L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL

2020 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	2$^{\text{nd}}$ May 2020
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

1. (a) Using surds, express the fraction

$$\frac{\sqrt{3}+\sqrt{5}}{2\sqrt{3}-\sqrt{5}}+\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-2\sqrt{5}}$$

in the form $(a+b\sqrt{15})/c$, where a, b and c are positive integers. (3)

(b) Find the value of b if

$$\log_b 180+6\log_b 2-\frac{1}{4}\log_b 81-\log_b 30=\frac{7}{2}.$$

(3)

[Total: 6 marks]

2. The quadratic function $p(x)$ is given by $p(x)=2x^2+x-1$.

   (a) Find the equation of the tangent of the graph of $y=p(x)$ at $x=0$. (4)

   (b) Using partial fractions, find the constants A and B in the expression

   $$\frac{6x}{p(x)}=\frac{A}{x+1}+\frac{B}{2x-1}. \qquad (2)$$

   (c) Using the result obtained in part (b), find the integral $\int \frac{6x}{p(x)}dx$. (2)

   (d) If α and β are the roots of $p(x)$, find the quadratic function, $q(x)$, whose roots are $\alpha+3$ and $\beta+3$. (3)

[Total: 11 marks]

3. (a) A music class consists of 9 English, 10 Italian and 5 Spanish students. A committee of 3 English, 4 Italian and 2 Spanish students is to be selected from this class. In how many ways can this be done? (4)

   (b) A fair die has six faces, which are numbered respectively from one to six. Find the probability of obtaining a total score of 4 in three tosses of this die. (4)

   (c) Using the binomial expansion, expand the expression $(2+3x)^8$ in ascending powers of x up to the term in x^2. Hence obtain an estimate for 2.003^8. (4)

[Total: 12 marks]

4. A cubic polynomial is given by $h(x)=x^3-3x^2+ax+b$, where a and b are constants. The cubic $h(x)$ has $x-3$ as a factor. Besides, $h(x)$ leaves a remainder of 6 when divided by $x-1$.

   (a) Obtain a set of two linear equations for a and b. Solve for a and b using the method of the matrix inverse. (5)

   (b) By suitable division, or otherwise, express $h(x)$ as the product of three linear factors in x. (3)

   (c) Draw a sketch of $h(x)$ against x, showing where the graph intersects the coordinate axes. Deduce the values of x for which $h(x)\geq 0$. (3)

   (d) Find the range of $h(x)$ for $x\leq -2$. (1)

[Total: 12 marks]

5. (a) In an arithmetic progression, the sum of the first three terms is equal to the 5th term, whilst the average of the first 4 terms is equal to 6. Find the nth term of this series, and the sum of the first n terms. (4)

   (b) A geometric progression is given by 4, 2, 1, 1/2, ... Denote the sum to infinity of this series by S, and the sum of the first n terms of this series by S_n. Find S, S_n and the smallest value of n so that the difference between S and S_n is less than 0.001. (5)

[Total: 9 marks]

6. (a) Using the identity $\sin^2 x + \cos^2 x = 1$, or otherwise, find two values of x in the range $0 \leq x \leq 2\pi$, which satisfy the equation

$$10\sin^2 x - 7\cos x - 4 = 0. \quad (5)$$

   (b) An equilateral triangle has its sides equal to 1 cm. A sector of radius 1 cm subtends an angle θ at its centre. Find the value of θ if:

      (i) the triangle and the sector have equal perimeters; (3)

      (ii) the triangle and the sector have equal areas. (4)

[Total: 12 marks]

7. (a) Find the first derivative with respect to x of the functions:

      (i) $f(x) = \dfrac{x}{1+x^2}$, (2)

      (ii) $g(x) = e^{2x}(1+3x)$, giving your answer in the form $e^{2x}(ax+b)$, where a and b are integers. (2)

   (b) A rectangle has sides x and y cm. A thin wire of length 12 cm is bent to form three sides of this rectangle. Find by differentiation the values of x and y if the rectangular area enclosed by the wire is maximum. (5)

[Total: 9 marks]

8. (a) A differential equation is given by $\dfrac{dy}{dx} = \cos 2x$, given that $y = 0$ when $x = 0$.

   Solve this system by separating the variables. Draw, on the same graph, a sketch of y and $\dfrac{dy}{dx}$ in the range $0 \leq x \leq 2\pi$. (6)

   (b) Find the coordinates of the point where the curves $y = e^x$ and $y = 2e^{-x}$ intersect. Hence, find by integration, the area enclosed by these two curves and the y-axis. (6)

[Total: 12 marks]

9. The matrix **A** represents a clockwise rotation through $\pi/2$, whilst **B** represents a reflection in the line $y = x$.

   (a) Write down the matrices **A** and **B**. (4)

   (b) Using matrix multipliction, find the matrix **C** which represents a clockwise rotation through $\pi/2$, followed by a reflection in the line $y = x$. Interpret **C** geometrically. (2)

   (c) Find $\mathbf{C}^2$ and interpret it geometrically. (2)

[Total: 8 marks]

10. (a) Find the value of c if $\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} c & 1 & 2 \\ 1 & 5 & -1 \\ 2 & -1 & c \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 50.$ (3)

    (b) The matrices **K** and **L**, and the vector **m** are given by:

$$\mathbf{K} = \begin{pmatrix} 1 & -2 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & 2 \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} 4 & -5 & 3 \\ -2 & 0 & 1 \\ -3 & 5 & -1 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 10 \\ 5 \\ 15 \end{pmatrix}.$$

   (i) Find the matrix product **KL**. (2)

   (ii) Find $\mathbf{K}^{-1}$, the inverse of **K**, in terms of the matrix **L**. (2)

   (iii) Using the method of the matrix inverse, find the unknown vector **x** if **Kx**=**m**. (2)

[Total: 9 marks]