L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2021 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	12th June 2021
TIME:	9:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are **not** allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.

1. (a) Rationalise the denominator of $\dfrac{4-3\sqrt{2}}{5-3\sqrt{2}}$.

[2]

(b) A function f maps a real number x into the real number y given by

$$y=\frac{\sqrt{-x+2}}{(x+1)(x+9)}.$$

Find the domain for which the function f is defined.

[3]

(c) A and B are two points on the circumference of a circle centre O and radius r. The minor arc AB subtends an angle θ radians at O. If the area of the minor segment bounded by the chord AB and the circle is one quarter of the area of the minor sector AOB, show that $4\sin\theta=3\theta$.

[5]

[Total: 10 marks]

2. (a) Express $\dfrac{x+4}{(x+1)(x-2)^2}$ into partial fractions.

[5]

(b) Expand $(1+2u)^5$ in ascending powers of u.
Hence find the coefficient of x^{12} in the expansion of $(1+2x^3)^5$.

[5]

[Total: 10 marks]

3. (a) Let $k = \log_2 3$. Given that $2^{3u+1} = 3^{u+2}$, find u in terms of k.

[4]

(b) Two variables x and y are related by a rule of the form $y = ab^x$. For particular values of the constants a and b, the following table is obtained.

x	1	2	3	4	5
y	240	1920	15340	122880	983040

By plotting $\log y$ against x, find an estimate for the constants a and b, correct to one decimal place.

[6]

[Total: 10 marks]

4. (a) The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 3 \\ -1 & 3 & 3 \\ -2 & 5 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 0 & 5 & -3 \\ -1 & 16 & -9 \\ 1 & -14 & 8 \end{pmatrix}.$$

(i) Find the matrix product **AB** and deduce $\mathbf{A}^{-1}$, the inverse of **A**.

(ii) Using the method of matrix inverse, find the vector $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ if $\mathbf{Ax} = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$.

[3, 2]

(b) A closed box contains metal and plastic marbles of the same size but of two different colours, red and blue. A hole the size of a marble is drilled in the box and the box is shaken until one marble drops from the box. The total number of marbles in the box is 20. Half of them are made of metal and the number of red marbles is three times the number of blue marbles. The number of blue and plastic marbles is 2. Using a Venn diagram, or otherwise, find the probability that the marble that drops from the box is:

(i) made of metal;
(ii) either red or made of metal;
(iii) red and made of metal.

[1, 1, 3]

[Total: 10 marks]

5. (a) Given that $f(r) = r^3 - 3r + 2$ show that $r - 1$ is a factor of $f(r)$. Hence express $f(r)$ as a product of three linear factors and solve the equation $f(r) = 0$.

[3]

(b) The first, second and fourth terms of an arithmetic progression are respectively equal to the first, second and fourth terms of a geometric progression.

(i) Find the common ratio of the geometric progression given that its terms are not all the same. [Hint: Use the result of (a).]

(ii) Given that the third term of the geometric progression is –8, find the first term and the common difference of the arithmetic progression.

(iii) The sum of the first 31 terms of the arithmetic progression is four times the sum of the first n terms of the geometric progression. Find n.

[3, 1, 3]

[Total: 10 marks]

6. Let ℓ_1 and ℓ_2 be the lines given by the equations $y = 3x + 20$ and $2x + 3y = 5$, respectively. Let $f(x) = x^2 + px + q$ for some real numbers p and q.

(a) Find the coordinates of the point A where ℓ_1 and ℓ_2 intersect.

[2]

(b) Write down the equation of the line ℓ_3 having gradient -1 and passing through the point with coordinates $(0, 2)$.

[1]

(c) Determine the perpendicular distance of A from ℓ_3.

[2]

(d) Given that the graph of $y = f(x)$ cuts the x-axis at the same points where the lines ℓ_1 and ℓ_2 cut the x-axis, find p and q.

[3]

(e) For which value of x does $f(x)$ reach its minimum?

[2]

[Total: 10 marks]

7. (a) Find the range of values of k for which

$$x^2 + (2 - k)x + 1 - 3k = 0$$

has no real roots.

[4]

(b) If α and β are roots of $8x^2 - 2x - 15 = 0$, show that the quadratic equation whose roots are $\sqrt{2}\alpha + 1$ and $\sqrt{2}\beta + 1$ is

$$4x^2 - (\sqrt{2} + 8)x + \sqrt{2} - 11 = 0.$$

[6]

[Total: 10 marks]

8. (a) Show that

$$\frac{1}{y^2-1} \equiv \frac{1}{2(y-1)} - \frac{1}{2(y+1)}$$

and hence solve the differential equation

$$\frac{dy}{dx} = (1+e^{-x})(y^2-1),$$

given that $y = 2$ when $x = 0$.

[7]

(b) Let $u = \cos^4 t + \cos t^4$. Find $\frac{du}{dt}$.

[3]

[Total: 10 marks]

9. (a) By expressing in terms of $\cos\theta$ and $\sin\theta$, solve the equation

$$2\cos\theta - \sec\theta = \tan\theta,$$

in the range $-180° < \theta \leq 180°$.

[4]

(b) Two curves are given by the equations $y = -x^2 + 10$ and $y = (x-2)^2$.
   (i) Find the coordinates of the points of intersection of the two curves.
   (ii) Sketch the graphs of the curves on the same axes, showing clearly the points of intersection.
   (iii) Calculate the area bounded by the two curves.

[2, 1, 3]

[Total: 10 marks]

10. (a) The volume $V(t)$ of a balloon at time t is given by

$$V(t) = \frac{6\sqrt[3]{t}}{4t+1}.$$

Find the rate of change of $V(t)$ at time $t = 8$ and state if at this time the balloon is being filled with air or whether air is being released.

[6]

(b) Find the point of contact between the tangent and the curve with equation $y = 5x^2 - 2x + 3$ when the gradient is $\frac{4}{3}$.

[4]

[Total: 10 marks]