IM27.22s

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL
2022 SECOND SESSION

SUBJECT:	**Pure Mathematics**
DATE:	2nd September 2022
TIME:	4:00 p.m. to 7:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are **not** allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.

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1. Let $f(x)=2x^3+x^2-5x+2$.
   (a) Show that $(x-1)$ and $(2x-1)$ are factors of $f(x)$ and find the other factor of $f(x)$.

   [3 marks]

   (b) Express

$$\frac{3x^2-8x+2}{2x^3+x^2-5x+2}$$

   into partial fractions and show that

$$\int_2^5 \frac{3x^2-8x+2}{2x^3+x^2-5x+2}\,\mathrm{d}x=\ln\left(\frac{49\sqrt{3}}{64}\right).$$

   [4, 3 marks]

   [**Total: 10 marks**]

2. (a) Sketch the arc of the curve $y=2x-x^2$ for which y is positive and find the area of the region which lies between this arc and the x-axis.

   [3 marks]

   (b) The base of a triangle is half its perpendicular height. If the base is increasing at the rate of 5 cm/s, find the rate of change of the area of the triangle when the base is 10 cm.

   [3 marks]

   [**Total: 6 marks**]

3. (a) Find the set of values of $x \in \mathbb{R}$ for which $\dfrac{x+1}{3x+1} > 1$.

[4 marks]

(b) If α and β are the roots of the quadratic equation $2x^2 - 3x - 1 = 0$, find the quadratic equation whose roots are $\alpha^2 + \alpha$ and $\beta^2 + \beta$.

[4 marks]

[**Total: 8 marks**]

4. (a) x, $17 - x$ and $2x - 1$ are three consecutive terms of an arithmetic progression. Find x.

[2 marks]

(b) y, $y + 3$ and $5y - 3$ are three consecutive terms of a geometric progression. Find the value of y, given that it is an integer.

[3 marks]

[**Total: 5 marks**]

5. Let $f(x) = 1 - 2\sin^2 x$ where $-\pi \le x \le \pi$.

(a) Solve the equation $f(x) = 0$.

[2 marks]

(b) Using differentiation, find the stationary points of the curve $y = f(x)$ and determine their nature.

[4, 3 marks]

(c) Draw a sketch of $y = f(x)$, showing clearly the stationary points and the points where the curve cuts the coordinate axes.

[3 marks]

[**Total: 12 marks**]

6. (a) Let $f(x) = \ln(x^2 - 6x + 10)$. Find the coordinates of the point on the graph $y = f(x)$ at which the tangent has slope equal to 1.

[4 marks]

(b) Let $u = (2x-3)^{10}$. Determine the coefficient of x^3 in the expansion of $\dfrac{\mathrm{d}u}{\mathrm{d}x}$.

[4 marks]

(c) Let $g(x) = (x^2 + 3)e^{2x+3}$. Show that the function $g(x)$ has **no** stationary points.

[4 marks]

[**Total: 12 marks**]

7. (a) Let $y = te^{-t}$. Find $\frac{dy}{dt}$ and hence deduce that

(Eq. 1) $$\int te^{-t}\,\mathrm{d}t = -te^{-t} - e^{-t} + C,$$

where C is a constant.

[3 marks]

(b) An experiment was conducted to see how the number P of organisms in a population changes over time. The rate of change of P was found to satisfy the differential equation

(Eq. 2) $$\frac{\mathrm{d}P}{\mathrm{d}t} = (7-t)(10^5 e^{-t} + t^2 + 1),$$

where t is the time measured in seconds.

(i) Show that the population reaches a maximum when $t = 7$s.

[2 marks]

(ii) Given that $P = 10^6$ when $t = 0$s, show that when $t = 7$s the population reaches 1600315.
(Note that when integrating the right-hand side of (Eq. 2), you can make use of the integral given in (Eq. 1).)

[7 marks]

[**Total: 12 marks**]

8. The matrix $T = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ represents a transformation in the xy-plane. The line ℓ has equation $y - 2x - 6 = 0$.

(a) Sketch the line ℓ, marking clearly its intercepts.

[2 marks]

(b) The matrix T transforms the line ℓ into another line ℓ'. By considering the images of the intercepts of ℓ, find the equation of its image ℓ'.

[5 marks]

(c) T maps the point $P(a, b)$ into the point $(1, 2)$. Find a and b.

[3 marks]

[**Total: 10 marks**]

9. The line ℓ_1 passes through the points $A(1,1)$ and $B(5,13)$. The line ℓ_2 has equation $y=-x+6$.
   (a) Find the equation of ℓ_1.
   [2 marks]
   (b) (i) Find the coordinates of the point X where ℓ_1 and ℓ_2 intersect.
   [2 marks]
   (ii) Draw a diagram to show the lines ℓ_1, ℓ_2 and the point X.
   [2 marks]
   (c) Show that the lines ℓ_1 and ℓ_2 are **not** perpendicular.
   [1 marks]
   (d) Verify that the points $Q(-1,7)$ and $R(3,3)$ lie on ℓ_2 and that the line AR is perpendicular to ℓ_2.
   [2 marks]
   (e) Hence, or otherwise, find the distance of the point A from ℓ_2 leaving your answer in the simplest surd form.
   [2 marks]
   (f) Find the area of triangle QXA.
   [4 marks]
   [**Total: 15 marks**]

10. (a) In a room there are twelve workers; five are Maltese, four are Italian and three are English. Five workers are to be chosen to participate in some group-work. In how many ways can this be done if:
   (i) any five workers are to be chosen?
   [1 marks]
   (ii) **no** Italian worker is to be chosen?
   [2 marks]
   (iii) the oldest two workers are to be chosen? (Assume that none of the workers have the same age.)
   [2 marks]
   (b) The letters of the word INSTAGRAM are arranged randomly in a row.
   (i) How many different arrangements are possible?
   [2 marks]
   (ii) What is the probability that the arrangement has the vowels next to each other?
   [3 marks]
   [**Total: 10 marks**]