MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL
2023 SECOND SESSION

SUBJECT:	**Pure Mathematics**
DATE:	5th September 2023
TIME:	16:00 to 19:05

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

1. Let $f(x)=x^3-3x^2-4x+12$.

   (a) Evaluate $f(2)$ and factorize $f(x)$ completely.

   [4 marks]

   (b) Sketch the graph of $y=f(x)$ and find the range of values of x for which $f(x)>0$.

   [3 marks]

   (c) Express $\dfrac{1}{f(x)}$ into partial fractions.

   [4 marks]

   (d) Show that:

$$\int_{-1}^{1} \frac{1}{f(x)}\,\mathrm{d}x = \frac{3}{10}\ln 3 - \frac{1}{5}\ln 2.$$

   [4 marks]

   [**Total: 15 marks**]

2. Two matrices are given by:

$$A = \begin{pmatrix} 2 & 0 \\ a & b \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix},$$

where a and b are positive. Suppose that:

$$A^2 = \begin{pmatrix} 4 & 0 \\ 18 & 16 \end{pmatrix}.$$

(a) Show that $a = 3$ and $b = 4$.

[3 marks]

(b) Find $A - B$ and give a geometric interpretation of the transformation represented by it.

[2 marks]

[**Total: 5 marks**]

3. (a) The first and third terms of a geometric progression are a and $a - 30$, respectively. The common ratio is 0.25. Find the value of a and deduce the sum to infinity of this geometric progression.

[5 marks]

(b) The second term of an arithmetic progression is 10. The sum of the first ten terms of the progression is 345. Find the first term and the common difference. Also find the sum of terms from the eleventh term to the twentieth term.

[5 marks]

[**Total: 10 marks**]

4. (a) Solve the quadratic equation:

$$2u^2 + 3u - 2 = 0.$$

[2 marks]

(b) Use the identity $\cos^2 x + \sin^2 x = 1$ to solve for θ the equation:

$$2\cos^2(2\theta) = 3\sin(2\theta),$$

for values of θ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

[4 marks]

(c) Find x given that $2 \times 16^x - 3 \times 4^x = 2$.

[4 marks]

[**Total: 10 marks**]

5. A line ℓ with equation $y = x + 6$ intersects the curve $y = x(x-4)$ at the points A and B, where B has a positive x-coordinate.

   (a) Find the coordinates of A and B.

   [3 marks]

   (b) Find the distance AB and the distance of the origin O from the line ℓ. Give your answers in the simplest surd form.

   [2, 2 marks]

   (c) Show that triangle AOB is **not** a right-angled triangle.

   [3 marks]

   [**Total: 10 marks**]

6. The gradient $\frac{dy}{dx}$ of a curve at the point (x, y) is given by:

$$\frac{dy}{dx} = e^{9y} \sin 3x.$$

   (a) Solve the given differential equation by separating the variables.

   [3 marks]

   (b) Given that the curve passes through the point $(\pi, 0)$, find the equation of the curve.

   [2 marks]

   [**Total: 5 marks**]

7. (a) Peter has two fair dice, each with 12 faces numbered 1 to 12. One die is blue and the other is red. He tosses the two dice simultaneously.

   (i) What is the probability that the number 12 turns up on both dice?

   (ii) What is the probability that the sum of the numbers which turn up is less than 23?

   [2, 2 marks]

   (b) (i) Find the number of permutations of all the letters of the word MAURITANIA.

   How many of these arrangements have:

   (ii) the A's next to each other.

   (iii) the consonants next to each other.

   [2, 2, 2 marks]

   [**Total: 10 marks**]

8. (a) Differentiate the following functions with respect to x:

   (i) $f(x) = \dfrac{3}{(x^2-4)^3}$,

   (ii) $g(x) = e^{x(2+x)}$.

[2, 2 marks]

(b) Find the x-coordinate, and determine the nature of the stationary points of f and g.

[3, 3 marks]

[**Total: 10 marks**]

9. The rate of increase of the radius r of a circular stain is given by:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = kr^{-2}, \qquad (*)$$

where k is a constant.

(a) Find the rate of increase of the area of the stain in terms of r and k.

[4 marks]

(b) Find the value of k given that the area is increasing at the rate of 12 cm^2/s when the radius is 4 cm.

[2 marks]

(c) By separating the variables, solve the differential equation given above in (∗) and find the radius of the stain after 5 s, given that the radius of the stain at $t = 0$ is 1 cm. Give your answer correct to 2 decimal places.

[4 marks]

[**Total: 10 marks**]

10. The curve $\mathscr{C}_1$ is given by the equation $y = 2x - x^2$, and the curve $\mathscr{C}_2$ is given by the equation $y = x^2 - 3x + 2$.

(a) Find the x- and y-intercepts of $\mathscr{C}_1$ and $\mathscr{C}_2$, and the coordinates of the points of intersection of $\mathscr{C}_1$ and $\mathscr{C}_2$. On the same diagram sketch $\mathscr{C}_1$ and $\mathscr{C}_2$.

[6 marks]

(b) Find the area bounded between $\mathscr{C}_1$ and $\mathscr{C}_2$.

[4 marks]

(c) Find the equation of the normal to $\mathscr{C}_1$ at the point $(5/4, 15/16)$.

[5 marks]

[**Total: 15 marks**]