefficent of x^3 in this series. (4)

   (b) A child has 12 different vehicles in his toybox: 7 lorries and 5 cars. The child selects 3 toy models.

      (i) In how many ways can 3 vehicles be chosen? (1)

      (ii) In how many ways can 2 lorries and 1 car be chosen? (2)

      (iii) What is the probability of choosing 2 lorries and 1 car? (1)

   (c) A and B are events with $p(A) = 0.5$, $p(B) = 0.7$, and $p(A \cup B) = 0.85$. By drawing a Venn diagram, or otherwise, find $p(A \cap B)$ and $p(A \cup B')$, where B' is the complement of B. (3)

[Total: 11 marks]

5. (a) Using the identity $\sin^2 x + \cos^2 x = 1$, or otherwise, find two values of x in the range $0 \le x \le \pi$, which satisfy the equation $4\cos^2 x = 7 - 8\sin x$. (3)

(b) Two circles have radii 6 cm and 4 cm, and their centres are 7 cm apart. Find the perimeter and area of the overlapping region common to both circles. (7)

[Total: 10 marks]

6. (a) Find the first derivative with respect to x of the functions $f(x)$ and $g(x)$ given by:

$$f(x) = \frac{x}{1+e^x}, \quad g(x) = (1+x^2)\ln(1+x^2).$$

(4)

(b) A circular cylinder is to fit inside a sphere of radius 10 cm. Taking the height of the cylinder to be $2x$, express the radius r of the cylinder in terms of x, and obtain an expression for the volume of the cylinder in terms of x only. Find by differentiation, the maximum possible volume of the cylinder. (6)

[Total: 10 marks]

7. (a) A differential equation is given by $x\dfrac{dy}{dx} = y(y+1)$, given that $y = 4$ when $x = 2$.

Solve this system by first separating the variables and then expressing the y-term in terms of partial fractions. After integrating, express y as a function of x. (5)

(b) Sketch the area enclosed by the graph of $y = x^2 - x^3$ and the x-axis. Find by integration the value of this area. (5)

[Total: 10 marks]

8. (a) Draw a sketch of the graph of the function $y = \sin 2x$ over the range $0 \le x \le 2\pi$. Find the equation of the tangent to the graph at $x = \pi/8$. (5)

(b) The constants a and b are given by the equation

$$\frac{a\sqrt{7}+b}{2\sqrt{7}-3} = \frac{3\sqrt{7}+1}{4\sqrt{7}+2}.$$

Using surds, find a and b as two fractions in their lowest terms. (3)

[Total: 8 marks]

9. (a) The matrix **A** is defined by $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Find $\mathbf{A}^2$ and $\mathbf{A}^4$. Interpret these matrices geometrically. (5)

(b) If $\mathbf{Q} = \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \end{pmatrix}$, find $\mathbf{Q}^{-1}$, and hence find the matrix **P** such that $\mathbf{QP} = \mathbf{R}$. (5)

[Total: 10 marks]

10. (a) Show, using matrix multiplication, that the equation given by

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 9$$

simplifies to the linear equations $x + my = \pm n$, where m and n are integers to be determined. (3)

(b) A polynomial $q(x)$ is defined by $q(x) = x^3 + 2x^2 + ax + b$, where a and b are constants. The polynomial $q(x)$ has $x+1$ as factor, and has a remainder of 12 when it is divided by $x-2$. Set up a system of two linear equations for a and b. Solve this system using the method of the matrix inverse. (6)

[Total: 9 marks]