MATHSLINE

Issue No. 34 Mathematics Section, Directorate for Learning and Assessment Programmes April 2018

Maths Venture goes to GOZO!

The Mathematics Section within the Directorate for Learning and Assessment Programmes organised the fifth edition of the Maths Venture. This year it was held at the Cittadella on 17th April 2018. Year 9 students from state, church and independent schools in Malta and Gozo were invited for this activity and a total of 27 schools took part in this event.

The purpose of the Cittadella Mathematics Venture was to provide a context where students can apply the mathematics that is learnt in the classroom while walking around the streets and squares of Cittadella. Thus the activities were designed to give students the opportunity to experience mathematics outside the confines of the classroom.

During the activities, students had to tackle a series of problem-solving tasks in which they had to apply mathematical skills learnt in the classroom. Some activities also incorporated an element of Maltese history, emphasizing the importance of cross curricular activities.

During the Maths Venture students had to work in teams of four. It was very important that students work as a team and collaborate together. Besides the mathematical element, students needed to discuss and exercise collaborative skills to solve each activity.

The winning team of this year's Maths Venture Gozo edition consisted of Leon Pisani and Clyde Grech from Maria Regina College Secondary School (Lily of the Valley), Mosta and Laura Michaela Agius and Justine Galea from St Monica School, B'Kara.

The organisers would like to thank, ecoGozo, Heritage Malta and the Directorate for Learning and Assessment Programmes for their support.

Mathemagics

The Secretariat for Catholic Education is organising a series of five two-hour monthly sessions aimed at Year 8 (Form2) students with an aptitude in mathematics. These sessions, entitled '***Mathemagics***', are being prepared and conducted by Mr. Joseph Mamo, ex-Head of Department at St Ignatius College and visiting lecturer within the Faculty of Education at the University of Malta. The aim of these sessions is to bring out the fun element in Mathematics, through activities which are not normally done in a mathematics class. The subject content in these sessions is related to the subject curricula covered in schools.

The Mathemagics sessions started in January 2018, with the fifth and last session being held in May. A group of 42 students coming from both church and state schools are participating in these sessions which are being held at Stella Maris College. The first session focused on *Tangrams*, an old Chinese puzzle that resembles a jigsaw puzzle made up of just seven shapes. The activities gave students an excellent opportunity for creativity and problem-solving. *Mathematics with a Calendar*, the focus of the second session, enabled the students to take a look at the calendar from a mathematical perspective.

The topic of the March session was *Pi*, since Pi Day is celebrated on the 14th of this month. Students were encouraged to investigate the origins and use of Pi and solve a number of puzzles related to this very important number. During the April and May sessions the students will be exploring the intriguing world of *Pentominoes* and *Magic Squares*.

Students are showing a lot of enthusiasm during the Mathemagics sessions and they participate actively. They are encouraged to work in groups as this gives them the opportunity to share mathematical ideas, to engage in discussion with others and solve problems through collaboration.

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For further Mathematics Teaching Resources and Ideas please visit the Mathematics (Secondary) Room on Fronter.

Contributions to *MATHSLINE* are welcome. These **should not exceed 750 words** and preferably be presented in Rich Text Format (RTF) using the font **Times New Roman**.

Editorial

In this edition's last article, the reader is left with a doubt whether Fermat had in fact proved his theorem. This doubt is enhanced when Andrew Wiles, intrigued by this theorem, had to rely on aspects of mathematics which was done by others much later than Fermat's period. What Wiles achieved and contributed to mathematics was the result of collaboration between mathematicians coming from different countries in different eras. It seems that what different people can achieve collaboratively is by far superior to what the same people can individually achieve altogether.

The idea that teachers work disconnectedly from their fellow colleagues, without the need of sharing ideas, is becoming less popular with local teachers. Perhaps teachers are realising that the more you give to colleagues, the more you get – the more you share, the more other teachers are willing to share with you. In the sharing process teachers evolve; they become more knowledgeable and most importantly they get excited to try out new ideas with their students. The *Tikka Matematika* conference can be seen as a sharing celebration and is surely an occasion which sparks off in teachers that motivation to explore new ventures as mathematics educators. Very often, these initiatives do not require teachers to work individually but require a team mentality, a collective effort which gets teachers to collaborate to achieve the desired goal of the initiative. This team mentality is also seen in the MaSDiV project where teachers, together with their CPD leaders, learn together, discuss new ideas and share their experiences in implementing them. At the same time they are backed with continuous support from the University MaSDiV team.

Such collaboration is most welcome as teachers can learn a lot by interacting with other colleagues - not only those who teach the same subject area, but also those who teach other subject areas. Teachers who would like to benefit from such interactions should not wait for others to make the first step but should take the initiative. They can try out something small at first involving a few colleagues. They can discuss initiatives with colleagues who have tried them out in other schools. It is hoped that teachers will see the benefit that they can get from collaboration and this in turn will spark off more initiatives, healthier relationships and more positive attitudes.

The Editor

UPCOMING EVENTS

MaSDiV
Supporting mathematics and science teachers in addressing diversity and promoting fundamental values

January to July 2018
An Erasmus+ project aimed at supporting teachers to connect science and mathematics teaching to the learning of fundamental values in diverse and multicultural classrooms.
July 2018 - A celebration event to share the good practices carried out by teachers and CPD leaders participating in MaSDiV.

Learning Outcomes Framework

May 2018
Training workshops focusing on issues related to the implementation of the Learning Outcomes Framework and related assessment practices.

Mathematics The way Forward

9th - 10th July 2018
Inset sessions for mathematics teachers of Years 7 & 8.
Venue: St Nicholas College Secondary School Dingli.

tikka *matematika*

The two-day conference, aiming at disseminating examples of sound pedagogical practices in teaching mathematics, has now gone into its third year. **tikka** ***matematika*** *this year was held on the 20th and the 21st of February at St Benedict College Kirkop Secondary School where Pimary school teachers and/or administration and Secondary mathematics teachers had the opportunity to have a better glimpse of the good practices taking place in both sectors. Around 150 educators from the primary, secondary and tertiary sector have attended the* **tikka** ***matematika*** **2018** *seminar.*
This ***MATHSLINE*** *issue features a summary of some of the presentations which were shared during the conference.*

DR COLIN FOSTER

The keynote speaker for the **tikka** ***matematika*** **2018** was Dr Colin Foster. He is an Associate Professor of Education in the School of Education at the University of Leicester, England.

His research interests in mathematics education focus on the learning and teaching of mathematical problem solving and the design of rich tasks and their use in the mathematics classroom. Dr Foster has designed and published rich mathematical tasks and ways in which teachers can use and adapt them in the classroom to support students' conceptual understanding of mathematics. During the seminar Dr Foster has shared examples of these tasks and lead two workshops, one for the primary educators and one for the secondary educators. Dr Foster has been designing mathematical tasks and writing books and articles for mathematics teachers for over 15 years.

Beside his research, Dr Foster teaches pre-service teachers on the PGCE secondary mathematics course at the University of Leicester, supervise masters and PhD students and lead professional development for mathematics teachers. He also writes regularly for the magazine Teach Secondary and for mathematics teacher professional journals. Dr Foster was a member of the Department for Education Expert Group for the mathematics national curriculum and devised questions for University of Cambridge International Examinations. ⁜

(Retrieved and adapted from http://www.foster77.co.uk on 4th April 2018)

LESSON STUDY

Ian Buttigieg & Stephan Azzopardi

Lesson study is an initiative that the teachers at St Clare College Pembroke Secondary School took on last year. It involves in-depth planning and conducting a lesson which tackles concepts that students find difficult to understand and at the same time, hard for teachers to teach. This professional development process sees teachers as the most valuable resource in the classroom, as experts on students' learning and as learners of their own teaching.

The team involved was committed to prepare a lesson plan which would help them see links between different mathematical topics such as sequences, number machines, simultaneous equations, formulating and evaluating formulae, plotting table of values and drawing straight line graphs. A formal meeting was held once a week to discuss and share ideas in order to complete the lesson plan. Preparation continued also afterschool hours with the use of social media for communication so that we could make minor amendments to the lesson plan. When this was drafted, it was delivered to a Form 3 group, during which all participating teachers observed and evaluated the lesson.

This helped us make some adjustments to the lesson plan and then delivered the final lesson to another Form 3 group of the same level. A report on the lesson study was made. It includes the lesson itself, reflections of all participating teachers and some students' comments.

Overall, we were very satisfied with the outcome and we are currently planning our next lesson study. Besides other advantages, the whole experience has helped us grow stronger as a team. We believe that our students benefitted not only from a quality lesson but also from being led by example of seeing adults working as a team as such an opportunity is rarely seen.

We are also very pleased to have developed this lesson plan which can be adapted and used with other students of different performing levels. ⁜

MATHS TRAIL DESIGN

Grace Bonnici & Vanessa Tanti Rigos

A maths trail is a good opportunity for both teachers and students to do mathematics outside the classroom. This would make it possible to link mathematics to objects and places in everyday life and at the same time facilitate meaningful mathematical discussion while students work collaboratively on tasks and activities.

From past experiences of organising maths trails with students of different age groups, students say that such activities bring the element of fun into mathematics. On the teachers' part it is also an alternative way of assessment and revision of the mathematical concepts taught in class. A maths trail can also be linked to other subjects such as History and Geography when it is carried out in a historical site or Science when it is located in the natural environment.

These maths trails can be organised by the teachers themselves for the classes they teach. If time limits a maths trail outside school, it can also be prepared in the school building and playground. This would also be an excellent activity as part of an open day at school or mathematics fun day. Teachers can be creative regarding the topics and strands they could include in their trail, while keeping in mind certain logistics that are important for the activity to run smoothly.

We hope that more teachers can extend the mathematics they teach in class to situations outside the classroom, showing students that mathematics can be useful, relevant and enjoyable in everyday life.

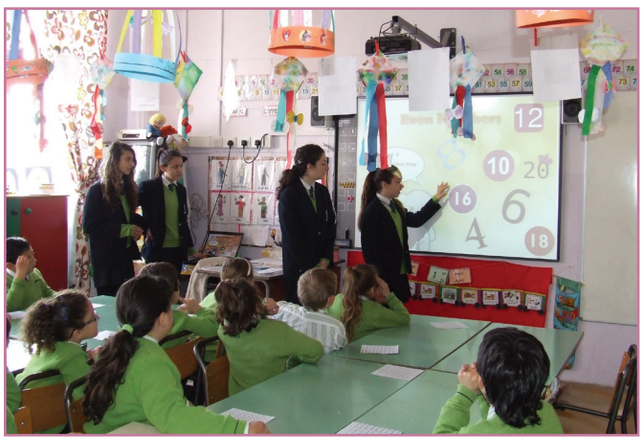
INTRA-COLLEGE EXPERIENCE

Joanne Cilia

The Maths department within Maria Regina College has been carrying out this activity for ten years. It's an experience offered to our secondary students where every year they go to the college primary school classes and be Maths teachers for an hour.

The idea came about when I was at home observing my son, who at that time was attending Year 3, while doing his homework. I noticed that some of the Maths topics he was learning were similar to some of the topics I was teaching to my Form 3 class. Therefore, I saw the possibility to give a chance to my students to act as teachers in the Primary classroom.

I contacted the Head teachers of both the Secondary and the Primary schools and they agreed that it was a good idea. The Head of the Primary school consulted all the Year 3 teachers and they suggested a topic which could be taught by our secondary school students. My class was split into six groups of four, each group doing the same lesson to six different classes.



We met during breaks to plan the lesson, holding brain-storming sessions to help students come up with real-life situations related to our topic. When we started this activity, the internet was not available, so we planned more hands-on games. Along the years, we have been changing the topics according to the primary school curriculum and updating our approach as the interactive white boards became available. Sometimes we also changed the year group with whom we did the lesson.

During the preparation, students became aware of the work involved in teaching and increased their appreciation towards our work. Some 'teaching tips' were given: a catchy introduction, motivating activities and a good conclusion summarising the whole lesson.

All in all, it is a positive vertical networking between the schools in our college. Both primary and secondary students enjoy and learn from this activity as it offers an opportunity for discussion. Secondary school students show a positive feeling towards teaching and gain self-confidence while conducting the lesson. Also the concepts are revised and consolidated. We are now looking forward to our next experience where students from both the secondary schools of our college will be conducting lessons in Mosta Primaries A and B.

NON-DIGITAL GAMES

Ann Marie Micallef

Some common phrases heard by students is that they hate or fear mathematics. Can something be done to overcome this fear? What if we teachers find something students love doing and combine it with mathematics?

For many years, our school has been preparing students for the SuperTmatik competition - a competition aimed at students capable of working out quick mental computations. What about the other students? Could they participate? Students from all levels of achievement tried this game out, yet most of them gave up after a few tries. It was then that the math teachers in our school decided to ask our students what they loved doing most, and gaming was the favourite of all.

We decided to set up a Maths Games Club – with a difference. We bought a number of board games, most of them chosen by the students themselves, with the aim of exploring mathematical aspects through their use. One of the favourite games is Jenga. This game requires a lot of patience but, surprisingly enough, those who usually hate sitting down, do so for several minutes as it keeps them engaged in not toppling their constructed tower.

Another favourite is Monopoly. Some students used to play it before while to others it was completely new - either because of today's heavy reliance on digital games or because they could not afford it. This particular game is suitable for all levels. Mathematical aspects tackled during the game usually involve money management; mental addition and subtraction for bills and change; and probability.

The game UNO is assigned to students who have difficulties with sequences. Some find it really difficult to choose which card to use, not knowing that they could use either a card of the same colour or one with the same number.

Everyone remembers the traditional Snakes and Ladders. I myself used to love playing it with relatives on a cold winter day. While playing this game, students consolidate counting. A more modern version is the 'Takes and Adders'. This one does not have a dice but has cards instead. Students pick 3 cards, two containing numbers and one containing an operation (+ or -). A positive result implies a move forward while a negative one, a move backwards. In this version, more mathematical aspects are employed making it ideal for secondary students.

Other games used are: Bingo (using addition, subtraction, multiplication, or division to get the number); the Dart board (addition); and the Spin and Fill (using shape recognition). All these and more are now played on a weekly basis during the break and in particular lessons, allowing students from all levels to participate. ⁜

INQUIRY-BASED LEARNING

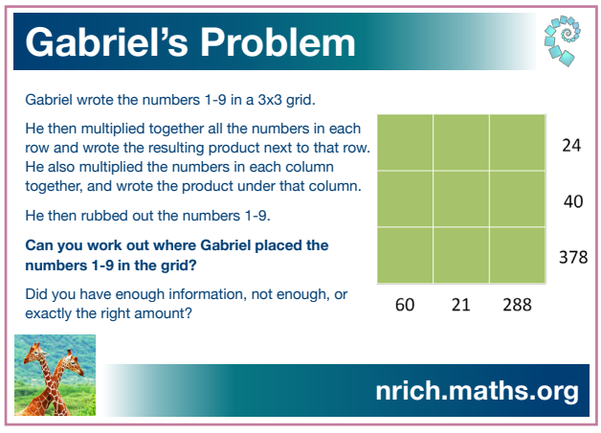
Claude Lautier

One may have certain concerns at attempting different teaching methods in the classroom. Teachers would wish to try new learning approaches but at the same time may feel anxious or discouraged about various things.

It is not the first time I ask questions like: Do learner-centred lessons work? Would such lessons be enough for students to acquire the skills I have in mind? How is it possible to cover the syllabus content by involving the students more? Will they learn from lessons of this kind, or would it simply be a game for them?

Inquiry-based Learning (IBL) is a dynamic learning process involving the teachers working alongside the students. IBL is a learner-centred pedagogy which values students' curiosity. Although there is no standard way of carrying out an IBL lesson, I adopted a particular approach, as explained hereunder.

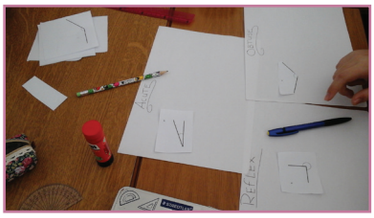
- *Introduction*
  Lesson is introduced through a short task, which would usually comprise one or two minutes thinking time, on an individual basis. This is followed by a brainstorming activity during which learners share their thoughts on mathematical knowledge and meanings they recall from previous lessons.



- *Explanation of Task*
  This stage involves the explanation of the lesson's main task, describing the situation in the task posed to students. I try to avoid giving hints on how the problem is to be solved. I prefer allowing students to use various methods which are then compared and contrasted during the plenary phase. From the information gathered in the introductory phase, many students would already know which topic/s or

skills need to be applied in order to solve the main task. Students are assigned to a particular group: sometimes I choose the groups myself but most of the times I let them choose whom to work with. The size of the groups varies, depending on the task but usually groups are composed of three to four students.

- *Plenary*
  In the plenary, groups are asked to come to the IWB and explain the different approaches used to tackle the situation posed. The other groups are free to ask questions and discuss constructively.



I feel that IBL lessons can be a fun and fruitful learning experience for all learners. Although a teacher may not adopt a design as outlined above, one can still include aspects of IBL in a traditional lesson by employing purposeful questioning, student discussion and presentations. Also, asking questions which are open-ended will create room for thinking and investigation. This way, a lesson becomes a more enriching learning experience both for learners and teachers alike. ⌗

MATHS FAIR

Analise Camilleri

Mirror mirror on the wall, what is Mathematics for?

This was the central question posed at the Maths Fair in St Theresa College Secondary School. What Sports Day is to P.E. the Maths Fair is to Mathematics. It was a festive day where students worked together to promote a better attitude and to share their appreciation and joy towards mathematics.

A class of year 9 students, together with their teacher, spent days preparing material like charts, games and activities to be eventually set up at the school foyer. A student constructed a Mathematics game based on the popular computer game "Minecraft" which was available for students to play with. Another student baked healthy treats in the form of different shapes and students had to find their area and perimeter.

The school foyer was packed with posters displaying proofs, explanations and history related to mathematics. Other posters exhibited material related to events such as Pi Day; others showed the use of recycled bottle caps to form words, shapes and equations.

Another exhibit featured a 'mirror' asking students when they use Mathematics in their daily lives. Students went all over the school taking photos to show that Mathematics is in fact everywhere around us.

The foyer was loaded with puzzles, logic games, and brain teasers. These included Sudoku challenges, magic squares, match stick puzzles, elastic cubes and tangrams. Students were provided with sticks and clay with which they had to form 3D shapes. Others were provided with circular objects as well as a piece of string with which to measure the circumference and diameter and they were also provided with instructions on how to find a value for pi.

Outdoors, the tasks involved physical activities. In one of them, students had to measure the speed of running students while other tasks consisted of water games involving volume.

Events such as this encourage students to collaborate together and show the enjoyable aspect of Mathematics to others. They see for themselves the application of mathematical concepts used in class; learn to be innovative and come up with their own solutions to mathematical problems. They also provide an opportunity for teachers to expose students to information which time usually does not allow in class - the lives of mathematicians, proofs and history of mathematics are just a few. Perhaps one day, Maths Fairs will be widespread across all schools and become as popular as Sports Days. ⌗⌗

MaSDiV

Supporting Mathematics and Science Teachers in addressing Diversity and promoting fundamental Values

James Calleja

MaSDiV is an ERASMUS+ project aimed at supporting teachers to connect science and mathematics teaching to the learning of fundamental values in diverse and multicultural classrooms. Spread over three years (2017-2019), this project involves the development of a research-based teacher CPD course to promote inclusive STEM education. This course is developed according to the latest research and standards related to CPD. Moreover, it is based on the well-researched STEM concept of IBL and it will be implemented across Europe.

The MaSDiV course features three modules presenting inquiry-based learning (IBL):

- as an approach for addressing achievement-related diversity
- in real-life, relevant contexts
- as a tool for intercultural learning.

This CPD course, provided by the University of Malta in collaboration with MEDE, offers two continuing professional development (CPD) opportunities – one for CPD leaders and another for secondary school teachers. MaSDiV has attracted 28 CPD leaders and 108 teachers from 32 different schools (from both state and non-state sectors).

For CPD leaders, the course started in January 2018 and is aimed to assist them in learning about inquiry-based learning, diversity and multicultural classrooms and in leading a team. It is also aimed at introducing the CPD materials provided by the MaSDiV project and engagement in learning how to implement each of its three modules with mathematics and science teachers.

During the period March to June 2018, CPD leaders will be supporting and facilitating the learning of teachers through school-based CPD. During these weekly sessions, CPD leaders will assist teachers and provide them with opportunities to engage in collaborative and reflective inquiry focused on the implementation of the MaSDiV modules. In planning for lessons, the use of IBL strategies to address diversity will be a key aspect of the school-based discussions.

A celebration event is scheduled to take place at the end of June 2018 to disseminate the good practices carried out by teams of teachers together with their respective CPD leaders.

Raising the Bar

Shaping Students' Science Career Aspirations for STEM teachers

Michel Spagnol

What careers do students aspire to? How can we encourage more students to undertake STEM related careers? These were some of the questions that were considered during the training seminar '*Raising the Bar: Shaping Students' Science & Career Aspirations for STEM teachers*' organized on Friday 9th February by the Directorate for Learning and Assessment Programmes.

This CPD, held at St. Nicholas College Secondary School Dingli, aimed to explore and discuss the factors that shape young students' participation and engagement with Science and Mathematics.

Professor Louise Archer, director at the Centre for Research in Education in Science, Technology, Engineering & Mathematics (CRESTEM) at the University College London, Institute of Education, coordinated this seminar and presented several strategies how STEM career awareness could be embedded during STEM lessons.

During the seminar, Professor Archer argued that STEM teachers and schools should be aware of the students' science capital in order to develop a holistic approach that inspires more individuals to consider a STEM related career.

Professor Archer also identified a number of approaches how schools can create the right ambience where students can enrich their science capital. These themes and other ideas, where ultimately discussed through Prof Archer's research and personal work experiences in the field of STEM education.

The seminar was well attended and STEM educators coming from the Primary, Secondary and Post-Secondary sectors managed to benefit from this CPD opportunity.

This training seminar was partly financed through funding from the European H2020 research and innovation programme – project Scientix 3 coordinated by European Schoolnet (EUN).

Beyond Pythagoras: Fermat's Last Theorem

Joseph Mamo

As all mathematics teachers know, formal assessment in mathematics often consists of a list of problems that students are expected to solve individually and in a limited amount of time. Indeed, one often assumes that the gifted child is the one who can solve problems in the shortest possible time. Sadly, this may differ from how real mathematics is done by mathematicians. Not only do they often work collaboratively with other mathematicians, but they frequently take a long time, indeed in some cases a considerably long time, to arrive at a proof to a theorem.

Probably the classic example of this is the theorem that has come to be known as *Fermat's Last Theorem*. We are all familiar with the theorem attributed to Pythagoras stating that the area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides. In short, if the three sides of a right-angled triangle are a cm, b cm and c cm long, the first being the hypotenuse, then

$$a^2 = b^2 + c^2$$

Of course, numerous proofs of the famous theorem had appeared over time, but no one had dared to push the famous theorem into the unchartered terrain that Pierre de Fermat ventured into. Fermat was not a professional mathematician but his main passion was mathematics. He conjectured that the equation

$$a^n = b^n + c^n$$

has no solution in the positive integers for $n > 2$. This means, for example, that there are no pairs of cubes whose sum is a cubic number as well. What I find so intriguing about the theorem is the fact that Fermat had ventured from the world of geometry into the world of pure number theory.

Pierre de Fermat (1607-1665)

Although Fermat stated that he had proved his conjecture, up to this day there is no evidence of this. What we have is just a note in one of the books that was found in his library after his death, stating the conjecture and that he had an elegant proof for it. And very little else!

Various attempts have been made over the years to prove the theorem, but with very little success. Indeed, the theorem has been proved for specific values of n. In 1770 Euler managed to prove the theorem for $n = 3$ and in 1885 Dirichlet and Legendre independently proved it for $n = 5$. However, no one had proved that the theorem is true for all the integers greater than 2. No one, that is, until a certain professor of mathematics delivered three of the most important lectures of mathematics on the 23rd June 1993. His name is Andrew Wiles - born in Cambridge in 1953 but had taken up a professorship at Princeton University in 1981.

Andrew Wiles (1953 -)

According to legend Professor Wiles had been intrigued, if not obsessed, by the theorem ever since, at the age of ten, when he came across a book by E.T. Bell entitled *The Last Problem* (1961). Now, sometimes we are given the impression that geniuses can just produce masterpieces by the simple stroke of a pen. "Talent does what it can, genius does what it must" stated Edward Bulwer-Lytton (1803 - 1873). But reality is not that simple, at least not in Wiles's case. It took him the best of seven, or rather eight years if one includes that extra year Wiles dedicated to amend a flaw in the initial proof, to finally scale the lofty heights of Fermat's last theorem. To prove the theorem, Wiles, not only had to make use of his knowledge of elliptic equations, but he had to familiarize himself with other aspects of incredibly difficult mathematics.

Perhaps one should ask whether Andrew Wiles would have arrived at his holy grail had it not been for the achievement of other mathematicians. Thus, fundamental to Wiles's proof is what has become known as the Taniyama-Shimura conjecture. Attributed to two Japanese mathematicians, Yutaka Taniyama and Goro Shimura, the conjecture proposed an association between elliptic equations - Wiles's area of expertise - and modular forms. What is perhaps equally important is Ken Ribet's assertion that anyone who could prove the Taniyama-Shimura conjecture would be proving Fermat's last theorem.

This, of course, is not to diminish the genius of Andrew Wiles. As Simon Singh (1997) points out in his highly recommendable book, "he had brought together virtually all the breakthroughs in twentieth-century number theory and incorporated them in one almighty proof. He had created completely new mathematical techniques and combined them with traditional ones in ways that had never been considered possible. In doing so he had opened up new lines of attack on a whole host of other problems." (p.304)

A mystery that surrounds the theorem is whether Fermat had a proof. This will probably never be known until someone unearths some document that demonstrates that he in fact had.

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